High-Fidelity Weather Forecasts in the **Grey Zone** of Convective Turbulence

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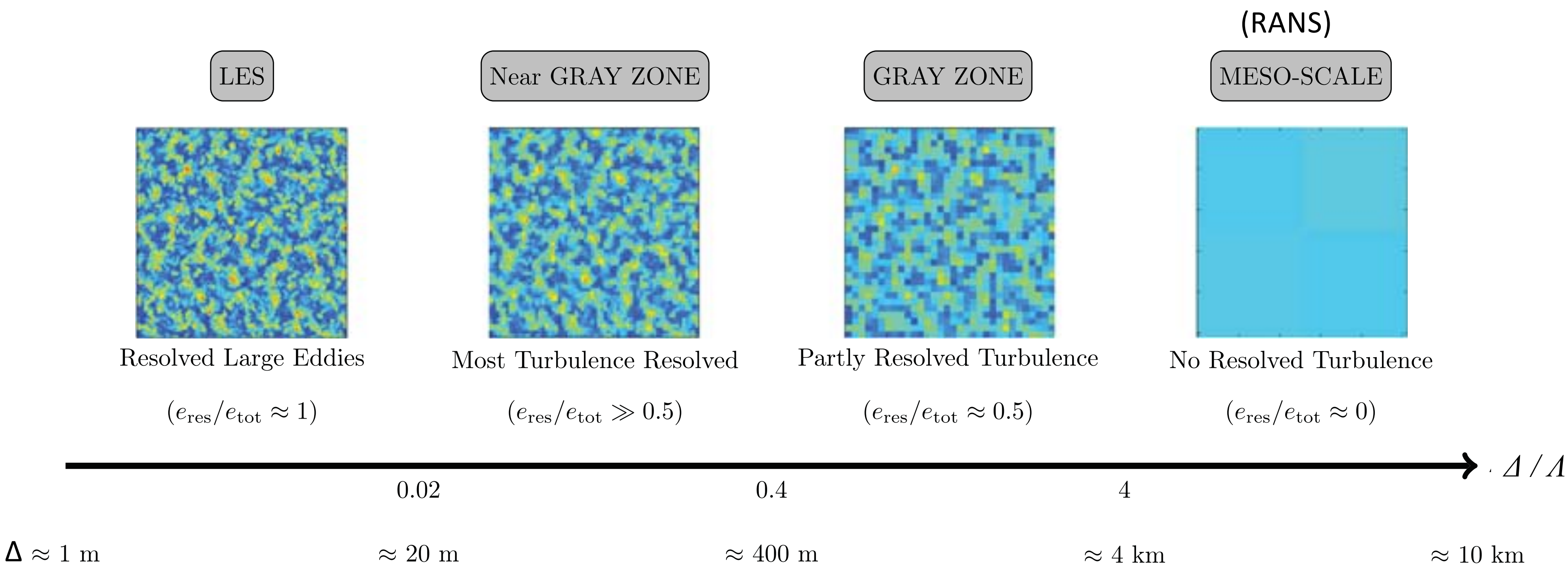
The 'Terra-Incognita' of turbulence modelling



$$\Lambda \approx \Delta$$

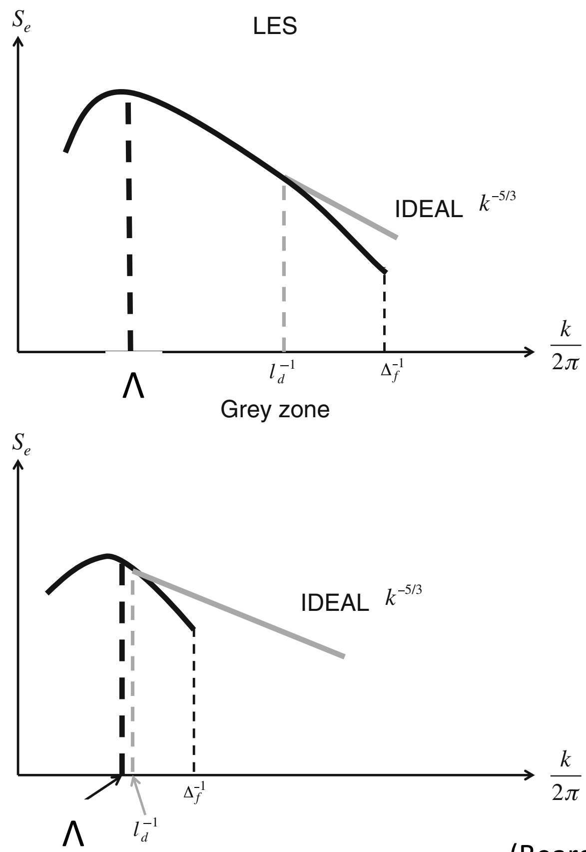
(Honnert et al. 2020)

The ‘Terra-Incognita’ of turbulence modelling

- **LES** : Clear inertial subrange of turbulence
- **Grey zone** : No clear separation of scales

$$\Lambda \approx l_d$$

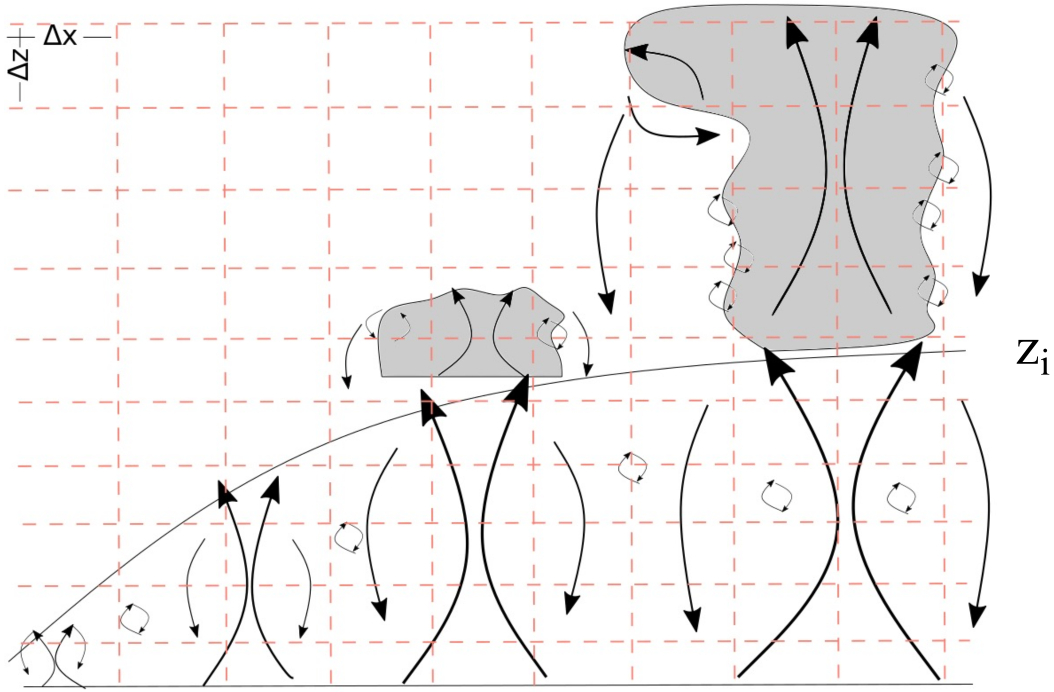
A new grey-zone definition (Beare, 2014)



(Beare, 2014)

