Recent changes for graupel and hail in the 2-moment microphysics

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1 Shedding during collisions of graupel and hail with cloud water and rain

Up to now, there is no shedding of water droplets from graupel and hail particles in ICON's 2-moment microphysics code. This means that if graupel and hail collects cloud- or rainwater, it will unconditionally stay attached to the collector particels leading to growth. This is unphysical in at least two respects. First, if the collected water does not immediately freeze during wet growth or if the ambient temperature $T_a > 0°\mathrm{C}$, in reality some of the water coating will be shedded into sub-millimeter droplets due to an aerodynamic instability caused by, e.g., wake turbulence. Second, in the 2-moment scheme immediate freezing and associated release of latent heat of the collected water is unconditionally assumed, even for $T_a > 0°\mathrm{C}$. While a meaningful parameterization of the shedding process can be derived from theory (as will be done below), the second point is not so easy to parameterize without having explicit mixed-phase particle categories.

In the old COSMO version of the scheme there was an option for a simple parameterization of shedding, but this option was not overtaken from there in ICON because it was not overly realistic. The simple parameterization did shed all of the accreted water during the timestep if the mean size of graupel or hail was larger than a critical diameter of 5 mm for hail and 3 mm for graupel and if the ambient temperature $T_a > -10°\mathrm{C}$. For $T_a > 0°\mathrm{C}$ all accreted water was immediately shedded. The mass of shed raindrops was assumed to be

$$x_{r,shed} = \begin{cases} \min[x_r(D_{r,shed} = 1.0\,\mathrm{mm}), \overline{x}_h] & \text{for } T_a \leq 0°\mathrm{C} \\ \max[\overline{x}_r, \overline{x}_h] & \text{for } T_a > 0°\mathrm{C} \end{cases} \qquad (1)$$

where $\overline{x}_h$ is the mean mass of graupel or hail particles. In essence, this parameterization strongly limited the growth of hail and graupel into larger particles above the melting level and acted as a mere transformer of water substances into raindrops with diameter $D_{r,shed} = 1.0\,\mathrm{mm}$. Below the melting level it inhibited accreation. Together with the instantaneous melt shedding, this leads to to too small, too slow and too quickly melting particles. The "truth" should be somewhere inbetween.

To overcome at least the instantaneous melt shedding problem, there were at least two attempts to introduce partially melted particle categories in COSMO's and ICON's microphysics: Frick et al. (2013) introduced prognostic water on snow flakes in COSMO's 1-moment scheme, and at MPI Hamburg Vivek Sant worked on prognostic water on hail and graupel in the 2-moment scheme. The latter reached a technically mature stage, but there were some remaining physical problems that could not be solved up to now and the development stopped around 2018. The last state of this development is part of ICON's 2-moment microphysics code (option `inwp_gscp=7`) but is currently not developed further. To the authors best knowledge, it does not consider collection of liquid water (either by wet growth or below the 0°C-isotherm) as a source for the prognostic water on graupel and hail.

Recently, a relatively simple shedding parameterization has been implemented for all flavors of ICON's 2-moment scheme, which covers at least some of the above aspects and is "somewhere inbetween" in the above sense. It can be optionally switched on and is described in the following.

Althoug the physics of the shedding process is a quite complicated aerodynamic problem (Rasmussen and Heymsfield, 1987), the same authors suggest a reasonable simplification from their lab measurements: if the diameter of a spherical ice particle is larger than about 9 mm and it has a liquid coating (e.g., by melting or by accretion in wet growth mode), the interplay of excitation by turbulent eddies with the surface tension of the water coating leads to shedding of most of the coating water into droplets of sub-mm size.

