

# Simulation of hail processes with a state-of-the-art 2-moment cloud microphysical scheme

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- **Hail formation on the end of a long process chain:**

Triggering of convective cells, smaller cloud- and precipitation particles form, under favorable conditions growth of smaller ice particles to hailstones by collisions with supercooled droplets.

- **Simulation in an NWP-model still a big problem, because relevant processes and time-/space scales only marginally resolved.**

- **Challenge for the microphysics parameterization:**

Compromise between physical accuracy and efficiency

→ Use of a 2-moment (bulk) cloud microphysical scheme (in this case Seifert and Beheng, 2006) enhanced by an additional hail particle class (Blahak, 2008; Noppel, 2010)



- On hail formation
- Parameterization within the 2-moment-scheme coupled to the COSMO-Model
- Case studies
  - 2 real cases,  $dx = 1$  km
  - Idealized study concerning influence of grid distance





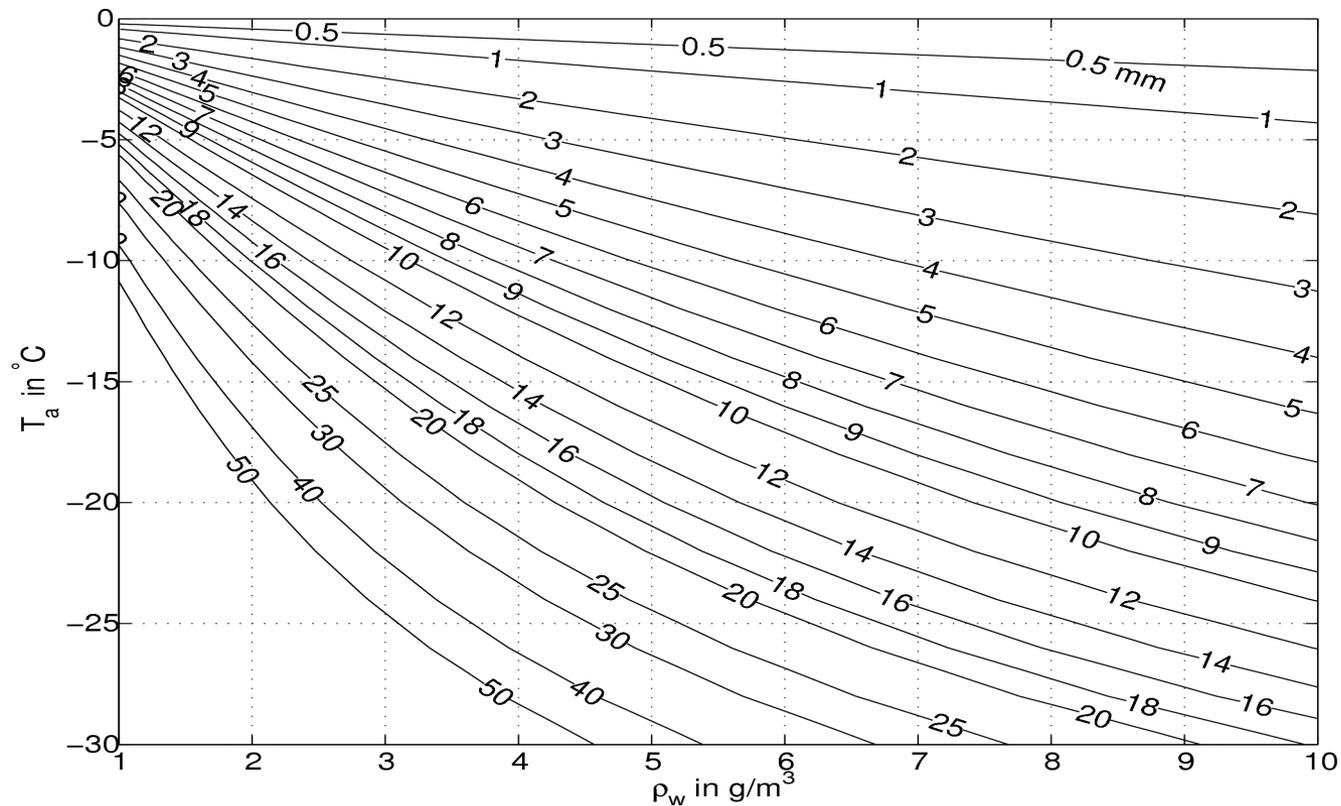
- **Ingredients:** Updrafts, supercooled droplets, hail embryos (e.g. frozen droplets, graupel, other ice particles)
- Hailstones grow large mainly by collection of supercooled droplets (riming)
- **Large stones** by **abundance of supercooled droplets** and/ or **long residence time** within such an environment
- Dry growth / wet growth  
( „intermediate“ / „high“ bulk density )
- **Also decisive: Ratio of available supercooled water to number of embryos / hailstones (Competition situation)**





## ■ Dry / wet growth:

Regime separation diameter for graupel,  $p = 600$  hPa

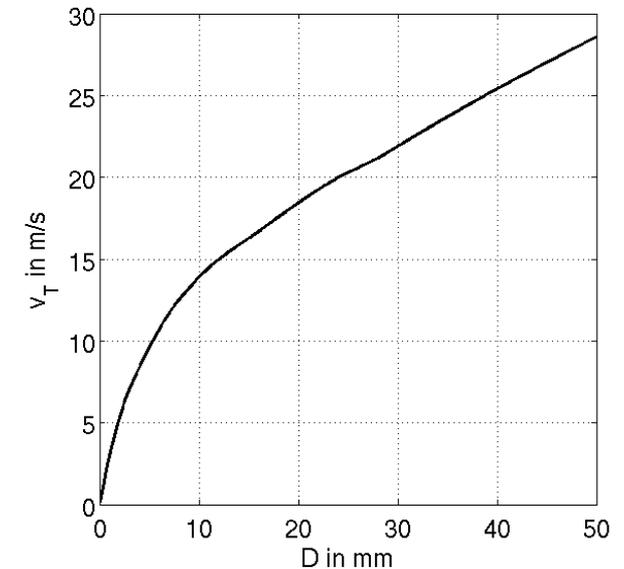




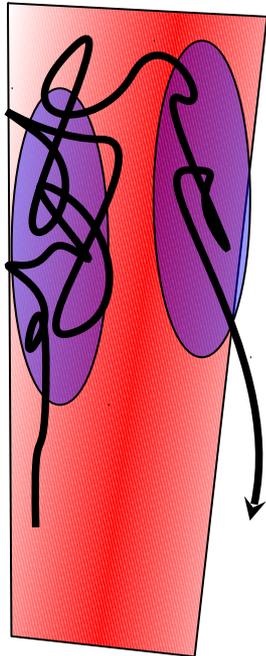
## Updraft structures and formation of large hail