The 'Terra-Incognita' of turbulence modelling

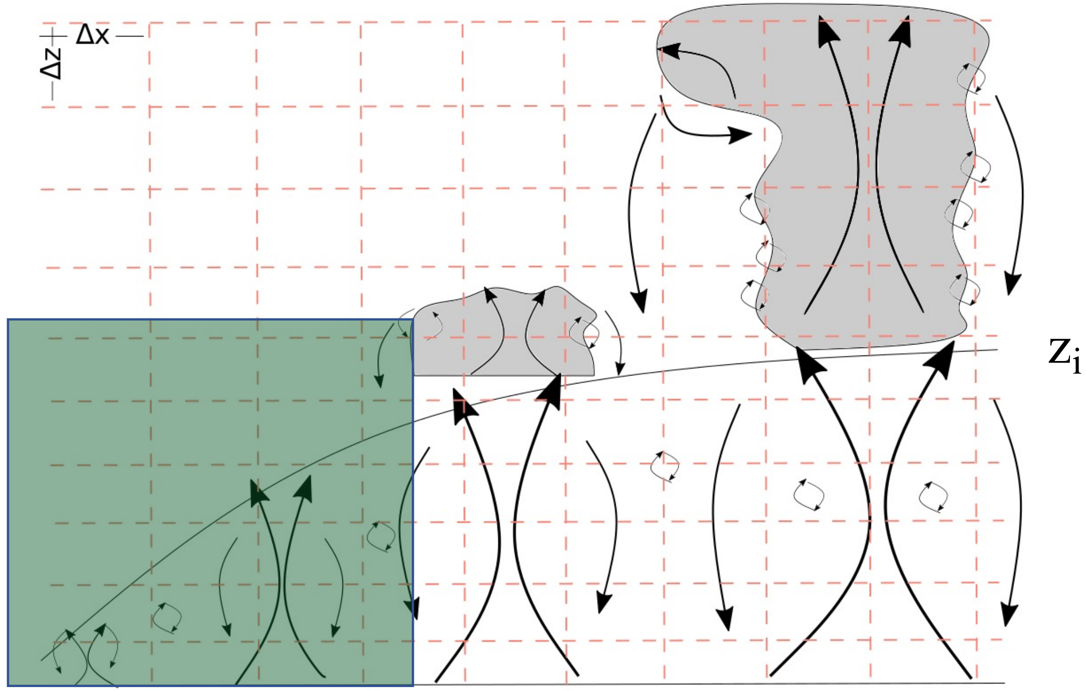
- Evolving turbulence scales
- Different in BL – clouds
- Varying in time-space
- Partially resolved entrainment /detrainment