Based on this assumption, a spectrally resolved but mathematically feasible parameterization for shedding from graupel and hail during wet growth and below the melting level has been developed and implemented in ICON's 2-moment microphysics. The idea is that

- for $T_a < 0°\mathrm{C}$: particles in wet growth mode with diameter $D_h \geq D_{wg}(T_a, p_a, q_r + q_c, q_i + q_s)$ will shed their accreted water mass into raindrops of a certain mean mass diameter $D_{r,shed}$ (e.g., 1.0 mm). If D_{wg} is smaller than a certain threshold diameter D_{RH87} following Rasmussen and Heymsfield (1987), where D_{RH87} was found to be ≈9 mm, shedding only occurs for $D_h > D_{RH87}$. Particles with $D_{wg} < D_h \leq D_{RH87}$ are assumed to keep their accreted water mass, which will subsequently freeze. D_{wg} is computed in ICON following the theory and lookup tables described in the appendix of Khain et al. (2011).
- for $T_a \geq 0°\mathrm{C}$, the accreted water on particles with $D_h \geq D_{RH87}$ is shed into raindrops of the same $D_{r,shed}$ (e.g., 1.0 mm).

Because D_{wg} is 0 for $T_a \geq 0°\mathrm{C}$, we can simply define $D_{shed} = \max(D_{wg}, D_{RH87})$ and use it as dividing diameter. This simplification enables a reasonable wet-growth- and shedding parameterization. Note however, that due to the lack of mixed phase particles there are some errors in the exact location where the latent heat of freezing/melting is consumed/released associated with the riming/ accretion/ shedding.

Let us start with the definition of the generalized gamma PSD

$$f(x) \;=\; N_0\, x^{\nu}\, \exp\left(-\lambda x^{\mu}\right) \tag{2}$$

with particle mass x and the parameters N_0, ν, λ, μ, the definition of the ith moment

$$M^{(i)} \;=\; \int_0^{\infty} x^i\, f(x)\, dx = \frac{N_0}{\mu}\, \frac{\Gamma(\frac{\nu+i+1}{\mu})}{\lambda^{\frac{\nu+i+1}{\mu}}} \tag{3}$$

and the power-laws for the diameter-mass-relation

$$D \;=\; a\,x^{b} \tag{4}$$

with parameters a and b and for the fallspeed-mass-relation

$$v \;=\; \alpha\,x^{\beta} \tag{5}$$

with parameters α and β.

For the mathematical derivation, we start from a quite general formulation of a partial microphysical collision integral in form of a two-dimensional partial moment. The Stochastic Collection Equation (SCE) for a collision rate term between two species (h and r below), leading to one (or even more than one) resulting species, can be written as a linear combination of the basic structure T:

$$T_{hr}^{(n,m)}\,(l_h, l_r, u_h, u_r) \;=\; \int\limits_{l_r}^{u_r}\int\limits_{l_h}^{u_h} f_h\, f_r\, K_{hr}\, x_h^n\, x_r^m\, dx_r\, dx_h \tag{6}$$

with

$$K_{hr} = \overline{E_{hr}}\frac{\pi}{4}(D_h + D_r)^2\,\overbrace{|v_h - v_r|}^{\sqrt{(v_h-v_r)^2}} \tag{7}$$

beeing the gravitational spherical collision kernel with collision-coalescence efficiency $\overline{E_{hr}}$. Let the subscripts h and r denote two different hydrometeor categories. The lower and upper mass integration limits, l_h, l_r, u_h and u_r are chosen to be finite.

Plugging Equations (2), (4), (5) into (6) and doing the math, one arrives at

$$T_{hr}^{(n,m)} \;=\; \overline{E_{hr}}\frac{\pi}{4}\overline{\left|\Delta v_{hr}^{(n,m)}\right|}\, M_h^{(n)}\, M_r^{(m)}\,\left[\delta_{hh}^{nm} D_h^2(\overline{x}_h) \;+\; \delta_{hr}^{nm} D_h(\overline{x}_h) D_r(\overline{x}_r) \;+\; \delta_{rr}^{nm} D_r^2(\overline{x}_r)\right] \tag{8}$$

using $\overline{x} = \dfrac{M^{(1)}}{M^{(0)}} = \dfrac{L}{N}$ with L and N mass- and number density.