### Hail term. fall speed

$$\rho_l = 1.0 \text{ kg/m}^3, \rho_b = 750 \text{ kg/m}^3$$

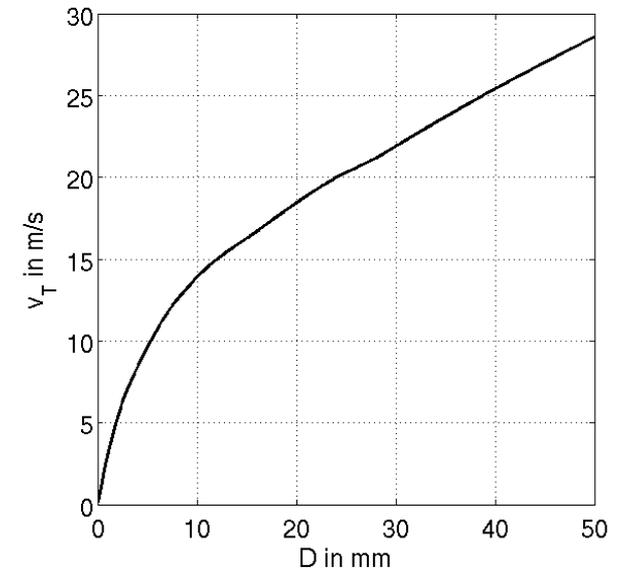


## Updraft structures and formation of large hail

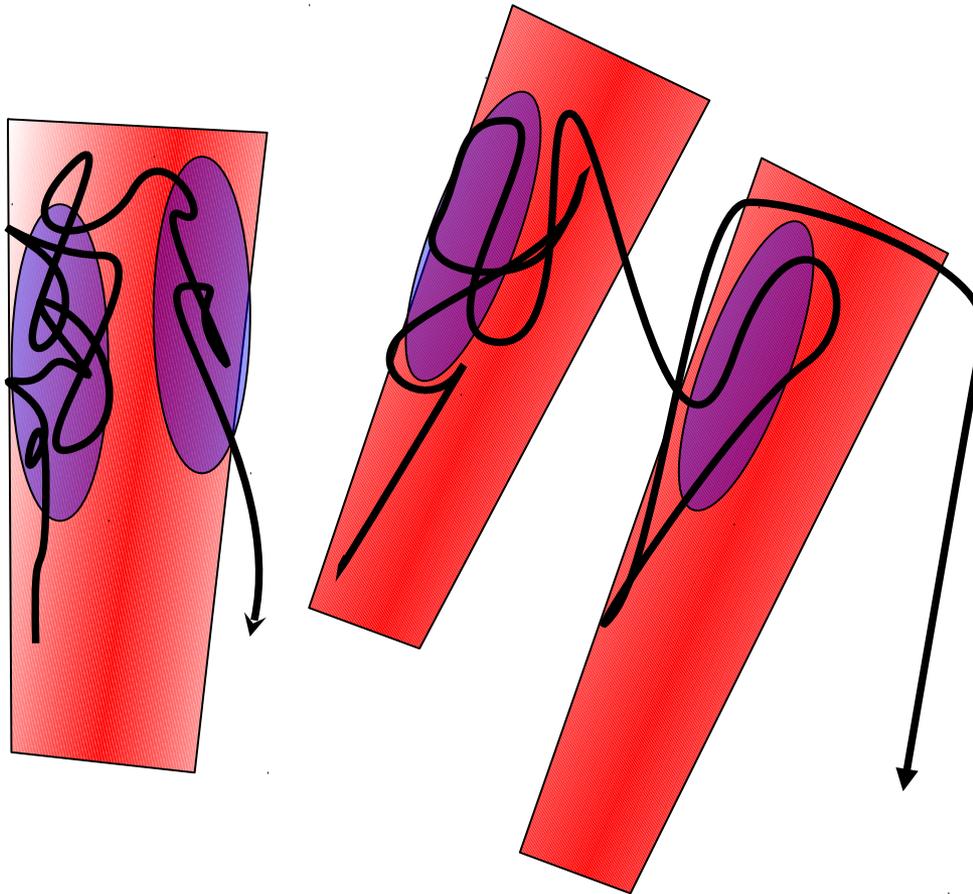


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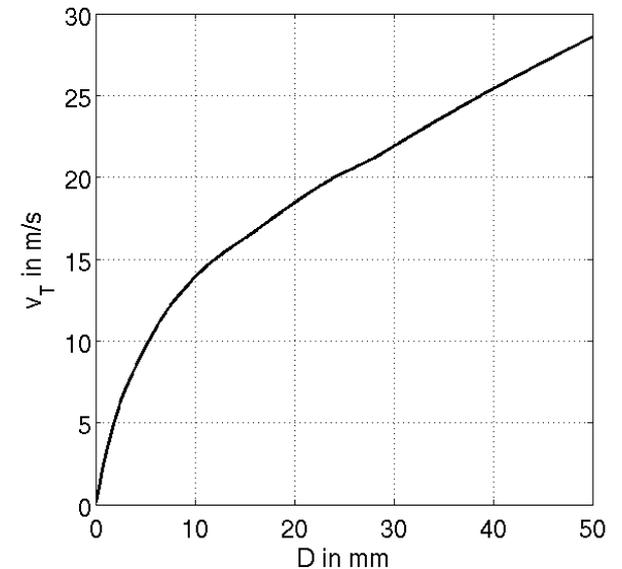


## Updraft structures and formation of large hail



Hail term. fall speed

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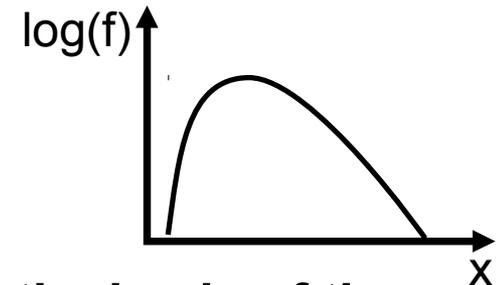
# 2-moment scheme



- Hydrometeor classes: cloud droplets, rain, cloud ice, snow, graupel, hail („simple“ extention, Noppel et al., 2006)

- For each class, budget equations for N and Q

$$N = \int_0^{\infty} f(x) dx \quad Q = \int_0^{\infty} x f(x) dx$$



- Parameterization of microphysical processes on the basis of the spectral budget equation(s) assuming a generalized gamma-distribution for f(x):

$$f(x) = N_0 x^{\nu} \exp(-\lambda x^{\mu})$$

$$\nu, \mu = \text{const.}, \quad \lambda = \left[ \frac{\Gamma(\frac{\nu+1}{\mu})}{\Gamma(\frac{\nu+2}{\mu})} \frac{Q}{N} \right]^{-\mu}, \quad N_0 = \frac{\mu N}{\Gamma(\frac{\nu+1}{\mu})} \lambda^{\frac{\nu+1}{\mu}}$$

- Coupling to the COSMO-Model





## Considered hail processes (new):

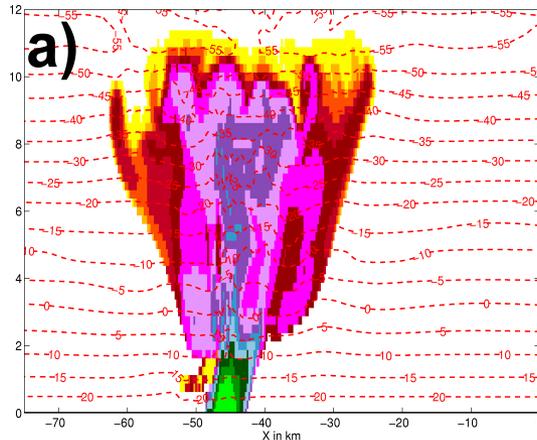
- Dry growth by collection of supercooled droplets
- ~~Wet Growth~~
- ~~Shedding of droplets~~
- Hail embryos: frozen raindrops
- Melting (instantaneous shedding of melt water into rain)
- Dynamic processes (Seeder-Feeder, recirculation, „balancing“ in updrafts) are taken into account, insofar the setup of the COSMO-Model allows that.
- „Competition“ for the available supercooled liquid