During the diurnal cycle the simulation enters multiple grey-zones

Convection evolution in hectometric models

- Late spin-up
- Inaccurate representation of the BL

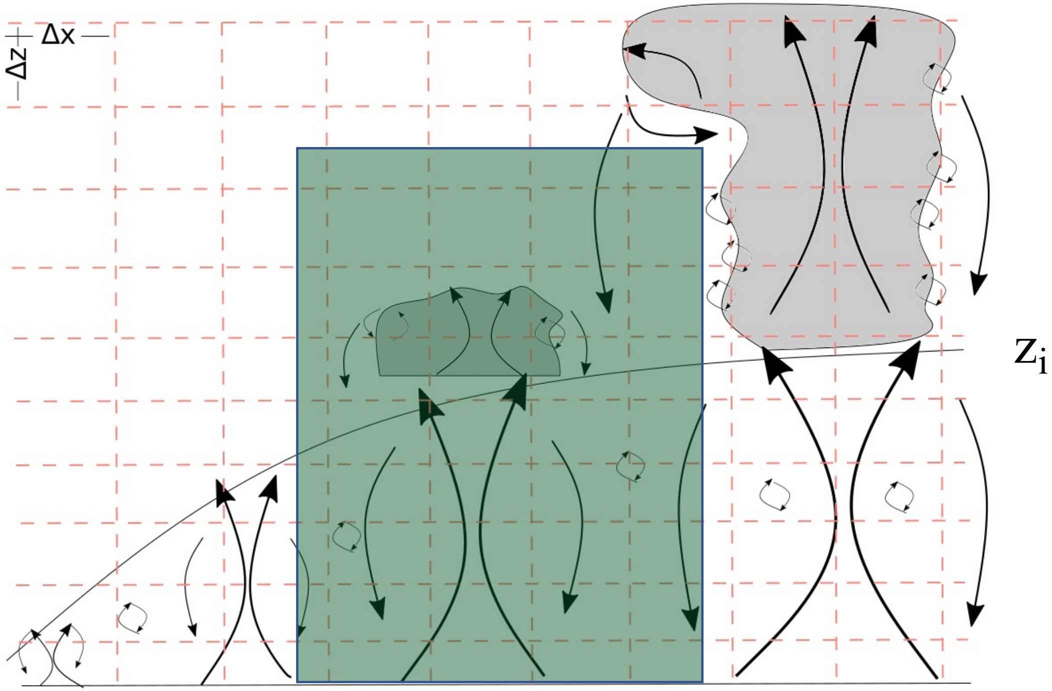


Late spin-up

Misrepresenting turbulence scales

Convection evolution in hectometric models

- Missing shallow convection stage
- A moistier and cooler BL

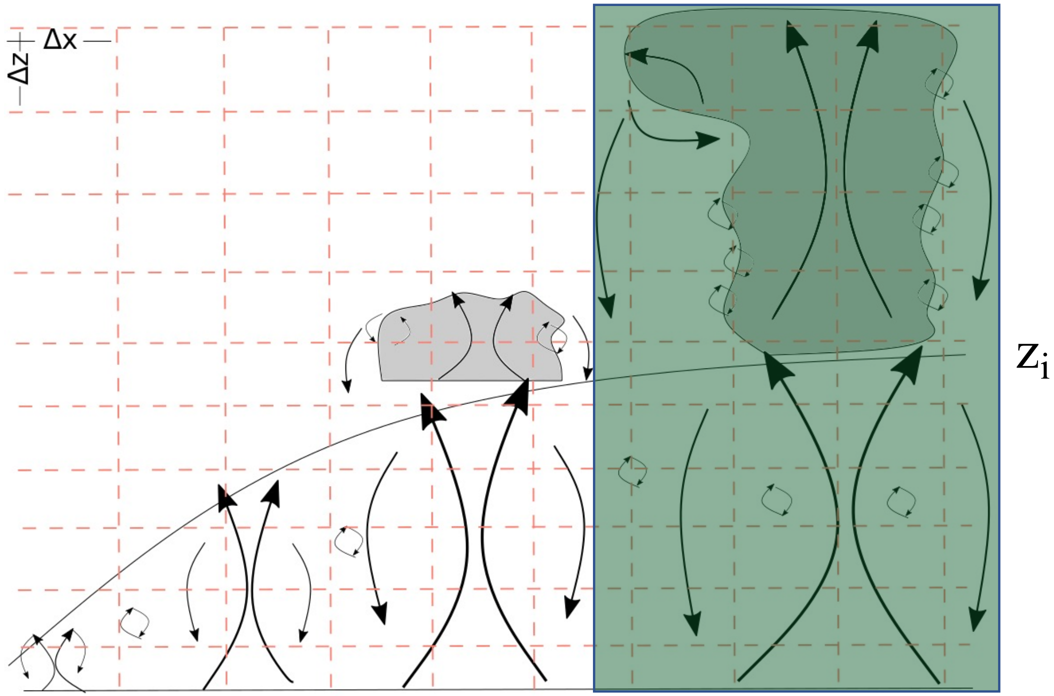


No clear shallow Cu stage

Misrepresenting turbulence scales

Convection evolution in hectometric models

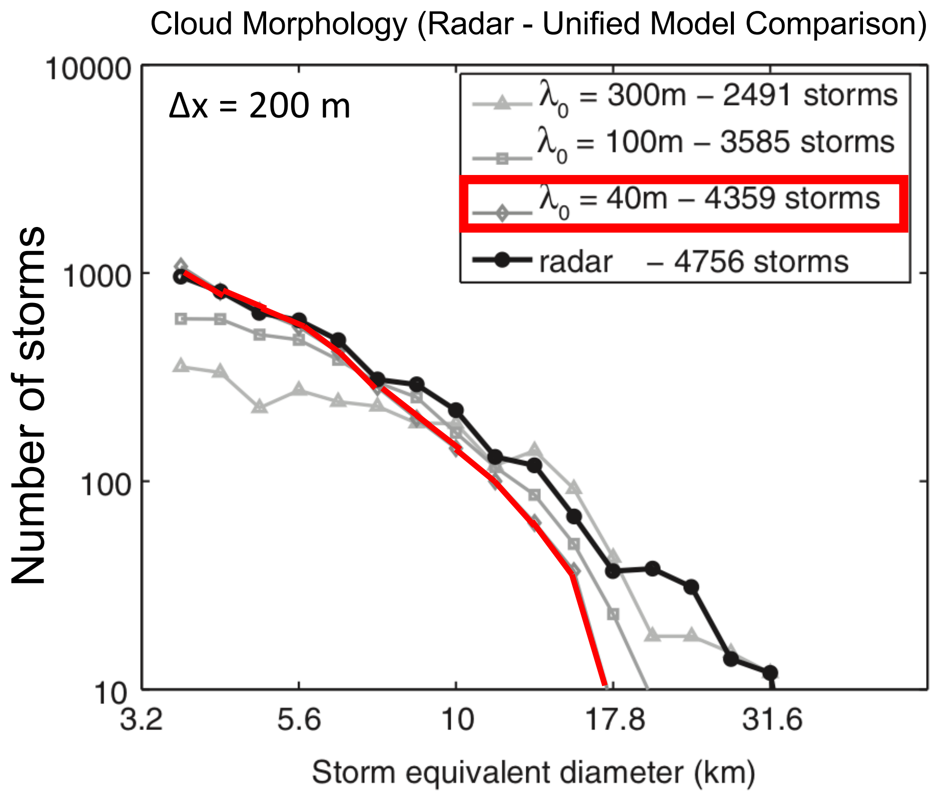
- ‘Blobby’ convection
- Non-monotonic behavior
- Lack of entrainment-mixing



‘Blobby’ deeper convection

Cloud scales in hectometric models

- Closure problem
- Very sensitive to subgrid turbulence length scales
- UM produces smaller storms
- Tuning not an option



(Hanley et al, 2015)

The prognostic turbulent transport equations

(Wyngaard 2004)

- **Subfilter θ flux (f_i) conservation equation** $f_i = \overline{u_i\theta} - \overline{u}_i\overline{\theta}$

Tilting Production — Gradient Production — Buoyant Production

$$\frac{\partial f_i}{\partial t} + \bar{u}_j \frac{\partial f_i}{\partial x_j} = -f_i \frac{\partial \bar{u}_i}{\partial x_j} - \tau_{ij}\frac{\partial \bar{\theta}}{\partial x_j} + \delta_{i3}\frac{g}{\theta_0}(\overline{\theta^2} - \bar{\theta}^2)$$

Turbulent transport

$$-\frac{\partial}{\partial x_j}(\overline{\theta u_i u_j} - \overline{\theta u_i}\bar{u}_j - \bar{\theta}\overline{u_i u_j} - \bar{u}_i\overline{\theta u_j} + 2\bar{\theta}\bar{u}_i\bar{u}_j) + \frac{1}{\rho_0}\left(\overline{p\frac{\partial \theta}{\partial x_i}} - \bar{p}\frac{\partial \bar{\theta}}{\partial x_i}\right)$$

$$\tau_{ij} = \overline{u_i u_j} - \overline{u}_i \overline{u}_j$$

Dynamic High-Order turbulence modelling

First-order closure

$$f_i = -l^2 \left|\bar{S}\right| \frac{\partial \bar{\theta}}{\partial x_i}$$

$$f_i = -l^2 \left|\bar{S}\right| \left(\frac{\partial \bar{\theta}}{\partial x_i} - \delta_{i3}\gamma_\theta \right)$$

First-order closure + Leonard terms (Mixed Model)
(Level 2 + tilting terms)

$$f_i = -l^2 \left|\bar{S}\right| \frac{\partial \bar{\theta}}{\partial x_i} + C\frac{(\gamma\Delta)^2}{12} \left(\frac{\partial \bar{u}_i}{\partial x}\frac{\partial \bar{\theta}}{\partial x} + \frac{\partial \bar{u}_i}{\partial y}\frac{\partial \bar{\theta}}{\partial y} \right)$$

Level 2.5/3

$$f_i = 3\frac{l_f}{(2e)^{1/2}} \left(-f_j \frac{\partial \bar{u}_i}{\partial x_j} - \tau_{ij}\frac{\partial \bar{\theta}}{\partial x_j} + \delta_{i3}\frac{g}{\theta_0}\tau_{\theta\theta} \right)$$