Because of the finite integration limits, the δ's are complicated factors involving the incomplete gamma function $\gamma\,(\,A\,, \lambda\, l^{\mu}, \lambda\, u^{\mu})$ with at least a non-constant λ, as well as the ordinary gamma function $\Gamma\,(A)$.

$$\begin{aligned}
\delta_{hh}^{nm} \;&=\; \frac{\gamma\left(\frac{\nu_r+m+1}{\mu_r}\,,\,\lambda_r l_r^{\mu_r}\,,\,\lambda_r u_r^{\mu_r}\right)}{\Gamma\left(\frac{\nu_r+m+1}{\mu_r}\right)}\;\frac{\gamma\left(\frac{\nu_h+2b_h+n+1}{\mu_h}\,,\,\lambda_h l_h^{\mu_h}\,,\,\lambda_h u_h^{\mu_h}\right)}{\Gamma\left(\frac{\nu_h+n+1}{\mu_h}\right)}\;\left[\frac{\Gamma\left(\frac{\nu_h+1}{\mu_h}\right)}{\Gamma\left(\frac{\nu_h+2}{\mu_h}\right)}\right]^{2b_h} \\
\delta_{hr}^{nm} \;&=\; \frac{\gamma\left(\frac{\nu_r+b_r+m+1}{\mu_r}\,,\,\lambda_r l_r^{\mu_r}\,,\,\lambda_r u_r^{\mu_r}\right)}{\Gamma\left(\frac{\nu_r+m+1}{\mu_r}\right)}\;\frac{\gamma\left(\frac{\nu_h+b_h+n+1}{\mu_h}\,,\,\lambda_h l_h^{\mu_h}\,,\,\lambda_h u_h^{\mu_h}\right)}{\Gamma\left(\frac{\nu_h+n+1}{\mu_h}\right)}\;\left[\frac{\Gamma\left(\frac{\nu_r+1}{\mu_r}\right)}{\Gamma\left(\frac{\nu_r+2}{\mu_r}\right)}\right]^{b_r}\;\left[\frac{\Gamma\left(\frac{\nu_h+1}{\mu_h}\right)}{\Gamma\left(\frac{\nu_h+2}{\mu_h}\right)}\right]^{b_h} \\
\delta_{rr}^{nm} \;&=\; \frac{\gamma\left(\frac{\nu_r+2b_r+m+1}{\mu_r}\,,\,\lambda_r l_r^{\mu_r}\,,\,\lambda_r u_r^{\mu_r}\right)}{\Gamma\left(\frac{\nu_r+m+1}{\mu_r}\right)}\;\frac{\gamma\left(\frac{\nu_h+n+1}{\mu_h}\,,\,\lambda_h l_h^{\mu_h}\,,\,\lambda_h u_h^{\mu_h}\right)}{\Gamma\left(\frac{\nu_h+n+1}{\mu_h}\right)}\;\left[\frac{\Gamma\left(\frac{\nu_r+1}{\mu_r}\right)}{\Gamma\left(\frac{\nu_r+2}{\mu_r}\right)}\right]^{2b_r}
\end{aligned} \tag{9}$$

Note the symmetry between δ_{hh}^{nm} and δ_{rr}^{nm} regarding exchange of $g \to r$ and $n \to m$.

Depending on the integration limits l and u, γ reduces to lower or upper incomplete gamma functions γ_l or γ_u or to $\Gamma()$,

$$\begin{aligned}
\lim_{l\to 0}\gamma(A,l,u) &= \gamma_l(A,u)\\
\lim_{u\to\infty}\gamma(A,l,u) &= \gamma_u(A,l)\\
\lim_{l\to 0,u\to\infty}\gamma(A,l,u) &= \Gamma(A)
\end{aligned}$$

and the following relations apply:

$$\begin{aligned}
\gamma_u(A,u) &= \Gamma(A)-\gamma_l(A,u)\\
\gamma(A,l,u) &= \gamma_l(A,u)-\gamma_l(A,l) \;=\; \Gamma(A)-\gamma_u(A,u)-\gamma_l(A,l)
\end{aligned}$$

γ_l or γ_u may be efficiently calculated either by the fit from Blahak (2010) or by efficient lookup-tables, as is done in ICON.