# 2-moment scheme

## First experiences in 2006

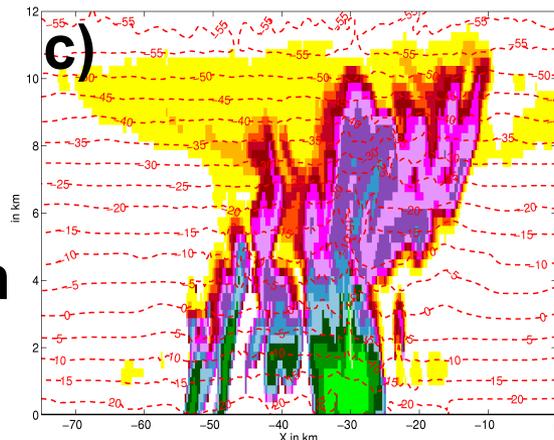
00:50 h



Reflectivity in Ice region  
much too low

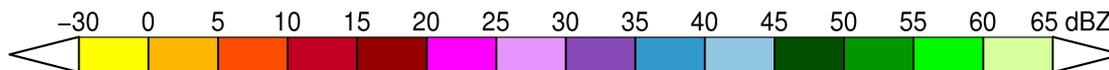
Graupel and „hail“  
much too small

02:00 h



High sensitivity of  
precipitation efficiency  
to aerosol regime  
(seemed unrealistic)

(2D idealized,  
warm bubble)



# 2-moment scheme



## Changes:

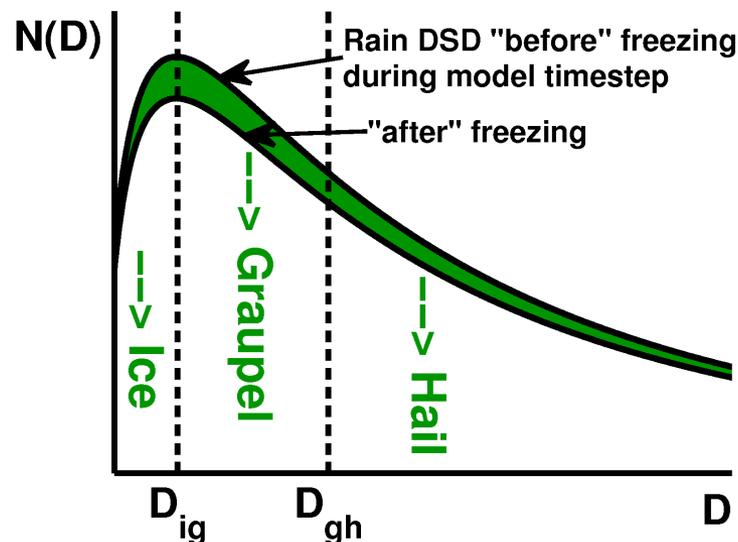
- 1) Assumptions about size-mass- and fallspeed-mass-relations altered (ice, snow, graupel, hail)



## Changes:

1) Assumptions about size-mass- and fallspeed-mass-relations altered (ice, snow, graupel, hail)

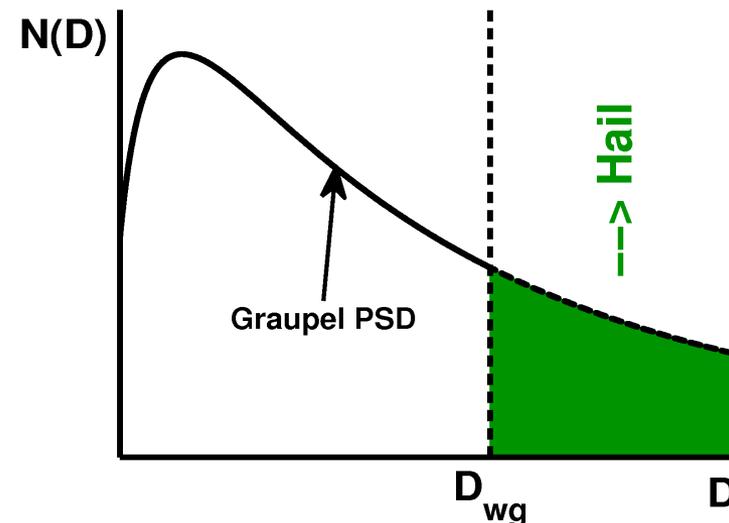
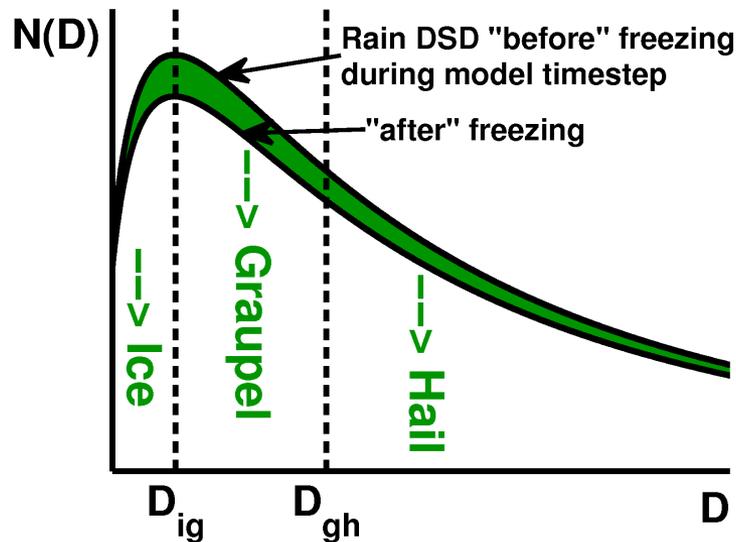
2) raindrop freezing changed:



# 2-Momenten-Schema

## Changes:

- 1) Assumptions about size-mass- and fallspeed-mass-relations altered (ice, snow, graupel, hail)
- 2) raindrop freezing changed:
- 3) New: conv. graupel  $\rightarrow$  hail by wet growth of graupel

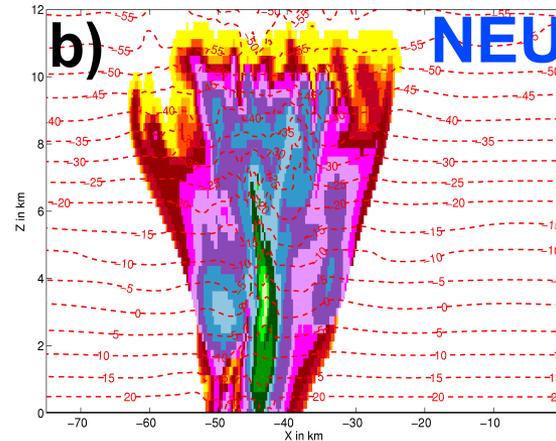
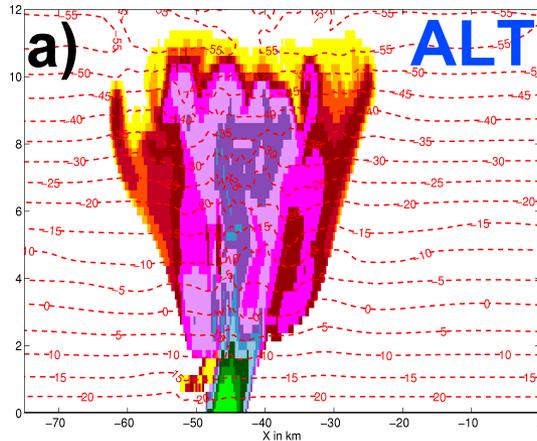