γ_θ : countergradient term

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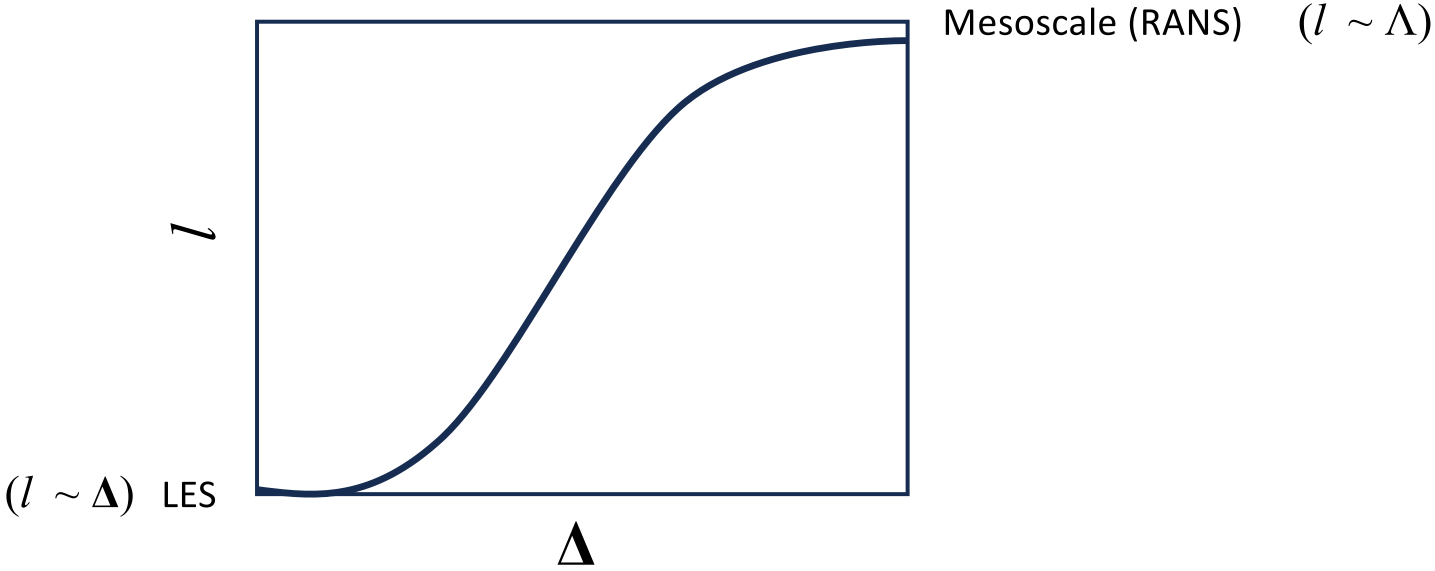
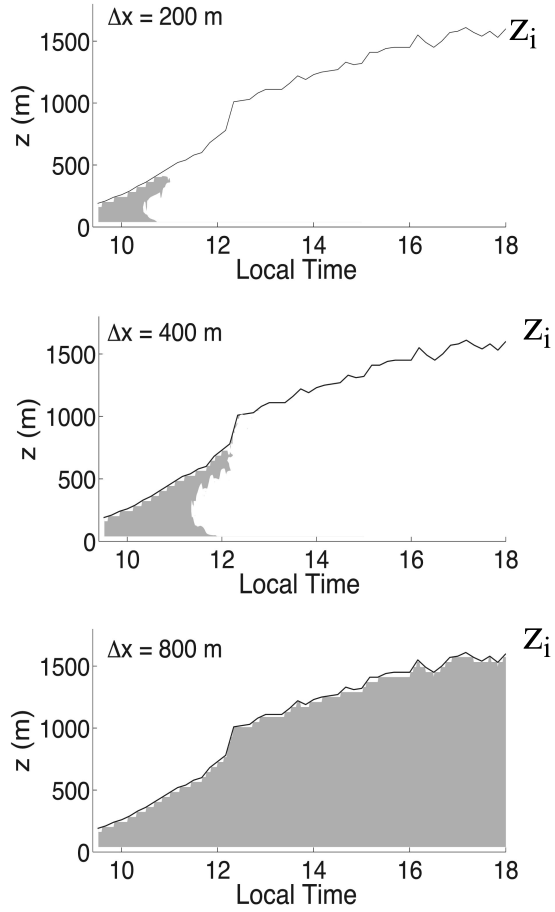
Turbulence length scales across the grey zone



(Wyngaard 2004; Honnert 2011; Boutle et al. 2014)

Convection evolution in hectometric models

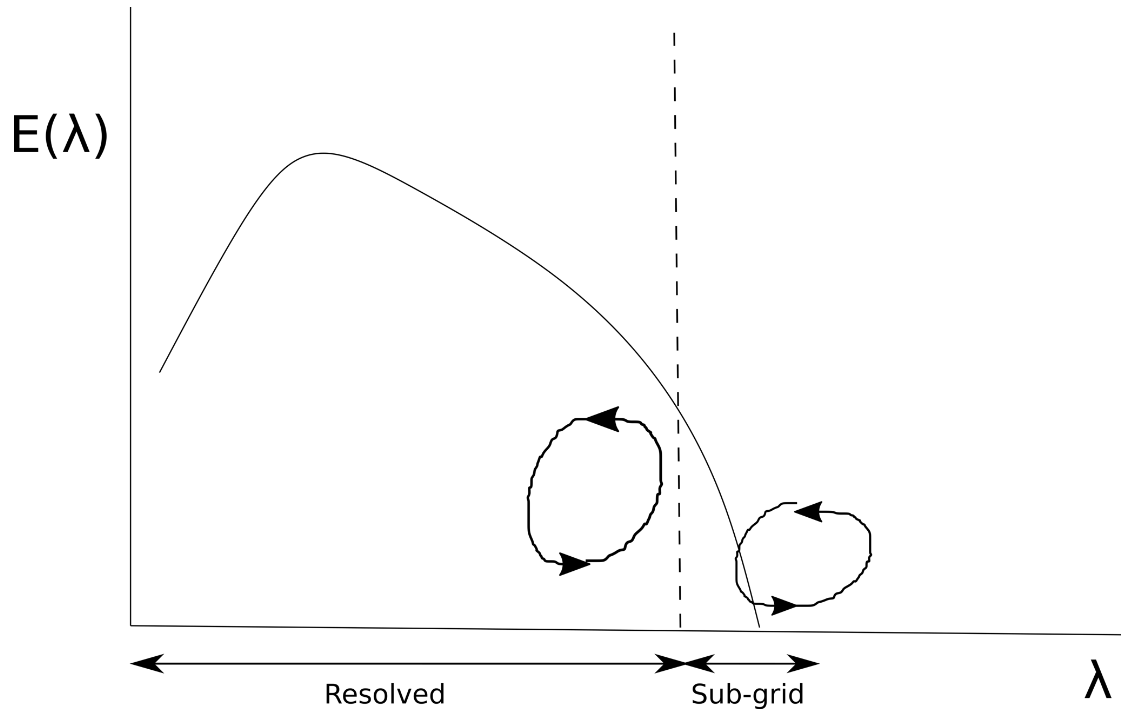
- Dissipation scale to diagnose the **grey zone**



(Honnert et al. 2020)

Dynamic Turbulence Modelling

- **Scale similarity** between resolved and subgrid eddies
- **Use smallest resolved fluxes** to diagnose the subgrid scales



Dynamic Turbulence Modelling

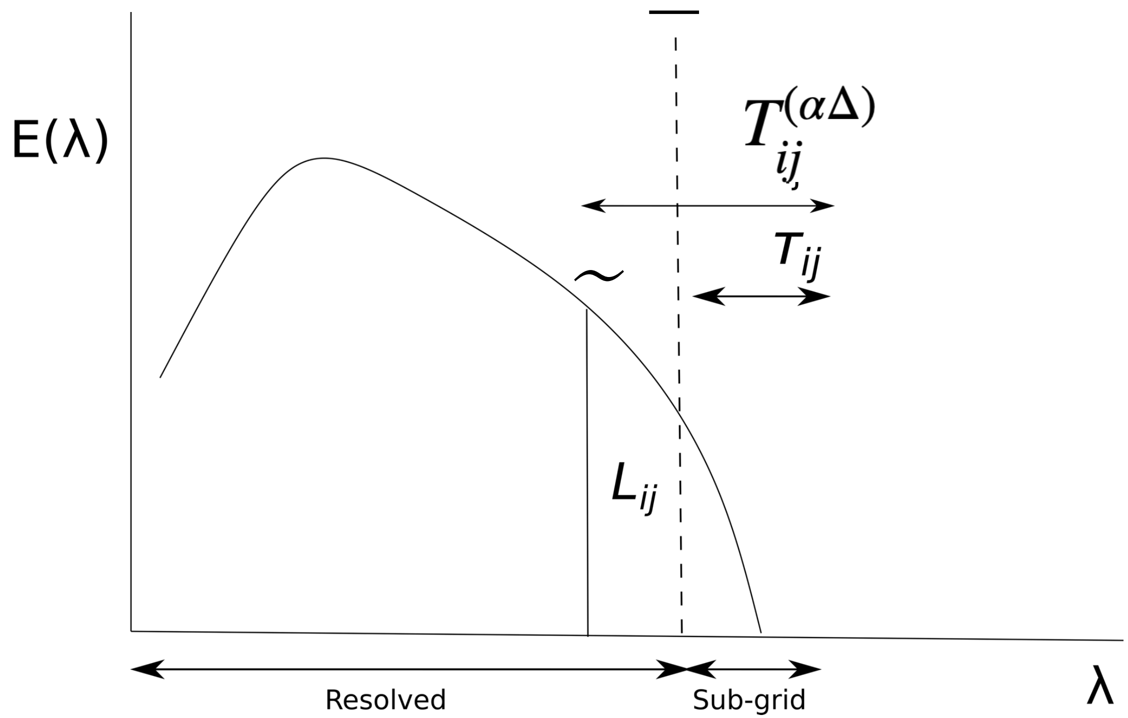
Germano Identity

$$L_{ij} = \widetilde{\bar{u}_i \bar{u}_j} - \tilde{\bar{u}}_i \tilde{\bar{u}}_j = T_{ij}^{(\alpha\Delta)} - \tilde{\tau}_{ij}$$