Eq. (8) contains the characteristic fall speed difference

$$\overline{\left|\Delta v_{hr}^{(n,m)}(l_h,l_r,u_h,u_r)\right|} = \frac{\int_{l_r}^{u_r}\int_{l_h}^{u_h} f_h\, f_r\,(D_h+D_r)^2\sqrt{(v_h-v_r)^2}\, x_h^n\, x_r^m\, dx_r\, dx_h}{\int_{l_r}^{u_r}\int_{l_h}^{u_h} f_h\, f_r\,(D_h+D_r)^2\, x_h^n\, x_r^m\, dx_r\, dx_h} \tag{10}$$

In most of today's bulk cloud microphysics schemes, also in ICON, the collision integrals are taken as complete integrals over the entire size range, e.g., $l_h = l_r = 0$ and $u_h \to \infty$, $u_r \to \infty$. For the collection of, e.g., cloud water (index $r \to c$) by graupel (index $h \to g$), the corresponding source- and sink terms in the budget equations for N and L read

$$\begin{aligned}
\left.\frac{\partial N_c}{\partial t}\right|_{coll,gc} &= -\,T_{gc}^{(0,0)}(0,0,\infty,\infty)\\
\left.\frac{\partial N_g}{\partial t}\right|_{coll,gc} &= 0\\
\left.\frac{\partial L_c}{\partial t}\right|_{coll,gc} &= -\,T_{gc}^{(0,1)}(0,0,\infty,\infty)\\
\left.\frac{\partial L_g}{\partial t}\right|_{coll,gc} &= +\,T_{gc}^{(0,1)}(0,0,\infty,\infty)
\end{aligned} \tag{11}$$

$\gamma(A,\dots)$ in Eq. (9) reduces to $\Gamma(A)$ and some of the terms in these coefficients cancel out. Because the parameters ν, μ, a, b α and β are constant, the coefficients can be pre-computed once at the start of the model run.

Let us now have a closer look at the characteristic fall speed difference Eq. (10). There the term $\sqrt{(v_h-v_r)^2}$ is somewhat problematic for an analytic evaluation. A solution involving hypergeometric functions is given in Verlinde et al. (1990), but is far too costly in numerical models. Therefore, a number of approximations have been presented in the literature and are in use today.

Again for the case of complete integrals Wisner et al. (1972) proposed to approximate the entire Eq. (10) by the absolute difference of the mass-weighted mean fallspeeds

$$\left.\overline{\left|\Delta v_{hr}^{(n,m)}(0,0,\infty,\infty)\right|}\right|_{Wisner} \approx |\overline{v}_{3h}-\overline{v}_{3r}| \tag{12}$$

However, the Wisner-approximation is known to strongly underestimate collision rates when the characteristic speeds of both species are nearly equal. Seifert and Beheng (2006) devise another approximation (index 1) based on the assumption that

$$\int\int \sqrt{(\dots)^2}\, dx\, dy \;\approx\; \sqrt{\int\int (\dots)^2\, dx\, dy} \tag{13}$$

and postulate

$$\overline{\left|\Delta v_{hr}^{(n,m)}\right|}_1 \;\approx\; \left[\frac{\int\limits_0^\infty\int\limits_0^\infty f_h\, f_r\, D_h^2 D_r^2\, (v_h - v_r)^2\, x_h^n\, x_r^m\, dx_r\, dx_h}{\int\limits_0^\infty\int\limits_0^\infty f_h\, f_r\, D_h^2 D_r^2\, x_h^n\, x_r^m\, dx_r\, dx_h}\right]^{1/2} =$$
$$\left[\theta_{1hh}^{nm} v_h^2(\overline{x}_h) \,+\, \theta_{1hr}^{nm} v_h(\overline{x}_h) v_r(\overline{x}_r) \,+\, \theta_{1rr}^{nm} v_r^2(\overline{x}_r)\right]^{1/2} \tag{14}$$

This formulation is implemented for many of the binary collision processes of different hydrometeor categories ICON's 2-moment scheme, where n and m are mostly either 0 or 1.