# 2-moment scheme

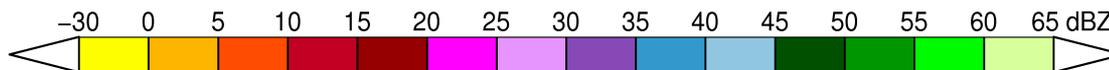
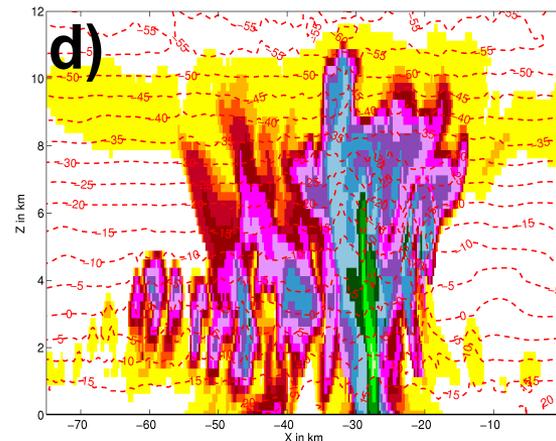
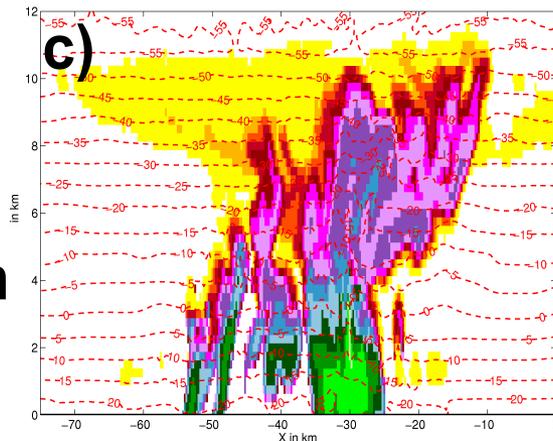


## Effect of changes:

00:50 h



02:00 h



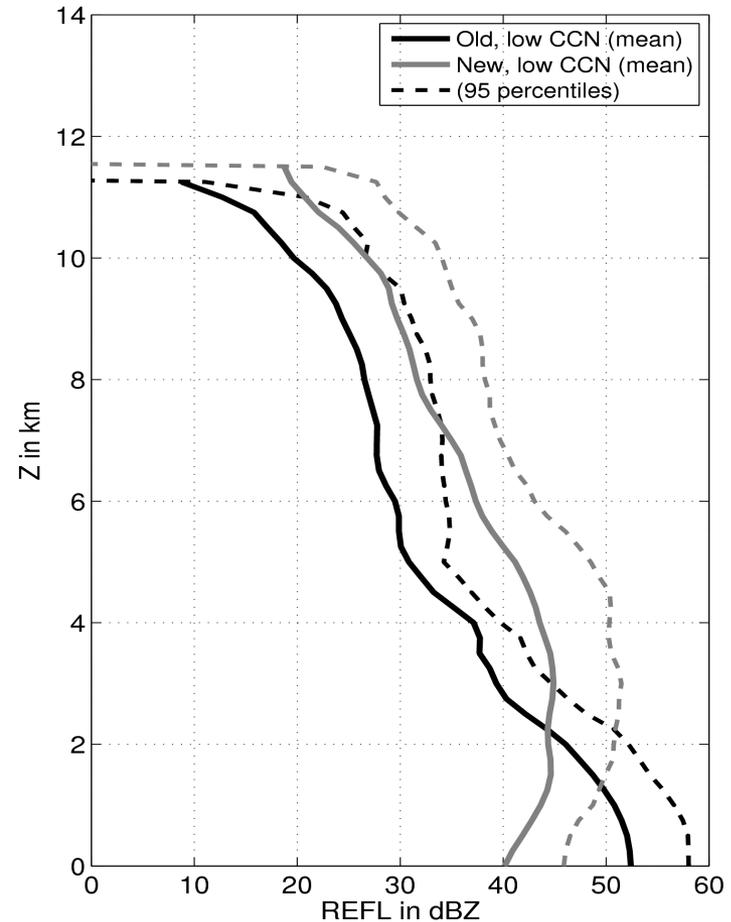
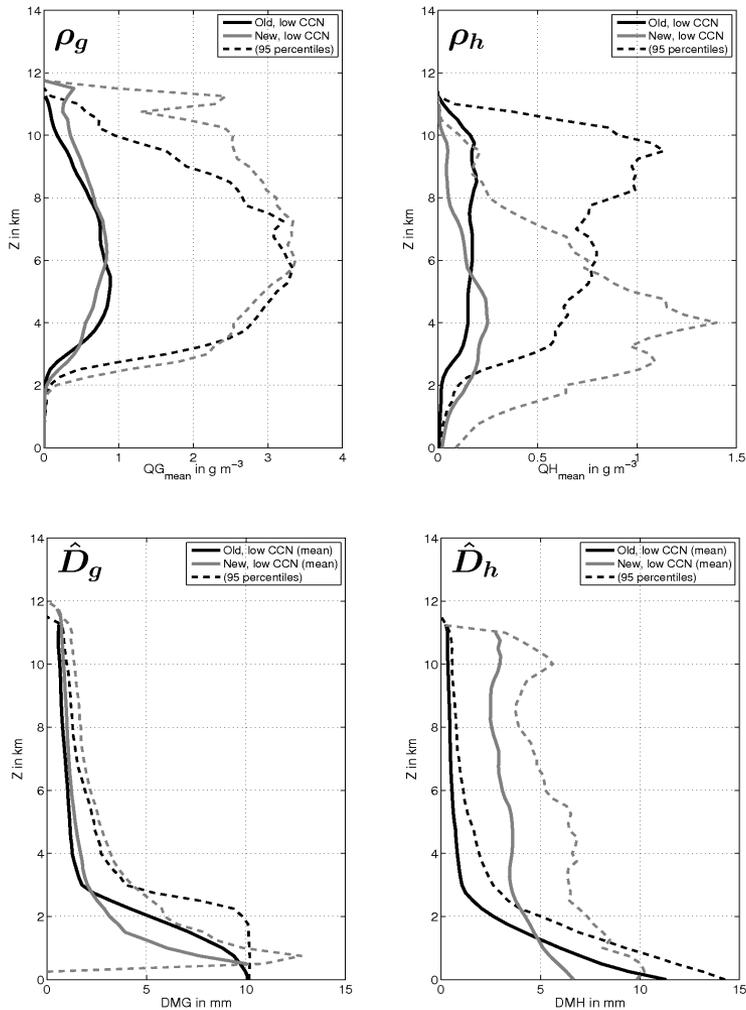
(2D idealized,  
warm bubble)



