M. Denhard, A. Rhodin, J. T. Ambadan, H. Anlauf, A. Fernandez del Rio, A. Cress, G. Zängl, H. Frank, T. Hanisch, C. Primo, F. Fundel, M. Buchhold, R. Potthast

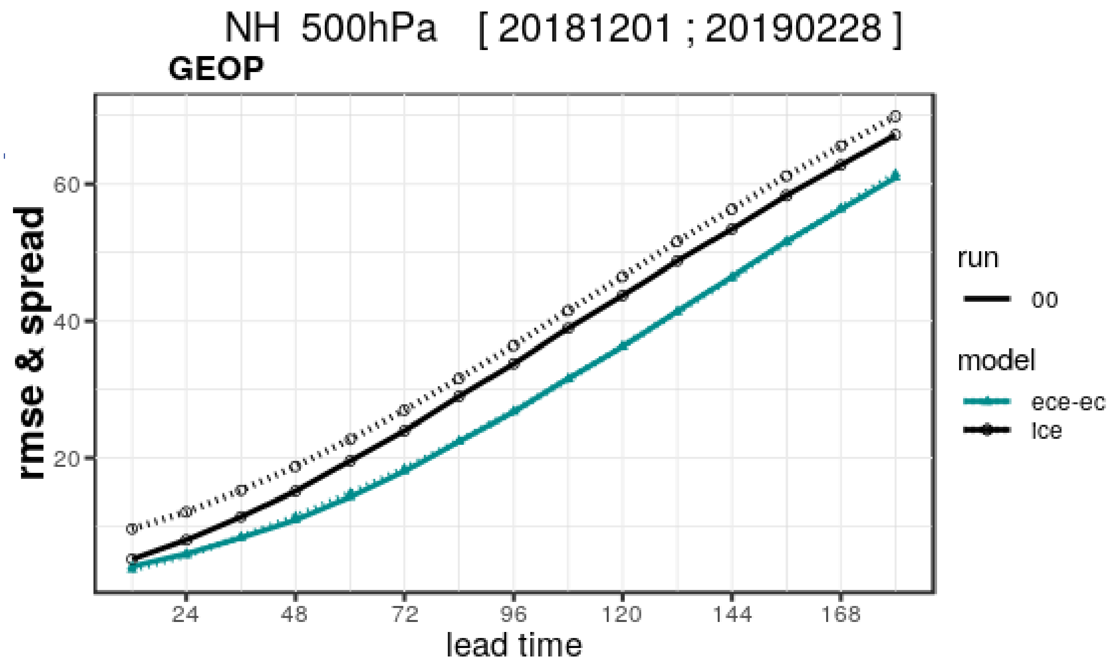
The ICON-EPS

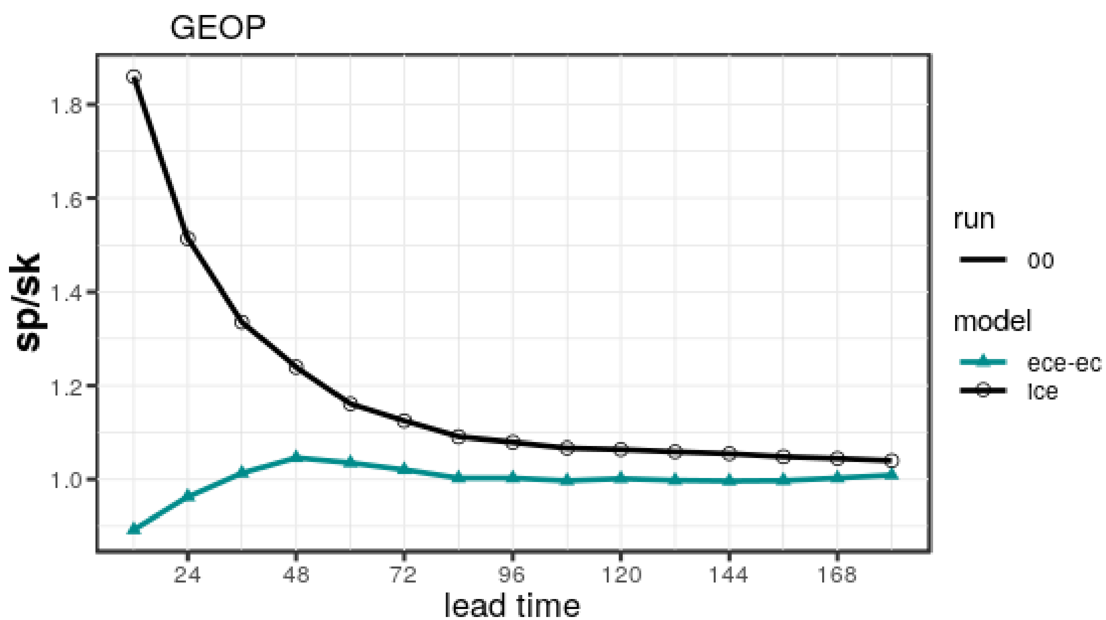
operational suite (since 18th January 2018)