Before we detail the coefficients θ, we generalize to finite integration limits. Eq. (14) becomes

$$\overline{\left|\Delta v_{hr}^{(n,m)}\right|}_1 \;\approx\; \left[\frac{\int\limits_{l_r}^{u_r}\int\limits_{l_h}^{u_h} f_h\, f_r\, D_h^2 D_r^2\, (v_h - v_r)^2\, x_h^n\, x_r^m\, dx_r\, dx_h}{\int\limits_{l_r}^{u_r}\int\limits_{l_h}^{u_h} f_h\, f_r\, D_h^2 D_r^2\, x_h^n\, x_r^m\, dx_r\, dx_h}\right]^{1/2} \tag{15}$$

and the θ's are given by

$$\begin{aligned}
\theta_{1hh}^{nm} &= \frac{\gamma\left(\frac{\nu_h+2b_h+2\beta_h+n+1}{\mu_h},\, \lambda_h l_h^{\mu_h},\, \lambda_h u_h^{\mu_h}\right)}{\gamma\left(\frac{\nu_h+2b_h+n+1}{\mu_h},\, \lambda_h l_h^{\mu_h},\, \lambda_h u_h^{\mu_h}\right)} \left[\frac{\Gamma\left(\frac{\nu_h+1}{\mu_h}\right)}{\Gamma\left(\frac{\nu_h+2}{\mu_h}\right)}\right]^{2\beta_h} \\
\theta_{1hr}^{nm} &= \frac{\gamma\left(\frac{\nu_h+2b_h+\beta_h+n+1}{\mu_h},\, \lambda_h l_h^{\mu_h},\, \lambda_h u_h^{\mu_h}\right)}{\gamma\left(\frac{\nu_h+2b_h+n+1}{\mu_h},\, \lambda_h l_h^{\mu_h},\, \lambda_h u_h^{\mu_h}\right)} \frac{\gamma\left(\frac{\nu_r+2b_r+\beta_r+m+1}{\mu_r},\, \lambda_r l_r^{\mu_r},\, \lambda_r u_r^{\mu_r}\right)}{\gamma\left(\frac{\nu_r+2b_r+m+1}{\mu_r},\, \lambda_r l_r^{\mu_r},\, \lambda_r u_r^{\mu_r}\right)} \times \\
&\qquad \times \left[\frac{\Gamma\left(\frac{\nu_h+1}{\mu_h}\right)}{\Gamma\left(\frac{\nu_h+2}{\mu_h}\right)}\right]^{\beta_h} \left[\frac{\Gamma\left(\frac{\nu_r+1}{\mu_r}\right)}{\Gamma\left(\frac{\nu_r+2}{\mu_r}\right)}\right]^{\beta_r} \\
\theta_{1rr}^{nm} &= \frac{\gamma\left(\frac{\nu_r+2b_r+2\beta_r+m+1}{\mu_r},\, \lambda_r l_r^{\mu_r},\, \lambda_r u_r^{\mu_r}\right)}{\gamma\left(\frac{\nu_r+2b_r+m+1}{\mu_r},\, \lambda_r l_r^{\mu_r},\, \lambda_r u_r^{\mu_r}\right)} \left[\frac{\Gamma\left(\frac{\nu_r+1}{\mu_r}\right)}{\Gamma\left(\frac{\nu_r+2}{\mu_r}\right)}\right]^{2\beta_r}
\end{aligned} \tag{16}$$

Note again the symmetry between θ_{1hh}^{nm} and θ_{1rr}^{nm} regarding exchange of $h \to r$ and $n \to m$.

Coming back to Eq. (14), the incomplete gamma functions $\gamma(A, \dots)$ in the θ's in Eq. (16) have to be again replaced by ordinary $\Gamma(A)$ and the coefficients can be pre-computed once at the start of the model run.

However, to now take into account shedding in graupel-cloud-collection, the total collided cloud mass of the spectral part where graupel mass is larger than $D_{shed,g}$ =

$\max\left(D_{wg,g}, D_{RH87}\right)$ is transferred to the rain category (index r). With $x_g(D_{shed,g}) = x_{shed,g}$, Eq. (11) then becomes

$$\left.\frac{\partial N_c}{\partial t}\right|_{coll,gc} = -\, T_{gc}^{(0,0)}\left(0,0,\infty,\infty\right) \tag{17}$$

$$\left.\frac{\partial N_g}{\partial t}\right|_{coll,gc} = 0$$

$$\left.\frac{\partial L_c}{\partial t}\right|_{coll,gc} = -\, T_{gc}^{(0,1)}\left(0,0,\infty,\infty\right)$$

$$\left.\frac{\partial L_r}{\partial t}\right|_{shed,gc} = T_{gc}^{(0,1)}\left(x_{shed,g},0,\infty,\infty\right) \tag{*}$$