τ_{ij} : subgrid stress tensor
(Turbulence model)

$$\tau_{ij} = -l^2 |\overline{S}| \overline{S}_{ij} f_m(\mathrm{Ri})$$

$$h_i = -l_\theta^2 |\overline{S}| \frac{\partial \overline{\theta}}{\partial x_i} f_h(\mathrm{Ri})$$



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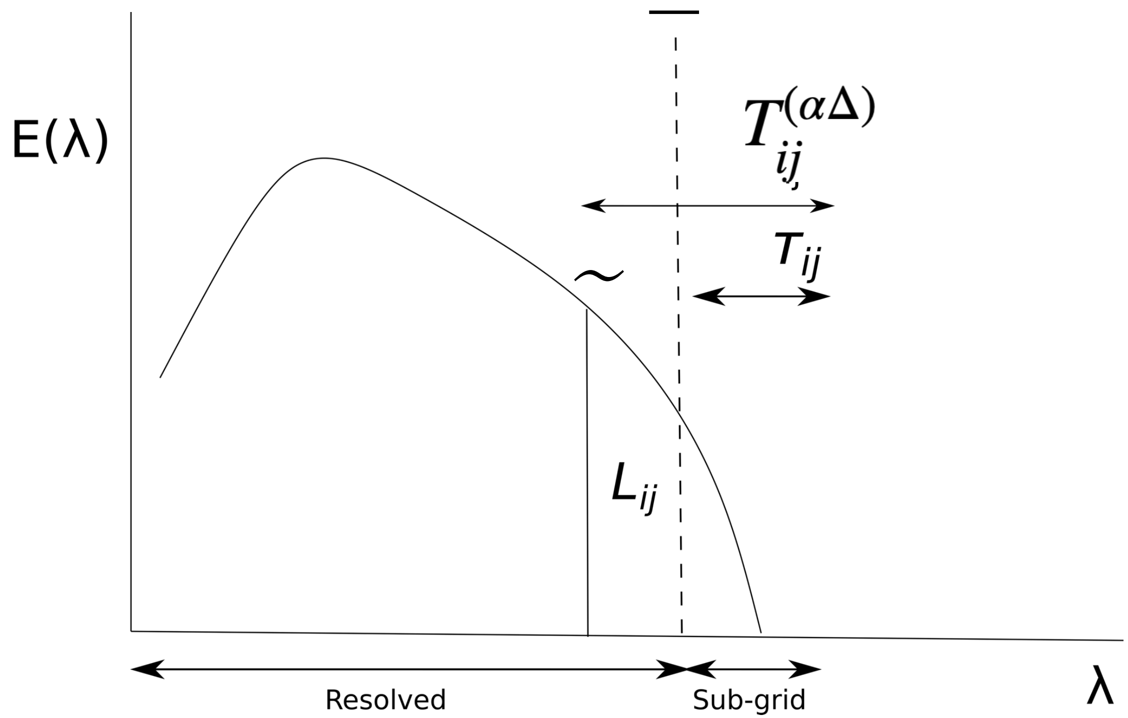
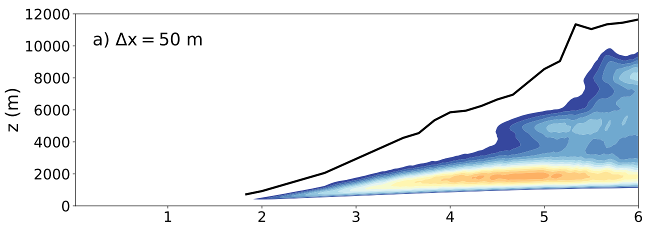