- 40 Member
- Global, 40 km / ICON-EU Nest, 20 km
- 00/12 UTC → +180h / 06/18UTC → +120h
- 03/09/15/21 UTC → +30h Boundary Conditions for COSMO-D2-EPS
- Perturbing physics tuning parameters fixed during the forecast
- Initial perturbations by global EDA (LETKF)

www.dwd.de
-> ICON database reference manual

Deutscher Wetterdienst
Wetter und Klima aus einer Hand





Verification against analysis

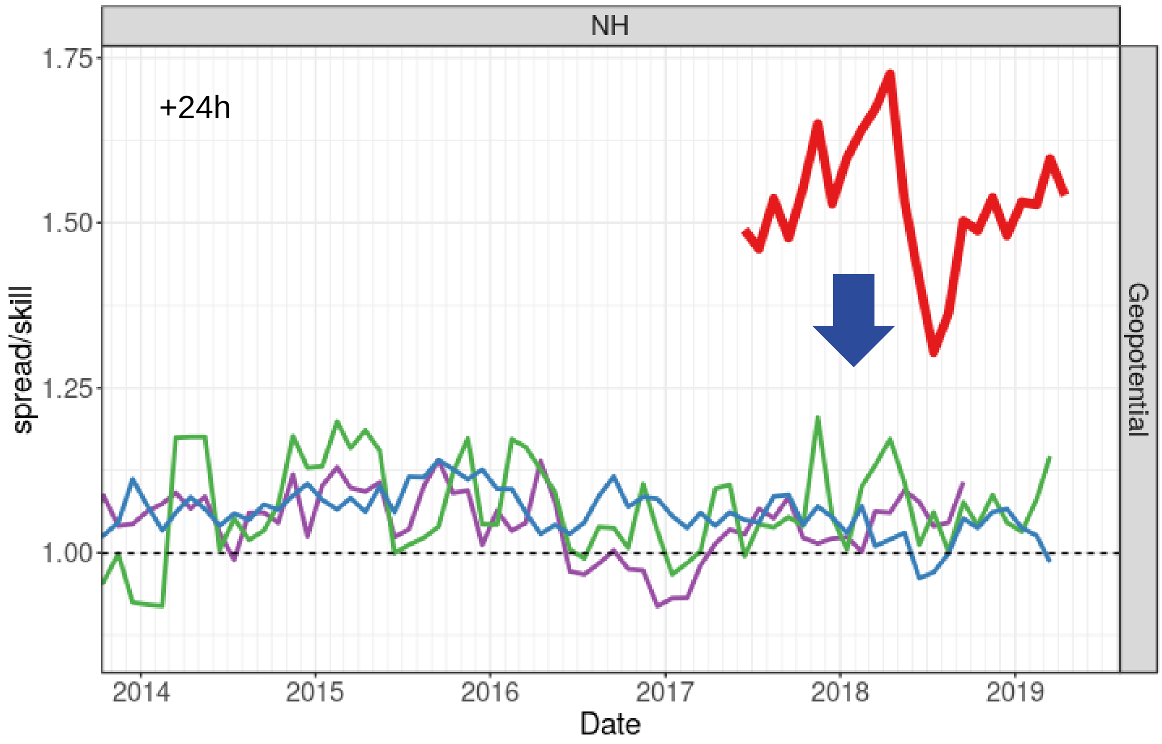
- ECMWF-EPS
- ECMWF-EPS (spread)
- ICON-EPS vs LETKF
- ICON-EPS vs LETKF (spread)

- **Spread / Skill => 1**

ICON-EPS

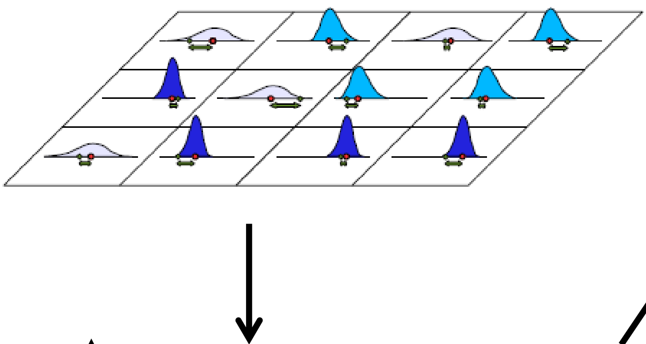
WMO EPS verification against analysis
lead-time: 24h
level: 500hPa