$$\left.\frac{\partial L_g}{\partial t}\right|_{coll,gc} = T_{gc}^{(0,1)}\left(0,0,\infty,\infty\right) - \left.\frac{\partial L_r}{\partial t}\right|_{shed,gc} = T_{gc}^{(0,1)}\left(0,0,x_{shed,g},\infty\right) \tag{*}$$

$$\left.\frac{\partial N_r}{\partial t}\right|_{shed,gc} = \frac{\left.\frac{\partial L_r}{\partial t}\right|_{shed,gc}}{x_r(D_{r,shed})} \tag{*}$$

Shedded raindrops have a mean mass diameter $D_{r,shed}$, currently set to 1.0 mm. The starred terms have been added or modified due to shedding and can optionally be switched on.

The resulting integral from $x_{shed,g}$ to ∞ is computed by replacing the incomplete gamma functions γ in the δ's in Eq. (9) and θ's in Eq. (16) by upper incomplete gamma functions γ_u,

$$\gamma\left(A, \lambda_g\, x_{shed,g}^{\mu_g}, \infty\right) \implies \gamma_u\left(A, \lambda_g\, x_{shed,g}^{\mu_g}\right)$$

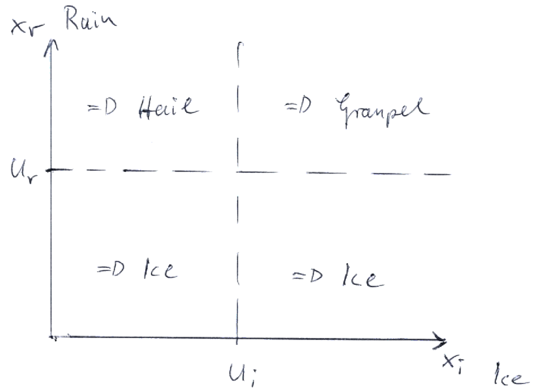
Collisions of graupel with raindrops, as well as of hail with cloud- and raindrops are treated analogously.

By default, shedding is not active in ICON. It can be activated by setting in the `/twomom_mcrph_nml/`:

- `itype_shedding_gh=2` (default: `0`)
- to specify D_{RH87}: set `D_shed_gh` to the desired diameter [m] (default: 9e-3)

It is however limited to $T_a > -10°$C. Note that for $T_a > 0°$C, the definition of $D_{shed,g}$ allows for some of the accreted water to stay at smaller collector particles, accepting the "false freezing" (latent heat release) to some degree but somewhat realistically keeping the size and fallspeed higher.

`itype_shedding_gh=1` is the more simple shedding parameterization along the lines of the 2-moment scheme in the old COSMO model. It just sheds all accreted water to raindrops with a size according to Eq. (1) below the melting level or if $T_a > -10°$C and the mean mass diameter $D(L/N)$ of graupel respectively hail is larger than a pre-definded size threshold. This size threshold can be given by the parameter `D_shed_gh` (default: 9e-3) in the `/twomom_mcrph_nml/` namelist and is assumed to be equal for graupel and hail.



- Small ice + small raindrops = ice
- Large ice + small raindrops = ice
- Small ice + large raindrops = hail
- Large ice + large raindrops = graupel

Figure 1: Conceptual sketch of the idea of splitting ice-rain-collision output into ice, graupel and hail, depending on the mass thresholds u_r and u_i. Note that this is NOT implemented in ICON.

The bigger picture of Eq. (6)

Apart from the above shedding process, the formulation Eq. (6) of collision integrals with finite integration bounds l and u could have the potential to also be applied to a more advanced parameterization of particle collision outcomes. The following is not implemented in ICON, but would in principle enable, as an example, to parameterize the riming of cloud ice by rain in a way that the resulting collision products are assigned to different frozen categories based on the sizes/masses of the mutual collision partners, instead of beeing assigned to one predefined species (graupel up to now). This is illustrated in Fig. 1, and the resulting conversion rates would be given by ($h \to i$, r=rain, i=ice, g=graupel, h=hail)