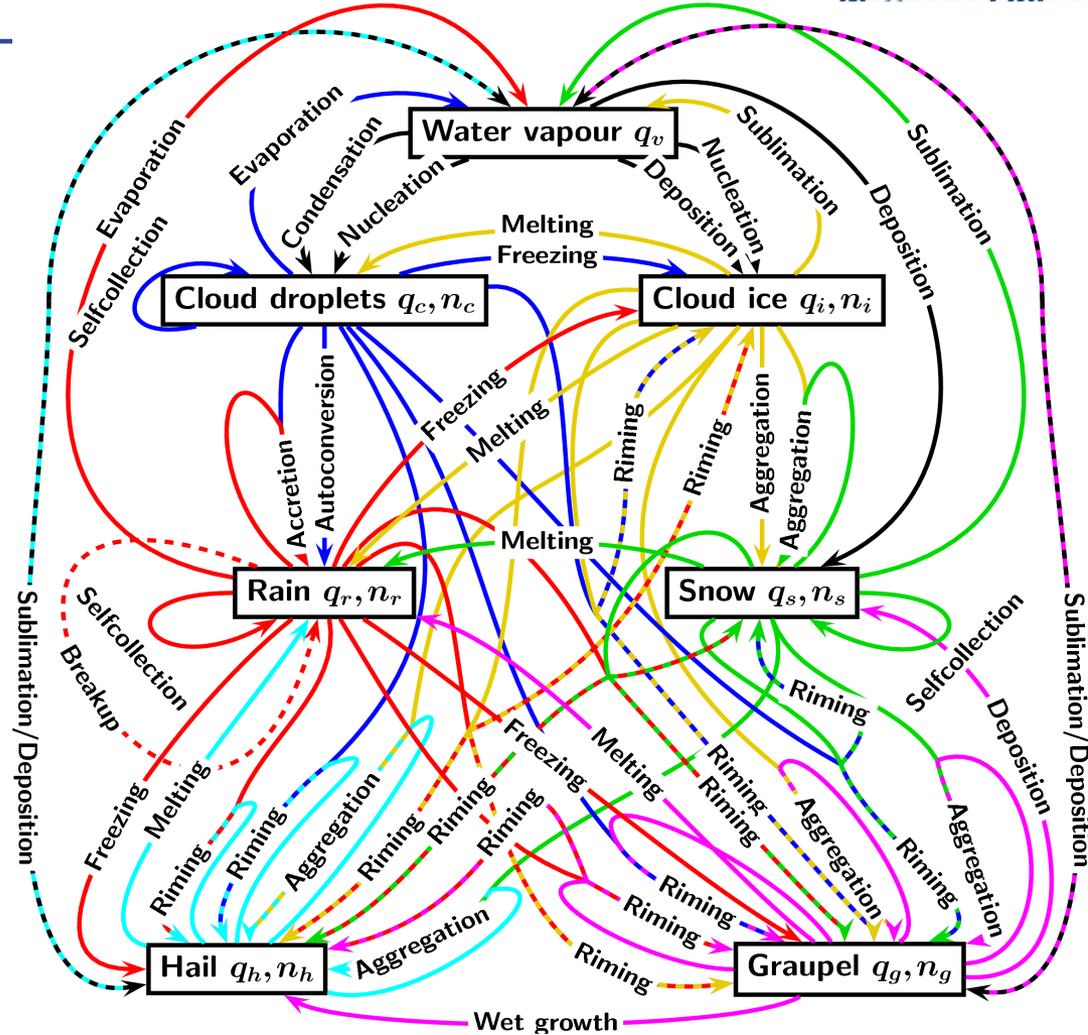
# 2-moment-scheme



## Effect of changes:



# 2-moment scheme

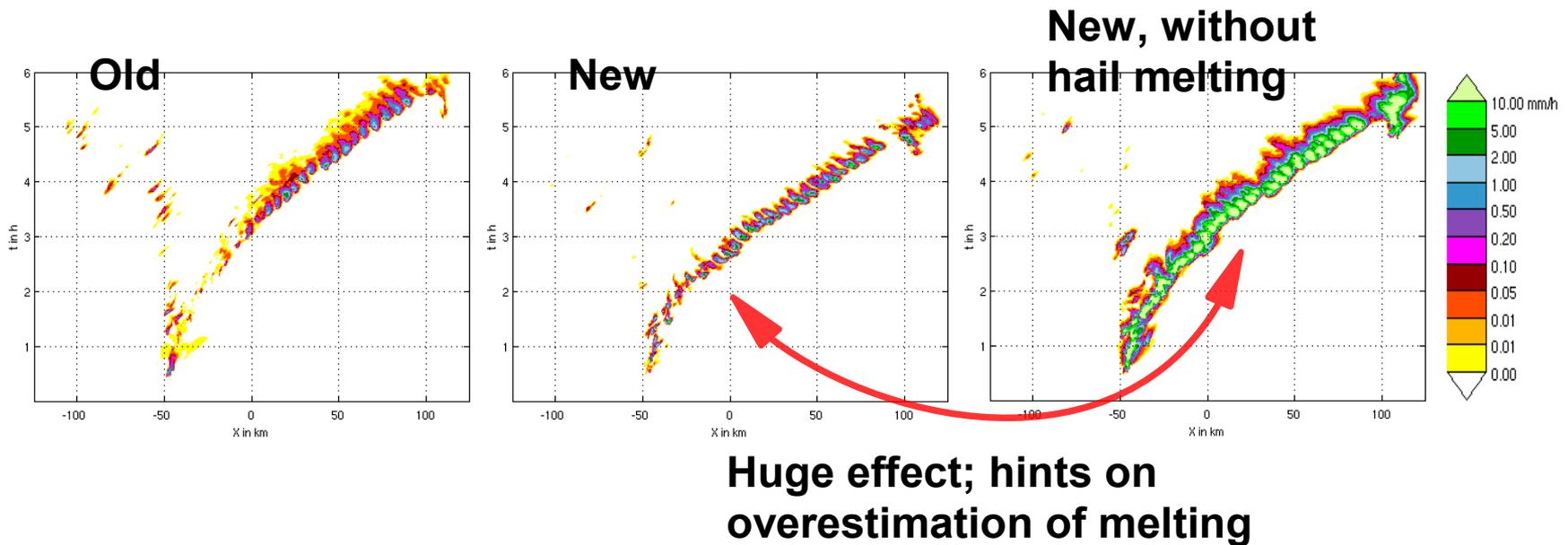


Seifert and Beheng (2006a,b), Seifert et al. (2006), Noppel et al. (2006), Blahak (2008), Seifert (2008)



## BUT: problem with too fast melting!

Hovmöller-Plots of the hail rate:





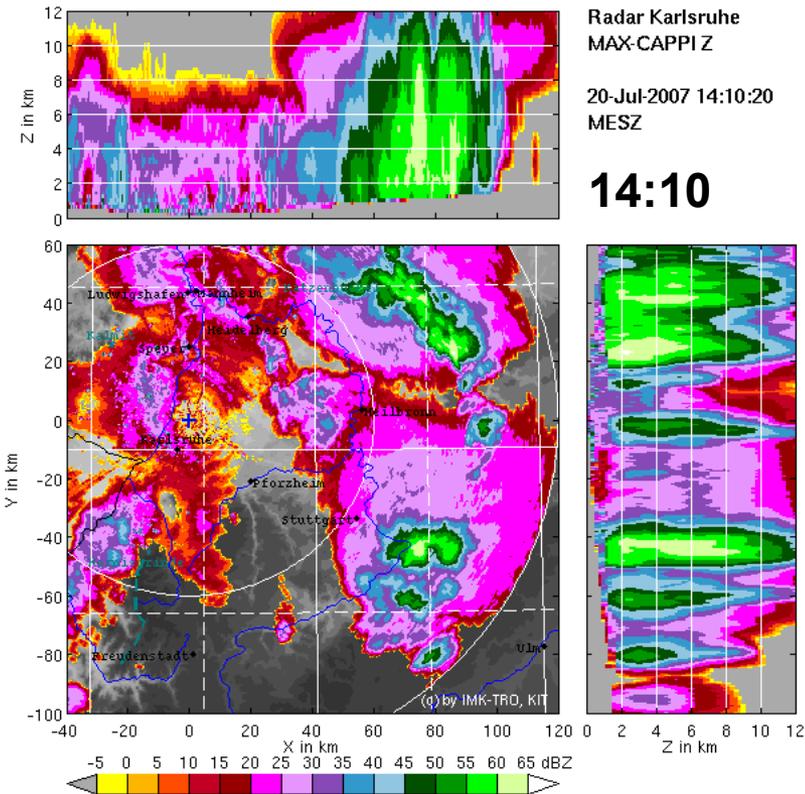
## Considered hail processes (new):