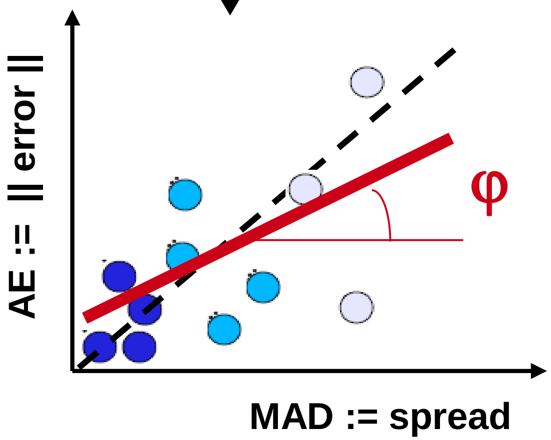
Z500

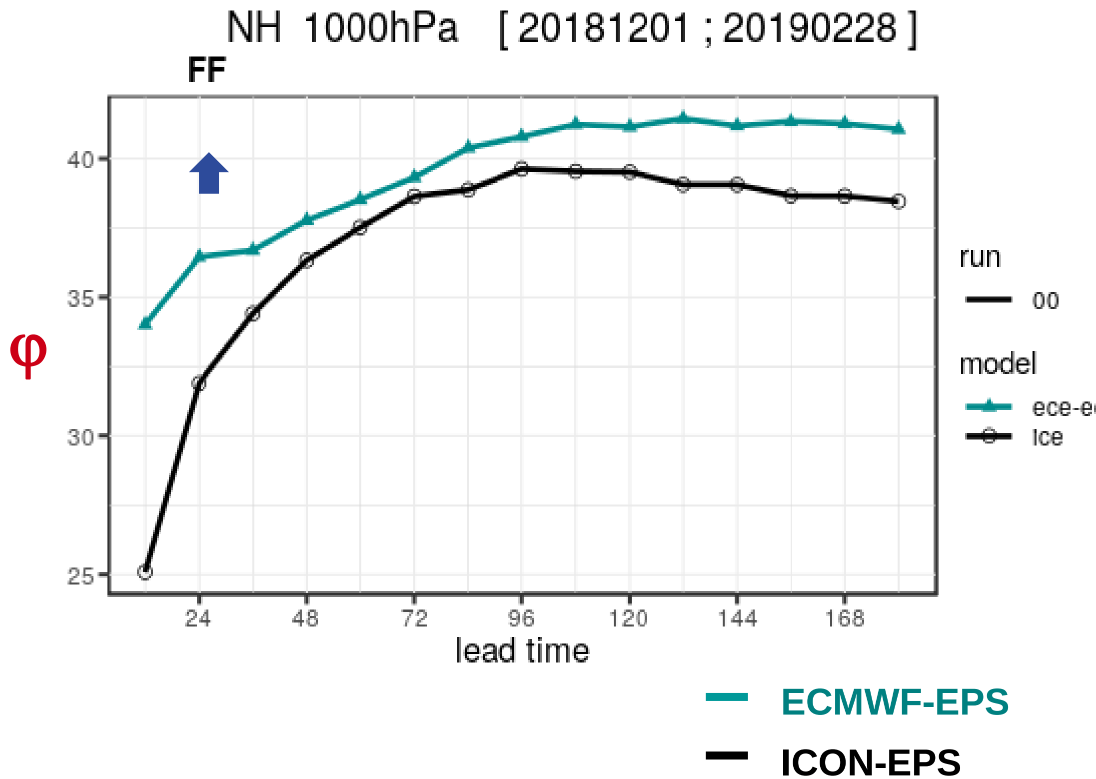


Resolution of the Spread

Leutbecher, M., 2009:
Diagnosis of Ensemble Forecasting Systems, ECMWF



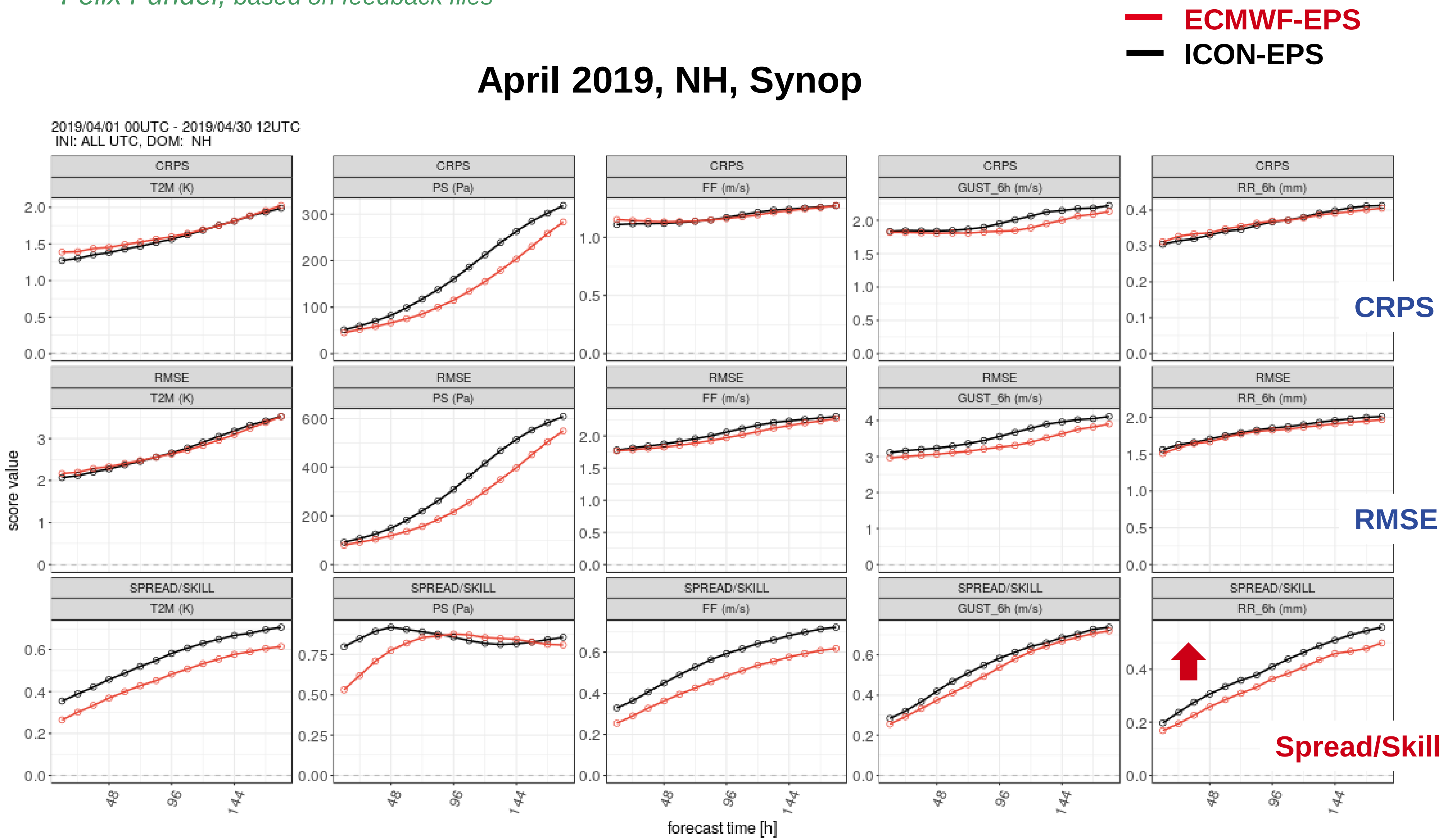




Linear Regression Model
Barker (1991)
Scherrer et al. (2004)
Grimit and Mass (2007)
Eckel et al. (2012)
van Schaevbroeck and Vannitsem (2016)

Felix Fundel, based on feedback files

April 2019, NH, Synop



Conclusion

- **Analysis verification:**
  - ICON-EPS is **over**-dispersive (spread/skill => 1)
  - the spread has almost no resolution

- **Verification with observations:**
  - ICON-EPS is **under**-dispersive (spread/skill => 1)
  - some more spread than ECMWF-EPS

Initial perturbations in the global ICON-EPS do not provide useful estimates of the short range forecast error

Differences between ICON-EPS and ECMWF-EPS

- **Forecast in the ICON-EPS are not centered on the best analysis (EnVar)**

- **Ensemble Data Assimilation (EDA)**

  Covariance Inflation in LETKF at DWD

  - multiplicative factor (0.9 to 1.5)
  - additive Inflation (+0,25Bclim)
  - relaxation to the prior“ (0.75) Zhang et al. (2004)
  - SST random perturbations 1°K, correlations of 100km/1000km and 1 day