$$
\begin{aligned}
\left.\frac{\partial N_r}{\partial t}\right|_{coll,ir} &= -\,T_{ir}^{(0,0)}\,(0, 0, \infty, \infty) \\
\left.\frac{\partial N_i}{\partial t}\right|_{coll,ir} &= -\,T_{ir}^{(0,0)}\,(0, u_r, \infty, \infty) \\
\left.\frac{\partial N_g}{\partial t}\right|_{coll,ir} &= T_{ir}^{(0,0)}\,(u_i, u_r, \infty, \infty) \\
\left.\frac{\partial N_h}{\partial t}\right|_{coll,ir} &= T_{ir}^{(0,0)}\,(0, u_r, u_i, \infty) \\
\left.\frac{\partial L_r}{\partial t}\right|_{coll,ir} &= -\,T_{ir}^{(0,1)}\,(0, 0, \infty, \infty) \\
\left.\frac{\partial L_i}{\partial t}\right|_{coll,ir} &= -\,T_{ir}^{(1,0)}\,(0, u_r, \infty, \infty) \\
\left.\frac{\partial L_g}{\partial t}\right|_{coll,ir} &= T_{ir}^{(1,0)}\,(u_i, u_r, \infty, \infty)\; + T_{ir}^{(0,1)}\,(u_i, u_r, \infty, \infty) \\
\left.\frac{\partial L_h}{\partial t}\right|_{coll,ir} &= T_{ir}^{(1,0)}\,(0, u_r, u_i, \infty)\; + T_{ir}^{(0,1)}\,(0, u_r, u_i, \infty)
\end{aligned} \tag{18}
$$

In that respect, Blahak (2012) developed an even more accurate approximation of the characteristic fall speed difference, especially for the case of finite integration bounds. Motivated by a different interpretation of Eq. (13), a number of similar type approximations have been tested, but with different exponents of the functions in the integrals. It has been found that a linear combination of an approximation 2

$$\overline{\left|\Delta v_{hr}^{(n,m)}\right|}_2 \approx \left[\frac{\int\limits_{l_r}^{u_r}\int\limits_{l_h}^{u_h} f_h^2\, f_r^2\, D_h^2 D_r^2\,(v_h - v_r)^2\, x_h^{2n}\, x_r^{2m}\, dx_r\, dx_h}{\int\limits_{l_r}^{u_r}\int\limits_{l_h}^{u_h} f_h^2\, f_r^2\, D_h^2 D_r^2\, x_h^{2n}\, x_r^{2m}\, dx_r\, dx_h}\right]^{1/2} =$$

$$\left[\theta_{2hh}^{nm} v_h^2(\overline{x}_h) + \theta_{2hr}^{nm} v_h(\overline{x}_h) v_r(\overline{x}_r) + \theta_{2rr}^{nm} v_r^2(\overline{x}_r)\right]^{1/2} \quad (19)$$

with Eq. (15) leads to very good results,

$$\overline{\left|\Delta v_{hr}^{(n,m)}\right|}_4 \approx \Omega\, \overline{\left|\Delta v_{hr}^{(n,m)}\right|}_1 + (1-\Omega)\, \overline{\left|\Delta v_{hr}^{(n,m)}\right|}_2 \qquad \text{with: } \Omega = \frac{\left|v_h^2(\overline{x}_h) - v_r^2(\overline{x}_r)\right|}{v_h^2(\overline{x}_h) + v_r^2(\overline{x}_r)} \quad (20)$$

To compute the δ_2's and θ_2's, one has to replace in Eq. (9) and Eq. (16)

$$\begin{aligned}
n &\to 2n \\
m &\to 2m \\
\nu_h &\to 2\nu_h \\
\nu_r &\to 2\nu_r \\
\mu_h &\to \mu_h \\
\mu_r &\to \mu_r \\
\lambda_h l_h^{\mu_h} &\to 2\lambda_h l_h^{\mu_h} \\
\lambda_r l_r^{\mu_r} &\to 2\lambda_r l_r^{\mu_r} \quad .
\end{aligned}$$

However, this is not done at present in ICON. Here, collisions of ice or snow with raindrops always lead to the formation of graupel. This brings us to the next change which has recently been introduced. It will be described in the next section.