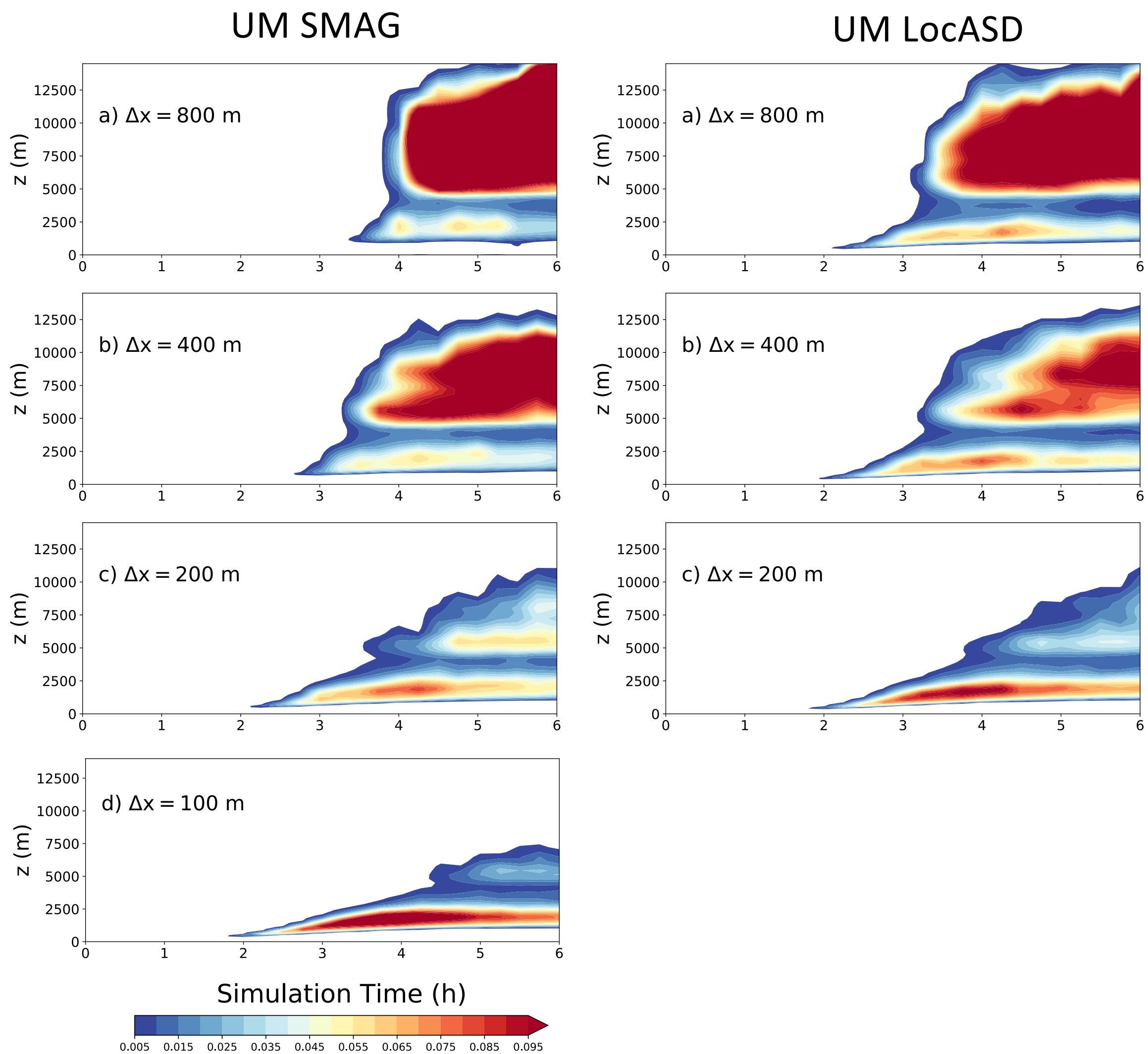
LBA Case study

Hydrometeor evolution

$(q_l + q_s + q_i + q_g)$

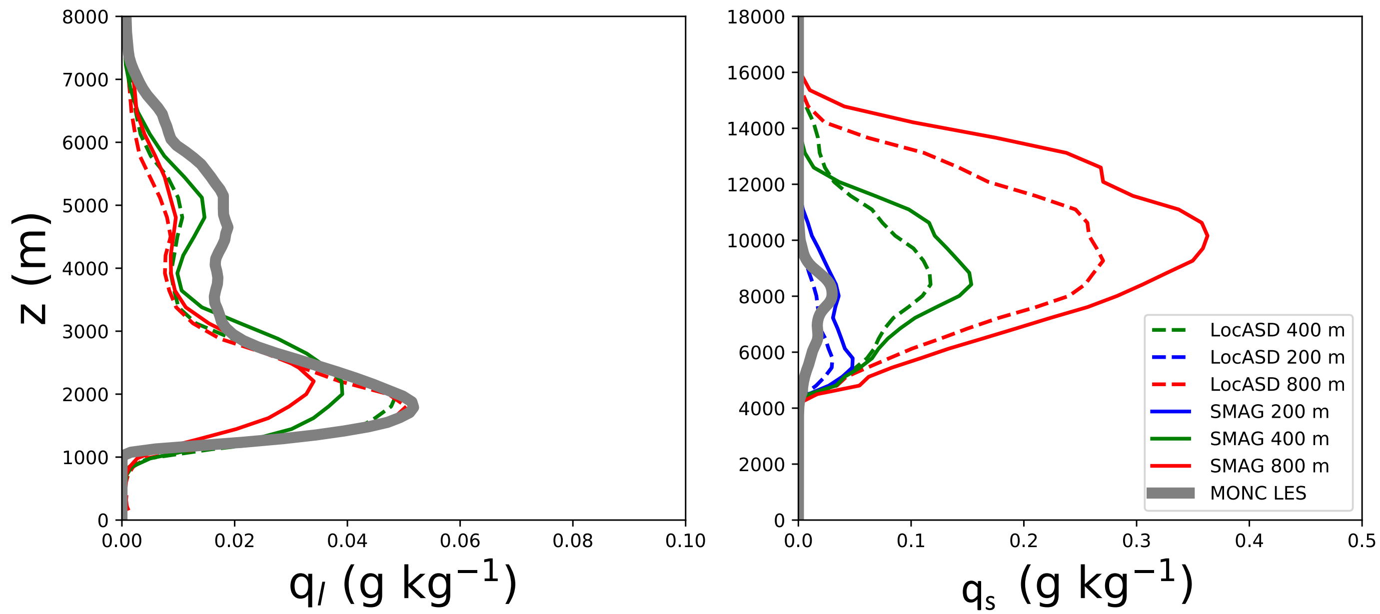
MONC LES





LBA Case study

Deep Cu (t = 6 h)