  Randomly pertubed observations in EDA at ECMWF (4Dvar)

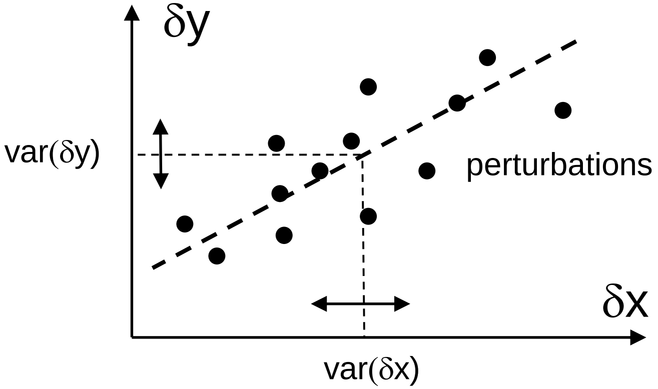
- **Singular Vector initial perturbations**

Leutbecher and Lang (2014) QJRMS:

„The reason d‘etre for singular vector perturbations would vanish, if the EDA, together with the representation of model uncertainties, generated enough variance in the space spanned by the leading singular vectors“

Can this happen, if perturbations in EDA and SPPT are of random nature and the ensemble size is limited to 50?

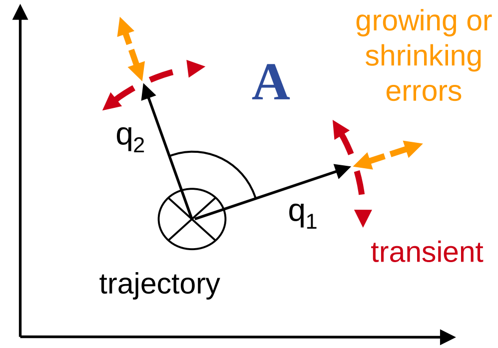
- LETKF optimizes the relation between **forecast -** and **observation error variances**, such that the long range forecast error of the ensemble mean is a minimum.
- LETKF cycles do not require growing perturbations, but it is important to keep a certain level of variance in the analysis ensemble. => **variance inflation**
- Hamill and Whitaker (2011): “What Constrains Spread Growth in Forecasts Initialized from Ensemble Kalman Filters?”
- Background error variances and co-variances do not relate to a specific set of perturbations



EDA schemes lack mechanisms for systematically exploring the error growth potential of the analysis ensemble perturbations.