- (Re-tuning of size-mass- and fallspeed-mass-relation parameters)
- Dry growth by collection of supercooled droplets
- ~~Wet Growth~~
- ~~Shedding of droplets~~
- Hail embryos: „large“ frozen raindrops + wet growing graupel
- Melting (but is at the moment systematically overestimated)
- Dynamic processes (Seeder-Feeder, recirculation, „balancing“ in updrafts) are taken into account, insofar the setup of the COSMO-Model allows that.
- „Competition“ for the available supercooled liquid

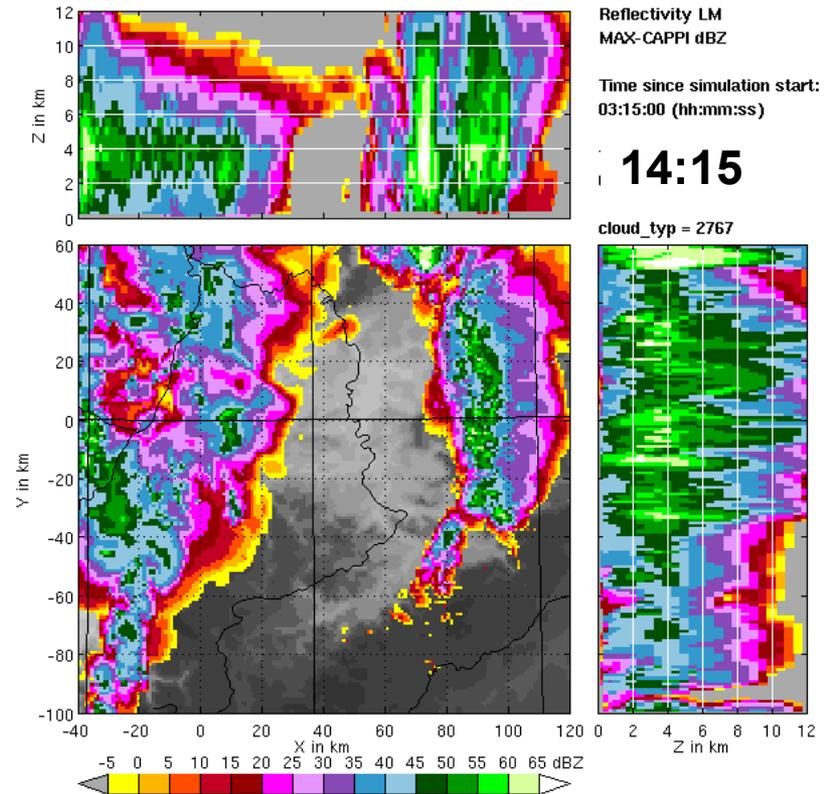