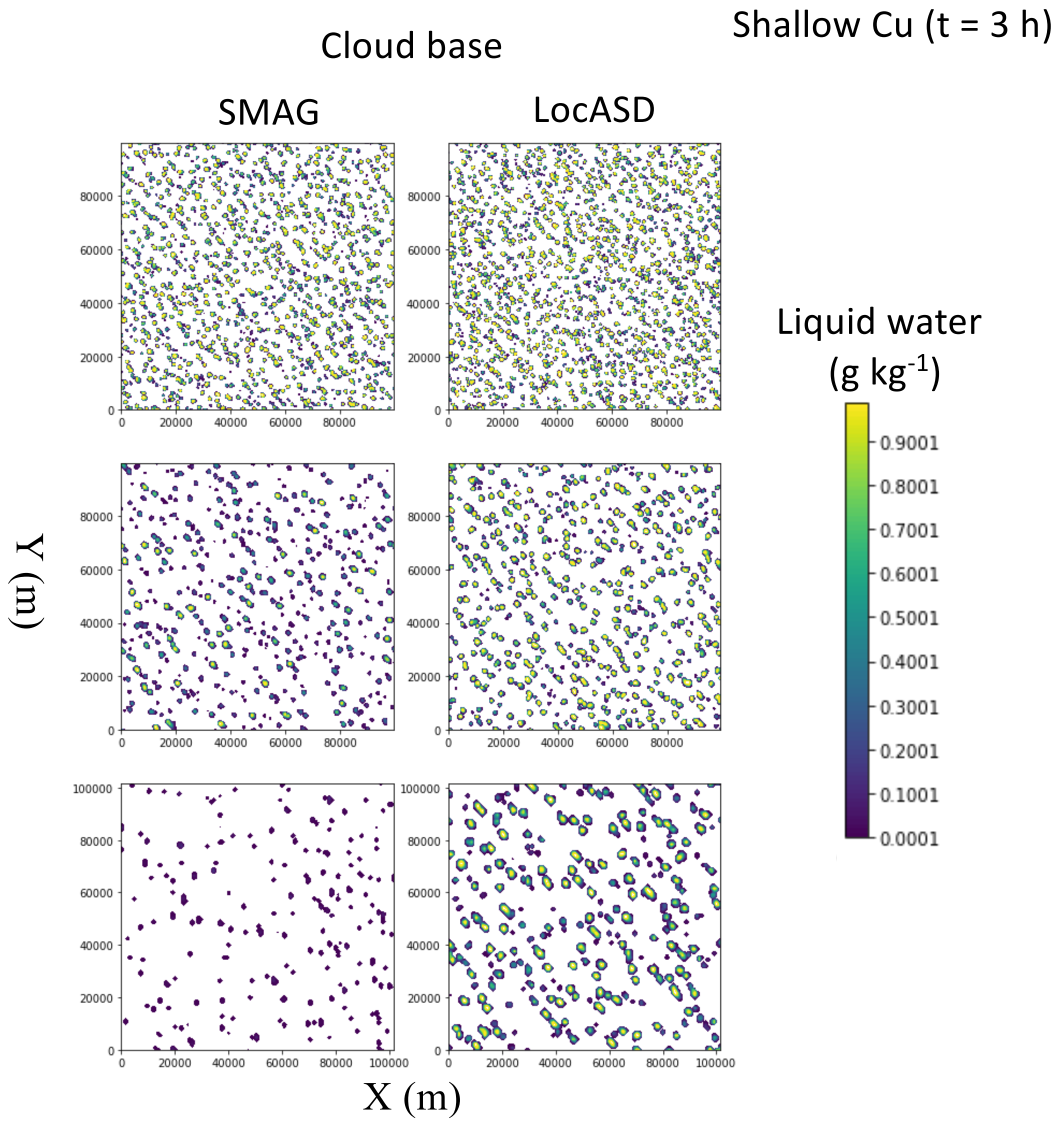


LBA Case study

Δx = 200 m

Δx = 400 m

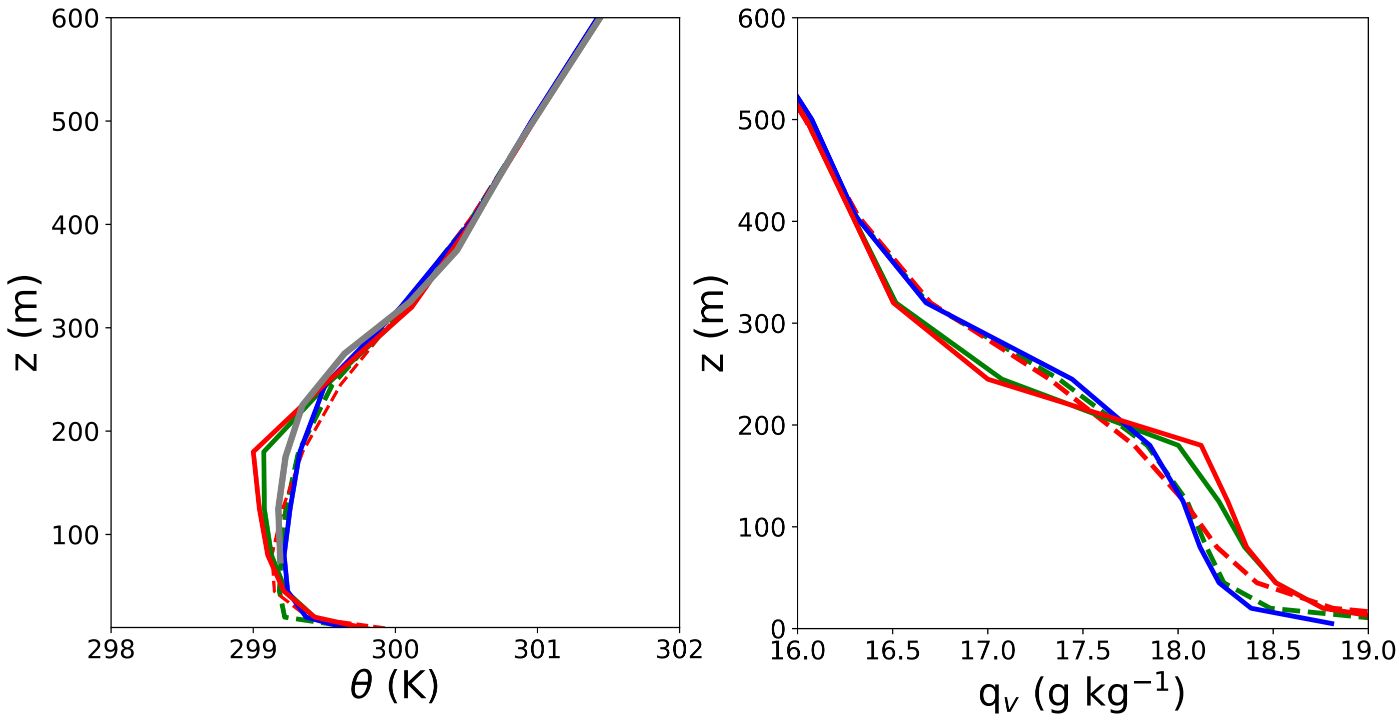
Δx = 800 m



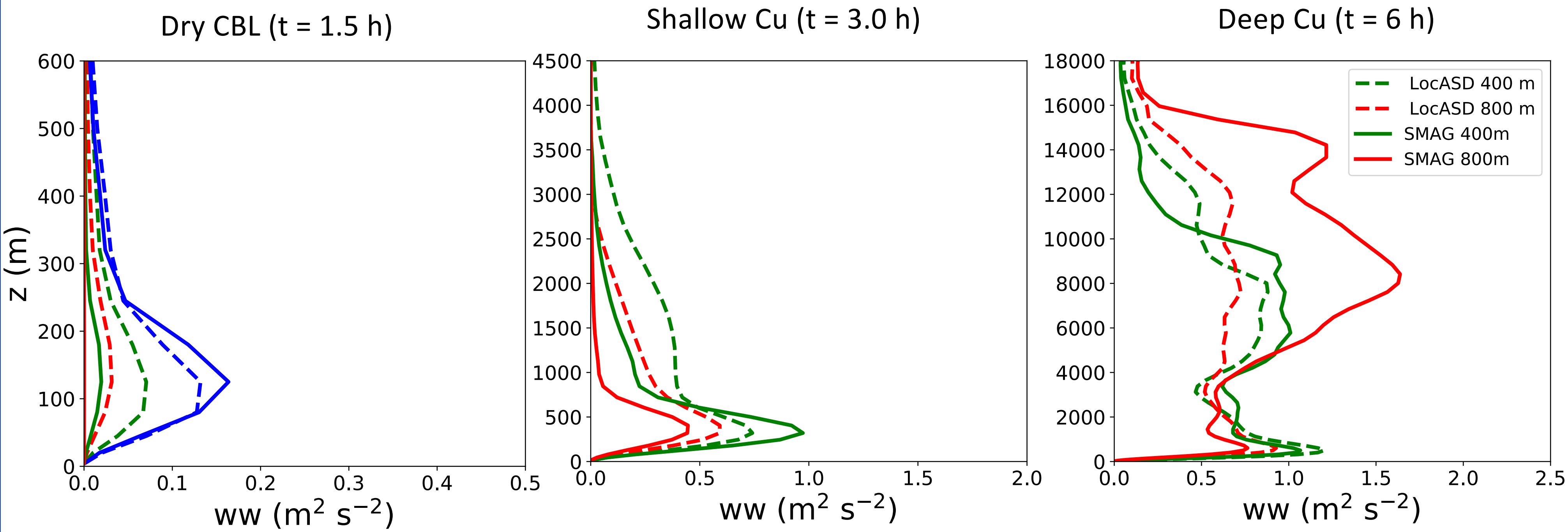
LBA Case study (BL representation)

Dry CBL (t = 1.5 h)

- **SMAG** BL lacks non-local mixing
- Lack of entrainment restricts ventilation of vapour