Singular Vector (SV) perturbations

- **Orthonormal system of perturbation vectors** pinned to a state **x** in the state space of the forecast model which describes the expansions and contractions in the neighbourhood of **x** such that the **1st SV is in line with the fastest growing direction**.
- **Model Property** only (no observations involved)
- **Expensive !**



Krylov Methods for Adjoint-free Singular Vector based perturbations

J. Winkler, M. Denhard, B. Schmitt, 2020: Q J R Meteorol Soc. 2020;146:225–239.

The classical way to estimate SV:

Lanczos-SV

Approximate SV …

… by first order Taylor approximation of perturbation dynamics

$$\mathbf{q}(t) = \mathbf{L}(t{:}0)\ \mathbf{q}(0)\,,$$

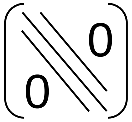
Growth rate of perturbations:

$$\frac{\|\mathbf{q}(t)\|_2}{\|\mathbf{q}(0)\|_2} = \frac{(\mathbf{L}\,\mathbf{q}(0))^{*}\ \mathbf{L}\,\mathbf{q}(0)}{\mathbf{q}^{*}(0)\ \mathbf{q}(0)}$$

estimate Eigenvalues and Eigenvectors of the **symmetric** matrix

$$\underbrace{\mathbf{A}}_{n\,x\,n} = \underbrace{\mathbf{L}^{*}}_{\text{adjoint}}\ \underbrace{\mathbf{L}}_{\text{linear}}$$

… by estimation of **A** using a Krylov subspace of dimension m<<n with the Lanczos algorithm



m x m

The Krylov subspace $\mathbf{K^m}$ based on a $n \times n$ matrix $\mathbf{A}$ which acts on a vector $\mathbf{q}$ is defined by

$$K^m(\mathbf{A}, \mathbf{q}) = \mathrm{span}\{\, \mathbf{q}, \mathbf{Aq}, \ldots, \mathbf{A^{m-1}q} \,\} \qquad m \ll n$$

Orthogonalisation of the Krylov vectors leads to $\mathbf{Q} = (q_1, q_2, \ldots, q_m)$, which is an orthonormal basis of $K^m(\mathbf{A}, \mathbf{q})$. Projection of $\mathbf{A}$ on the Krylov subspace gives

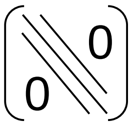
$$\mathbf{Q^*\, A\, Q} = \mathbf{H}$$

Lanczos

$\mathbf{A}$ *hermitian* => $\mathbf{A^*} = \mathbf{A}$

$$\underbrace{\mathbf{A}}_{n\,x\,n}\ \mathbf{Q} = \mathbf{Q}\ \underbrace{\mathbf{D}}_{m\,x\,m} + \varepsilon$$

$\mathbf{D}$ = *tridiagonal matrix*



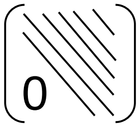
m x m

Arnoldi

(new!)

$$\underbrace{\mathbf{A}}_{n\,x\,n}\ \mathbf{Q} = \mathbf{Q}\ \underbrace{\mathbf{H}}_{m\,x\,m} + \varepsilon$$

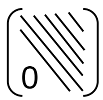
$\mathbf{H}$ = *hessenberg matrix*



m x m

Arnoldi-SV Approximate SV …

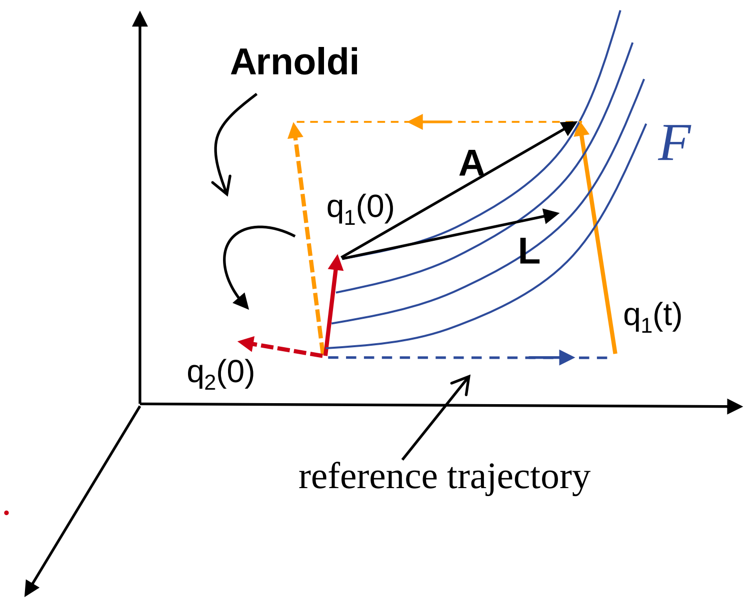
(see QJRMS 2020;146:225–239)



… by a Krylov Subspace of dimension m<<n,

$$\mathbf{A}\,\mathbf{q}_j \sim F(\,\mathbf{x} + \mathbf{q}_j\,) - F(\,\mathbf{x}\,)$$

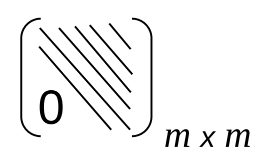
… by non-linear predictions from the orthonormal basis vectors $\{q_j\}$ of **Q**,



… by avoiding the adjoint integration (not solving an „equivalent eigenvalue problem") …

$$[U;\, S;\, \mathbf{V}\,] \leftarrow \text{svd}\,(\mathbf{H})$$

… what requires the calculation of the Singular Vectors of **H** and not its Eigenvectors.