# Case study: 20.7.2007

## Radar Karlsruhe

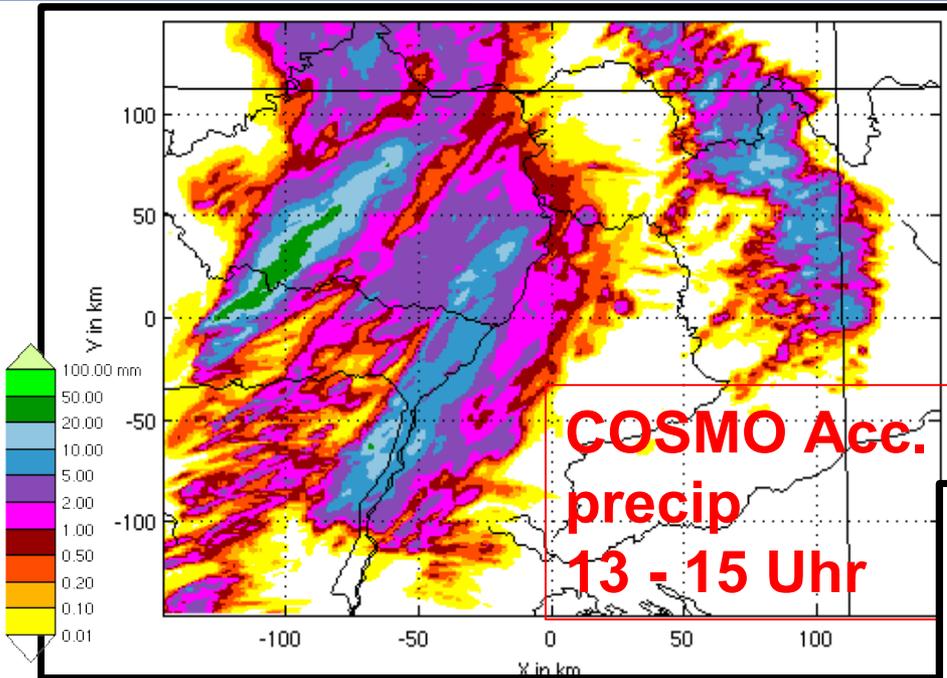


## COSMO - 2MOM

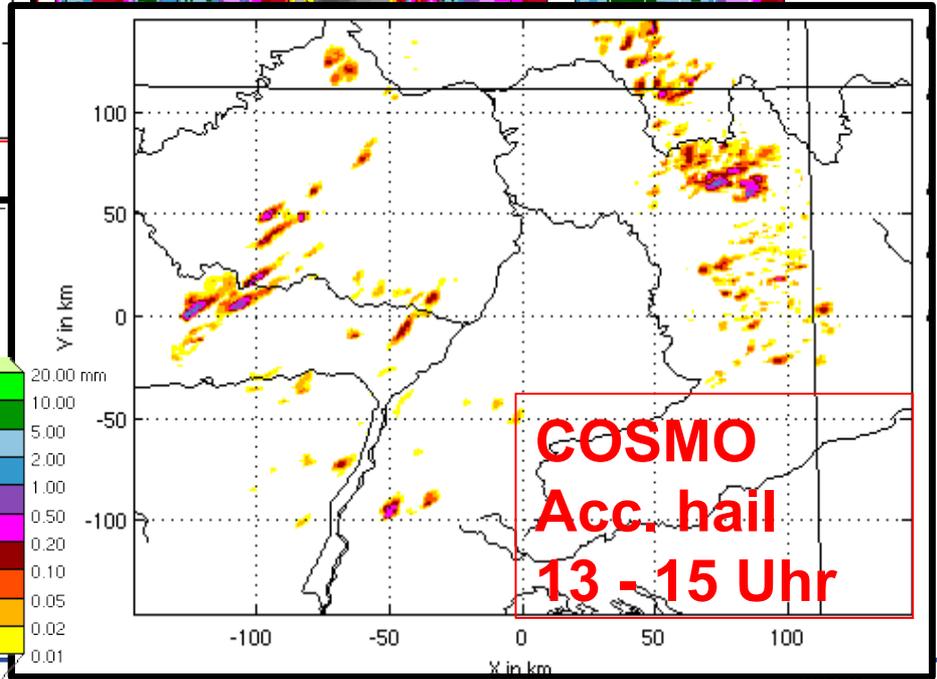
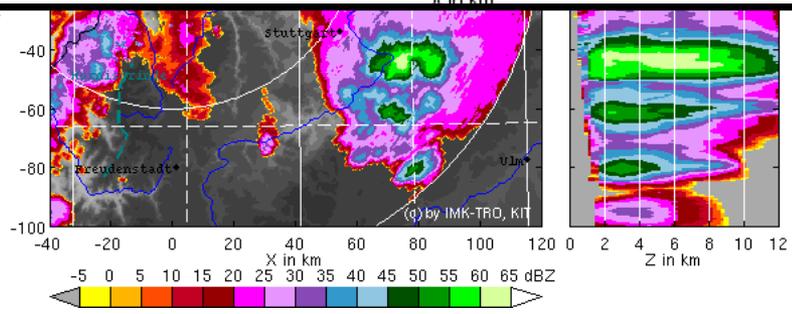
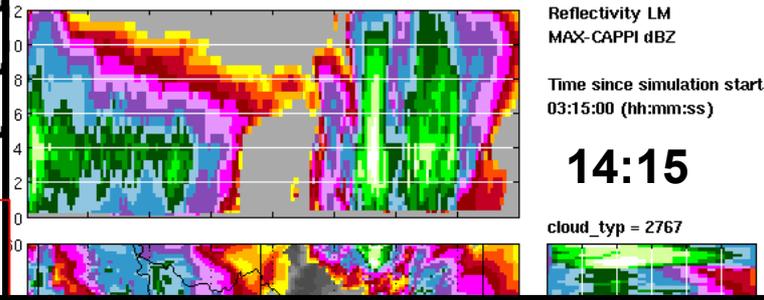


(dx = 1 km, nested into COSMO-DE, LES turbulence scheme, „low CCN“)

# Case study: 20.7.2007



## COSMO - 2MOM



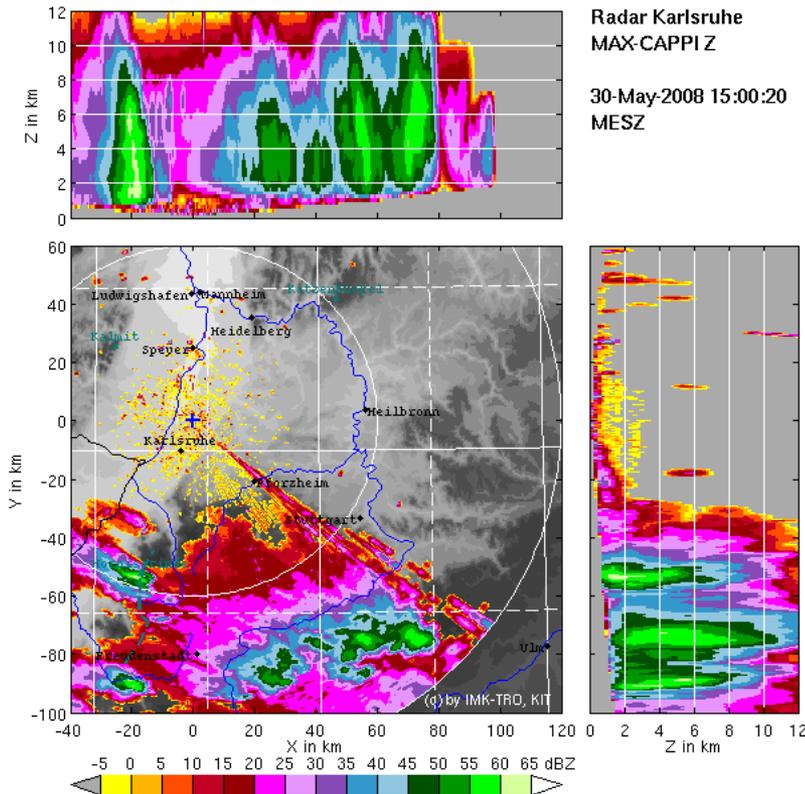
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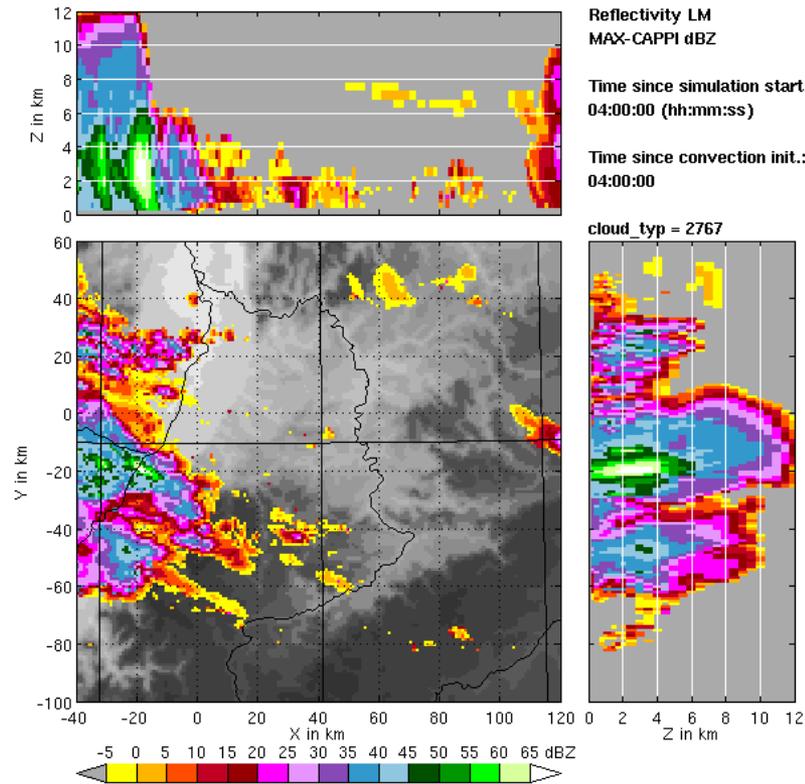
# Case study: 30.5.2008