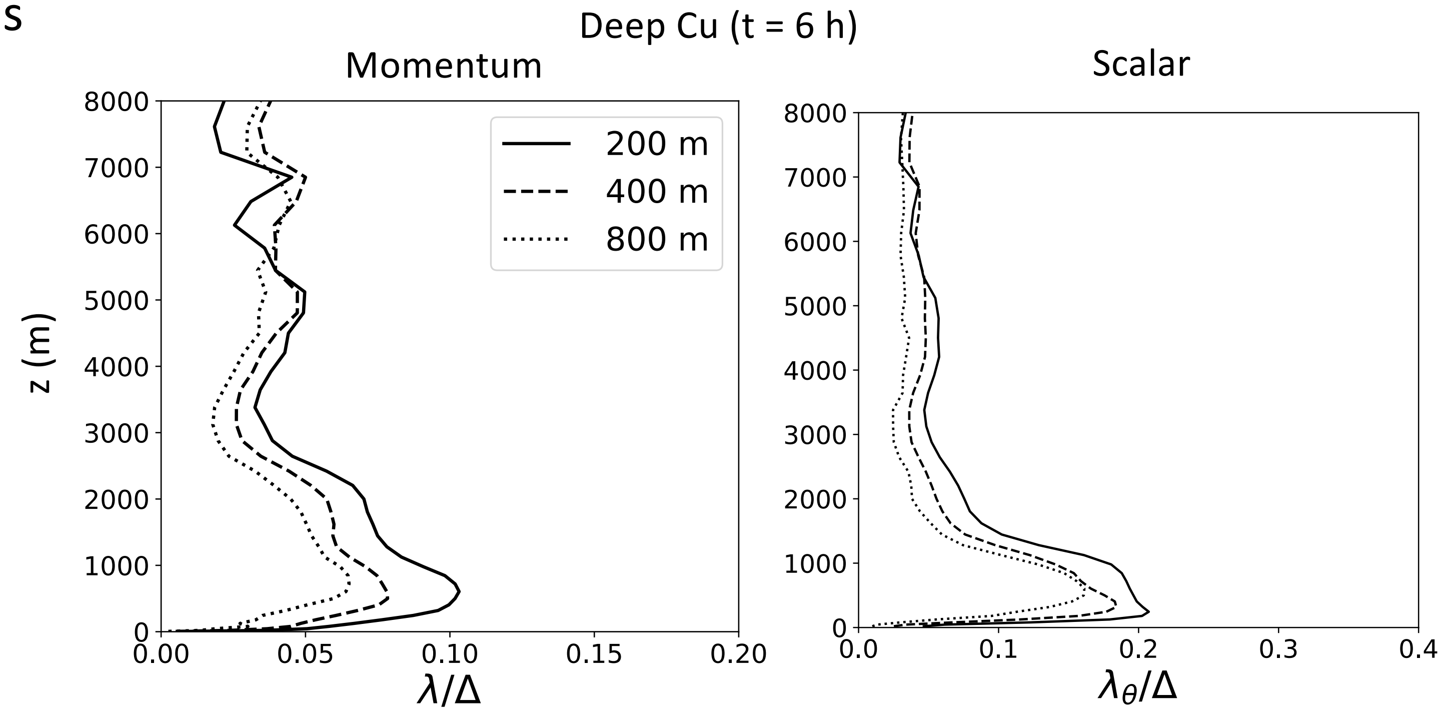


LBA Case study



LBA Case study

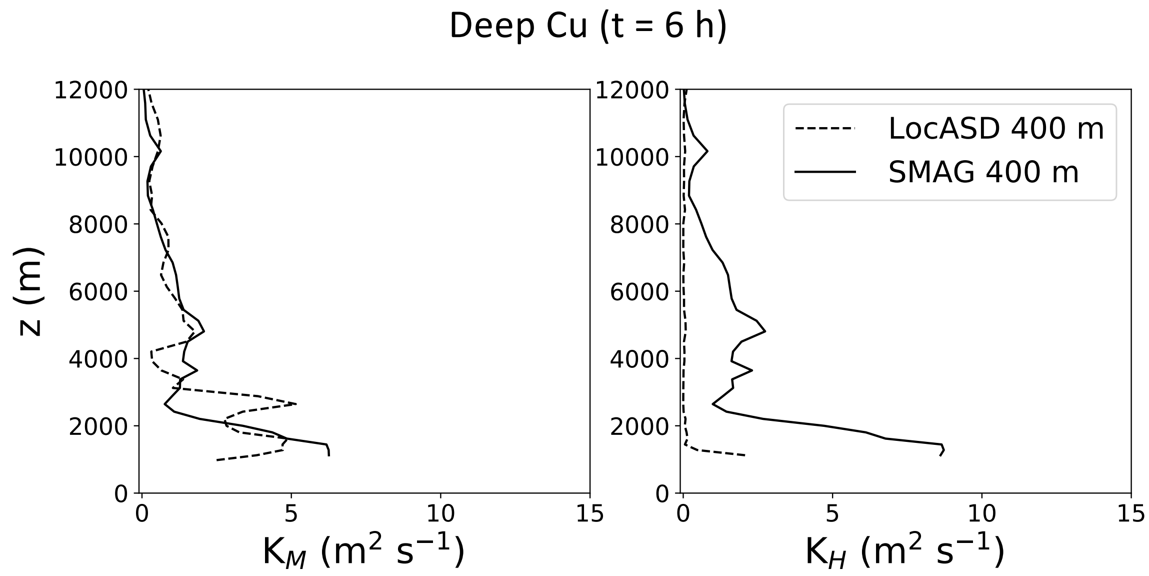
Dynamic length scales



LBA Case study

Turbulent Diffusivity
(where q_{hydro}.**ge**.10e^{-5})

- **LocASD-SMAG** maintain strong $\mathrm{K_M}$ in cloud
- **LocASD** clips $\mathrm{K_H}$ negative values in cloud (counter gradient fluxes)

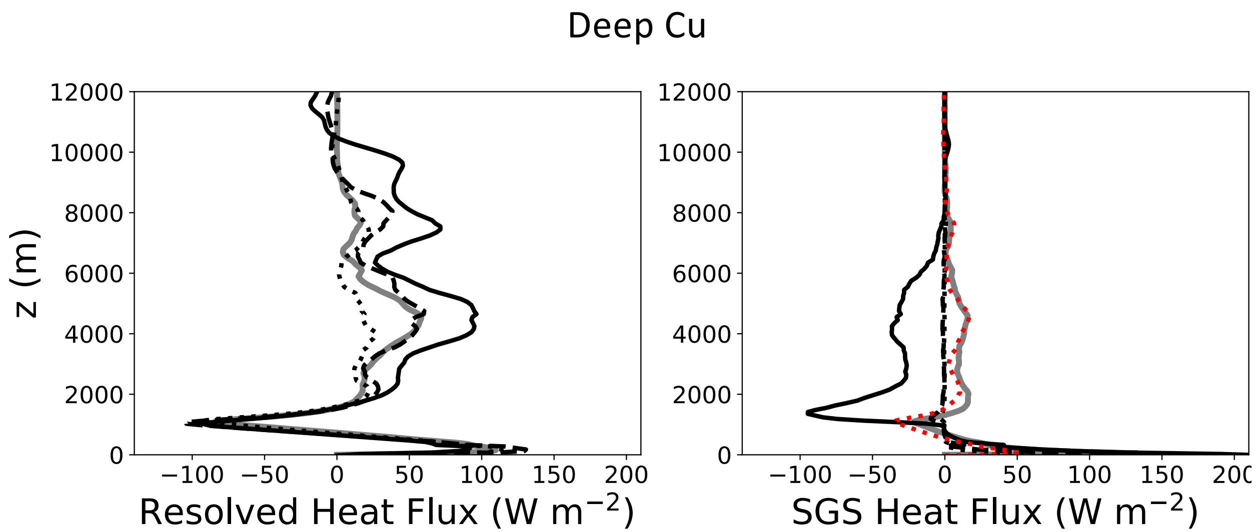


Countergradient fluxes - Leonard terms

Δx = 400 m

MONC LBA Simulations

- Coarse-grained LES
- SMAG
- Dyn SMAG
- Dyn SMAG + Leonard
- $\tau_{\theta 3}$ Leonard terms



(Efstathiou et al. 2024)

Summary

- The grey zone might stalling further improvement of NWP at hectometric scales
- Fundamental assumptions behind conventional schemes are no longer valid
- Full transport equations - Closure length scales not known

- Dynamic approach to derive closure parameters
  - Adapts to the evolving resolved flow field in time and space
  - Dynamic Smagorinsky relaxes the need for a clear inertial subrange
  - Ability to better represent the BL and cloud development in the near-grey-zone
  - Usability limit when the flow is poorly resolved (deep grey zone)

- Examine impact of extra production terms in connection to the dynamically derived closure parameters