$\mathbf{q}_{j=1}$ chose initial perturbation vector (=> $K^1(\mathbf{A}, \mathbf{q})$)

for $j = 1 : m$ **do**

Prediction step

↓

Gram-Schmidt orthogonalisation

↓

Construction of subspace model

extend Krylov space (=> $K^{j+1}(\mathbf{A}, \mathbf{q})$)

↓

final Perturbations **P**

Lanczos

$$\mathbf{w} \leftarrow \mathbf{L}^T \mathbf{L}\, \mathbf{q}_j$$

$$\alpha_j \leftarrow \mathbf{q}_j^T \mathbf{w}$$

$$\mathbf{w} \leftarrow \mathbf{w} - \mathbf{q}_j \alpha_j - \mathbf{q}_{j-1} \beta_{j-1}$$

$$\beta_j = |\mathbf{w}|$$

$$\mathbf{q}_{j+1} = \frac{\mathbf{w}}{\beta_j}$$

$\sim O(m \times n)$

Arnoldi

$$\mathbf{w} \leftarrow F(\mathbf{x} + \mathbf{q}_j(0)) - F(\mathbf{x})$$

for $i = 1 : j$ do

$$h_{i,j} \leftarrow \mathbf{q}_i^T \mathbf{w}$$

$$\mathbf{w} \leftarrow \mathbf{w} - \mathbf{q}_i h_{i,j}$$

end for

$$h_{j+1,j} = |\mathbf{w}|$$

$$\mathbf{q}_{j+1} = \frac{\mathbf{w}}{h_{j+1,j}}$$

$\sim O(m^2 \times n)$

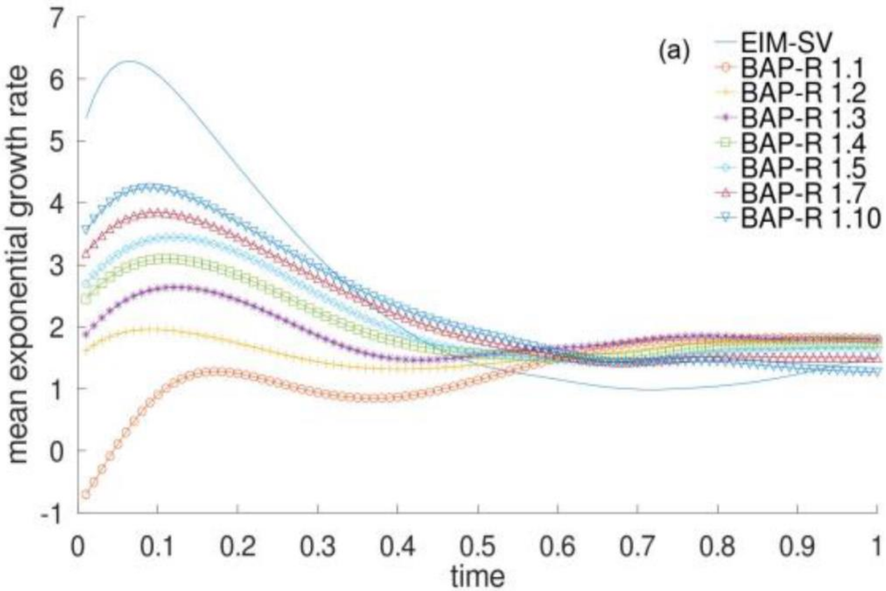
end for loop

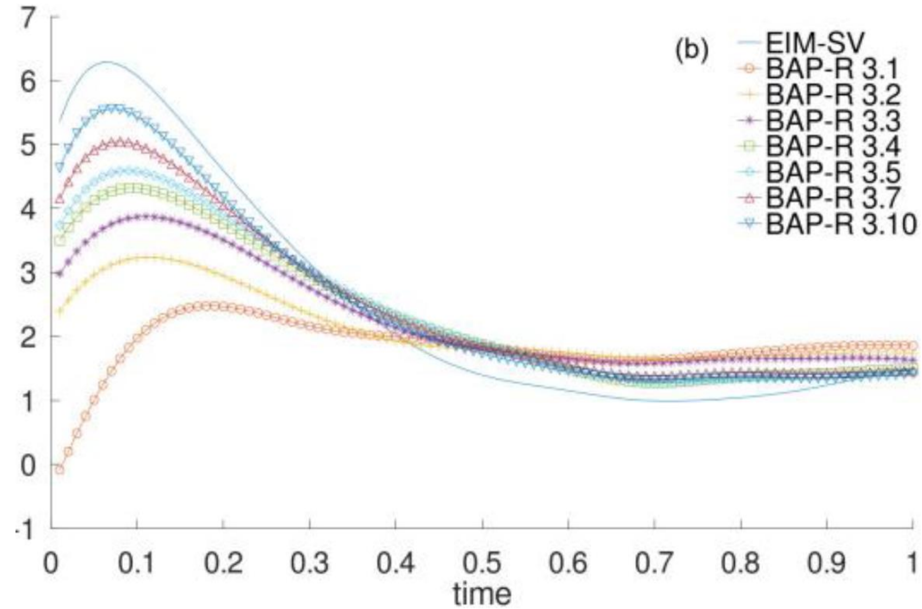
$[\lambda, \mathbf{V}] = \mathrm{eig}(\mathbf{D})$ $[U; S; \mathbf{V}] \leftarrow \mathrm{svd}(\mathbf{H}_{m,m})$