2 Graupel production by rain riming of cloud ice and snow

Up to now:

1. Graupel production due to riming of supercooled cloud droplets by snow and ice depends on $\alpha_{spacefilling}$.
2. Collisions of ice/snow with supercooled rain drops always leads to graupel.
3. For ambient temperature $T_a > 0°$C, there is full shedding of accreted cloud/rain water to the rain category, preserving mean rain size $\overline{x}_r$.

Bullet 2 leads to overly strong graupel production. Not every large snow flake collided with a tiny rain drop should be called graupel afterwards.

Idea:

- Parameterize graupel production by collisions of cloud ice and snow with supercooled rain by a bulk density criterion.
- If an ice or snow particle collides with a supercooled drop, the resulting rimed particle should not be converted to graupel per se, but only if it's bulk density ρ_{coll} is closer to the bulk density of graupel $\rho_{b,g}$ than to the original species $\rho_{b,y}$, $y \in i, s$.
- **In other words:** the decision whether the rimed mass- and number densities should be added to species y or graupel depends on ρ_{coll}.

For one such binary collision, we can define:

Collided mass: $x_{coll} = x_y + x_r$

Collided diam.: $D_{coll} = \max[D_y(x_y), D_r(x_r)] + w_D \min[D_y(x_y), D_r(x_r)]$

with $w_D \in [0, 1]$ (default: 0.5) to represent some overlap

Collided bulk density: $\rho_{coll} = \dfrac{x_{coll}}{\frac{\pi}{6} D_{coll}^3}$

Diam. of y-type particle having mass x_{coll}: $D_{y,coll} = D_y(x_{coll})$

Bulk density of this y-type particle: $\rho_{y,coll} = \dfrac{x_{coll}}{\frac{\pi}{6} D_{y,coll}^3}$

Diam. of graupel particle having mass x_{coll}: $D_{g,coll} = D_g(x_{coll})$

Bulk density of this graupel particle: $\rho_{g,coll} = \dfrac{x_{coll}}{\frac{\pi}{6} D_{g,coll}^3}$

where $D(x)$ is the max. diameter of the non-spherical particle defined by $D = a_{geo} x^{b_{geo}}$ and tbulk density is defined regarding the enclosing sphere.

Then we define a limiting bulk density for the decision for the conversion to graupel:

$$\rho_{lim} = (1 - w_{lim})\rho_{y,coll} + w_{lim}\rho_{g,coll}$$
$$\text{with } w_{lim} \in [0, 1] \quad \text{(default: 0.5)}$$

Note: Depending on the mass size relations and on x_{coll}, $\rho_{y,coll}$ can be smaller or larger than $\rho_{g,coll}$.

Conversion happens if ρ_{coll} is nearer to $\rho_{g,coll}$ than ρ_{lim}:

If $\rho_{y,coll} < \rho_{g,coll}$

Convert collided fraction of y and rain to graupel if $\rho_{coll} > \rho_{lim}$

Else

Convert if $\rho_{coll} < \rho_{lim}$

Because application of this criterion on a single-particle level in the collision integrals is mathematically not feasible, we approximate and use the mean mass particles:

Mean mass of species y: $\overline{x}_y = \dfrac{q_y}{n_y}$

Mean mass of rain: $\overline{x}_r = \dfrac{q_r}{n_r}$

Collided mass: $\overline{x}_{coll} = \overline{x}_y + \overline{x}_r$

Bulk density y: $\rho_{y,coll} = \dfrac{\overline{x}_{coll}}{\frac{\pi}{6} D_y(\overline{x}_{coll})^3}$

Bulk density graupel: $\rho_{g,coll} = \dfrac{\overline{x}_{coll}}{\frac{\pi}{6} D_g(\overline{x}_{coll})^3}$

This new ρ_{coll} criterion can be activated and configured by setting in ICON's `/twomom_mcrph_nml/`:

- `llim_gr_prod_rain_riming = .TRUE.` (default: `.FALSE.`)
- to specify w_D: set `wgt_D_coll_limgrprod` to a value between 0.0 and 1.0 (default: 0.5)
- to specify w_{lim}: set `wgt_rho_coll_limgrprod` to a value between 0.0 and 1.0 (default: 0.5)

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