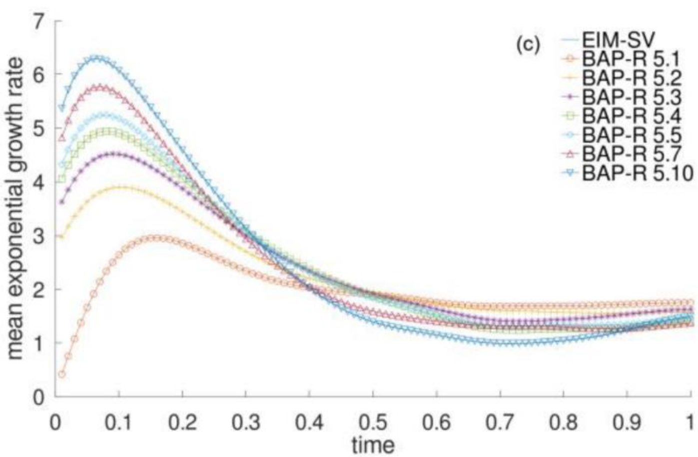
$$\mathbf{P} = \mathbf{Q}_{n,m} \mathbf{V}_{m,m}$$

$$\frac{dx_i}{dt} = -x_{i-2}x_{i-1} + x_{i-1}x_{i+1} - x_i + F$$

Lorenz96 with F = 8 and N=50







Mean exponential one step growth rates

EIM-SV = Evolved Increment Matrix singular vectors

BAP = Block Arnoldi Perturbations

BAP-R = random initial perturbation vector

BAP-R i.j = i initial vectors and j iteration loops

(a) BAP-R i = 1
(b) BAP-R i = 3
(c) BAP-R i = 5

Conclusions & Outlook

➢ Experiments at ECMWF

- implement the Arnoldi algorithm in the IFS
- run 3 experiments

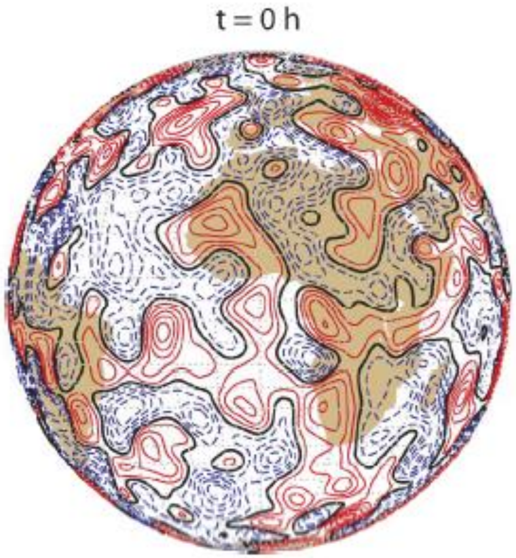
  1. **Reference with classical Lanczos SV**
  2. **Arnoldi SV using the linear version of the IFS**
  3. **Arnoldi SV using the full non-linear IFS model**

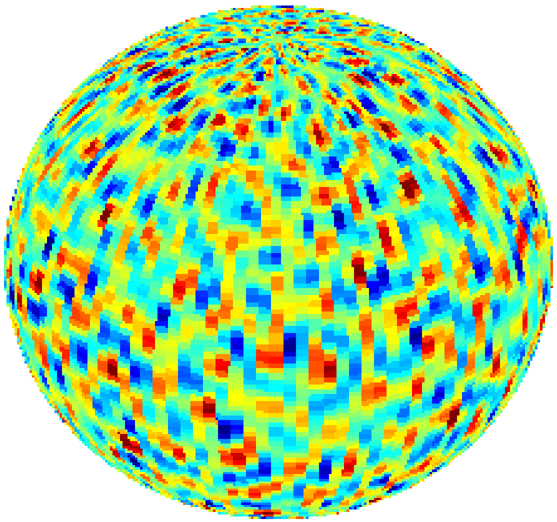
➢ Arnoldi SV for the short range in the ICON-EPS

- Strong localisation of SV calculations
- Reduce computational costs by caclulating Block Arnoldi SV (BAP) based on the background ensemble from the LETKF cycle
  (optimisation time Δt=6h)
- set perturbations by randomly combining the leading SV's
- determine subspace of growing perturbations
  (norm independent, because different norms generate different basis systems but describe the same subspace)

Representing Model Error in Weather and Climate Prediction:

Buizza (1999); Berner, Shutts, Leutbecher & Palmer (2009)





„Stochasticity should be introduced only where appropriate and not in every part of the model physics, otherwise physical meaning is lost.“

SRNWP workshop on physical parametrisation and ensemble prediction 18-20 June 2013, Madrid Spain

Additional Material

from "Initial Uncertainties in the EPS (I)", ECMWF Training Course, 2010, by Martin Leutbecher

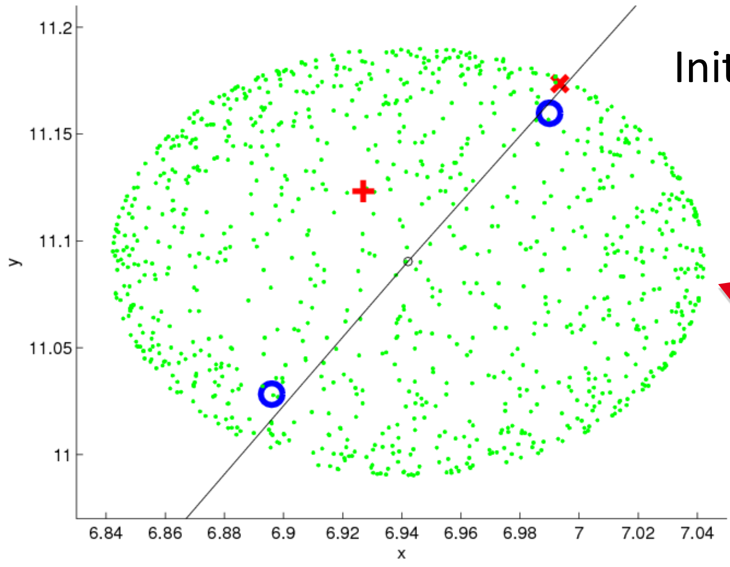
The structure of singular vectors depends on the choice of the norm, in particular the initial time norm.

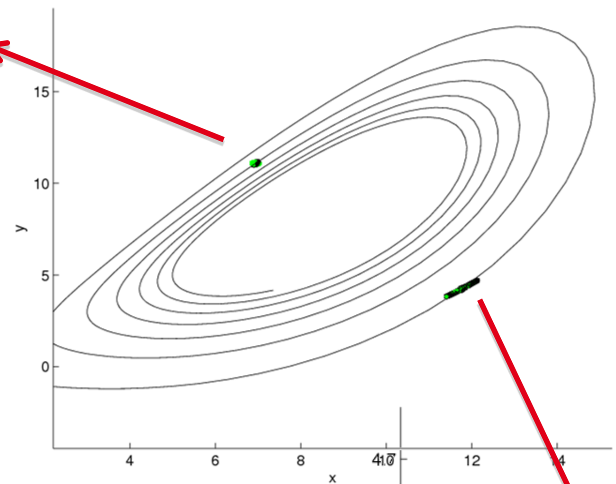
- An **enstrophy norm** at initial time penalises perturbations with small spatial scales, the initial SVs are planetary-scale structures.
- A **streamfunction variance norm** at initial time penalises the large scales and favours sub-synoptic scale perturbations.
- With a **total energy norm** at initial time, the energy spectrum of the initial SVs is white and best matches the spectrum of analysis error estimates from analyses dierences (Palmer et al. 1998)
- The **Hessian of the cost function of a variational assimilation scheme** provides an estimate of the inverse of the analysis error covariance matrix. (see Barkmeijer et al. (1998, 1999), Lawrence et al. 2009). This metric can account for spatial correlations of the initial errors and for the inhomogeneity of the observing network. Disadvantage: computationally much more expensive.

Perturbation methods Lorenz-63

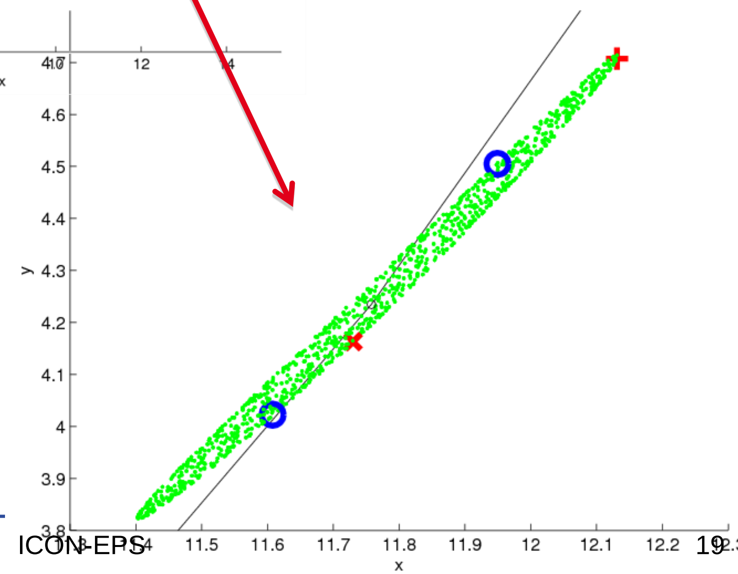
by Linus Magnusson

Initial point





After 1 time unit



- Random pert.
- 1st SV
- 2nd SV
- BV