## Radar Karlsruhe



## COSMO - 2MOM



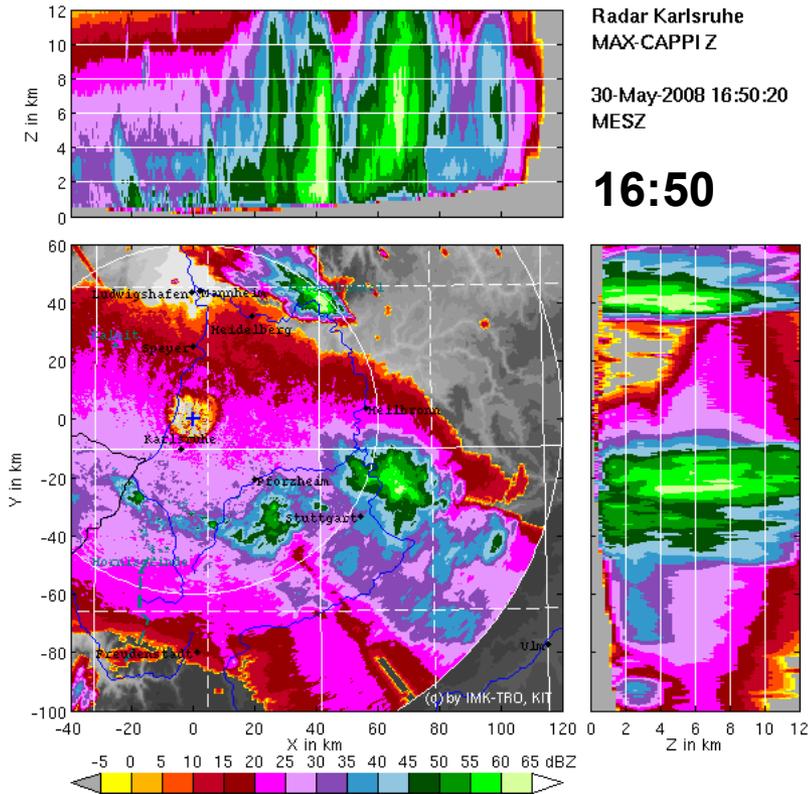
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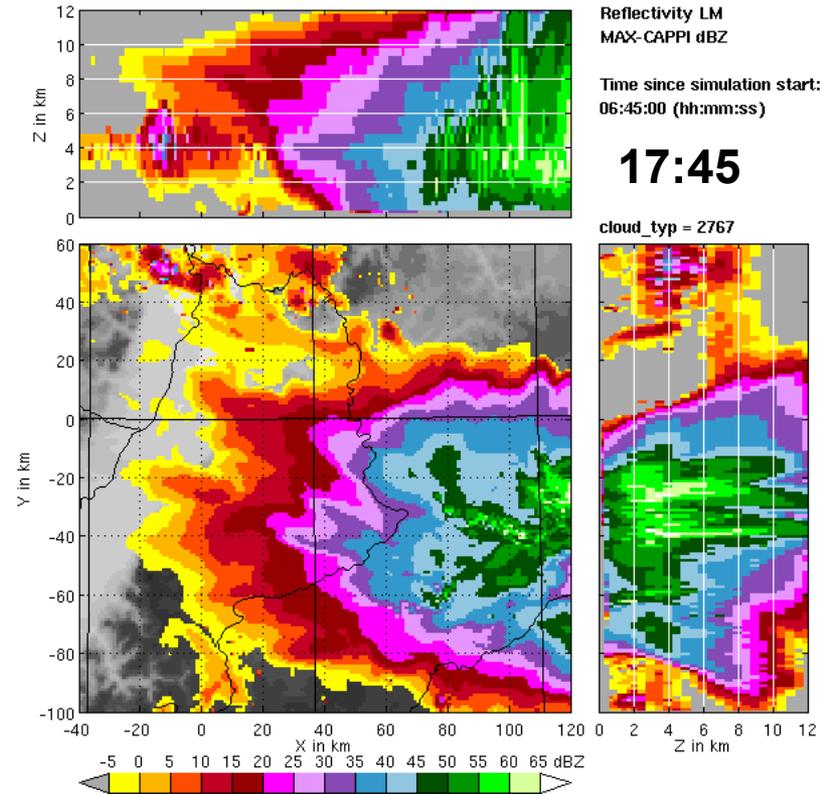
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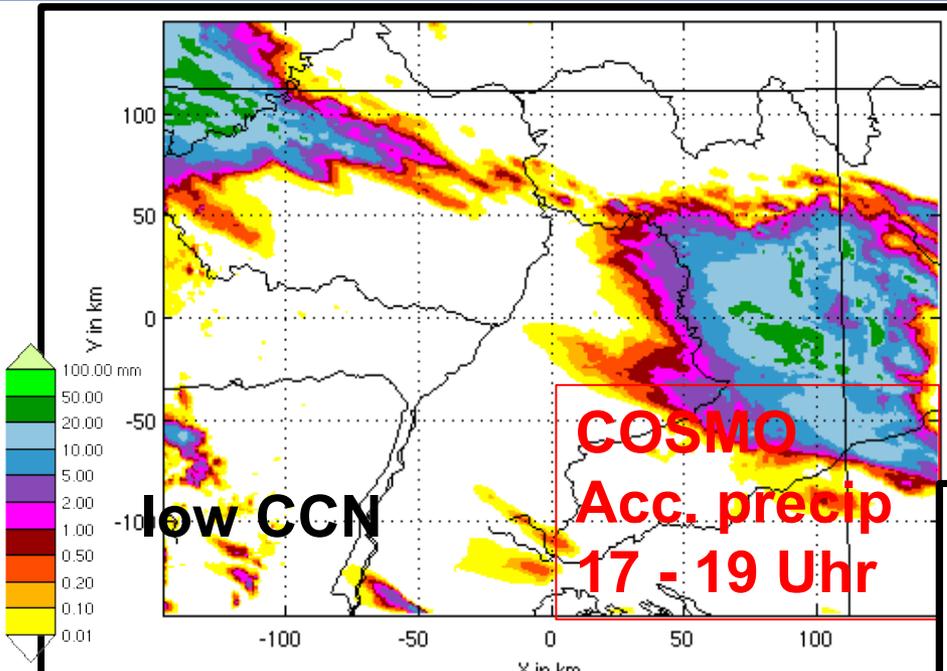
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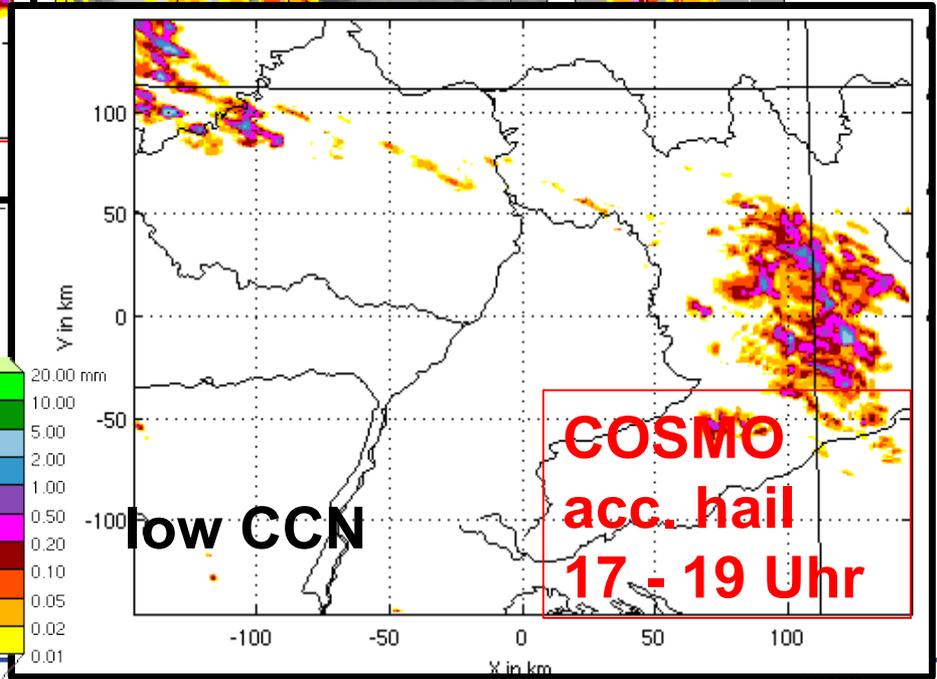
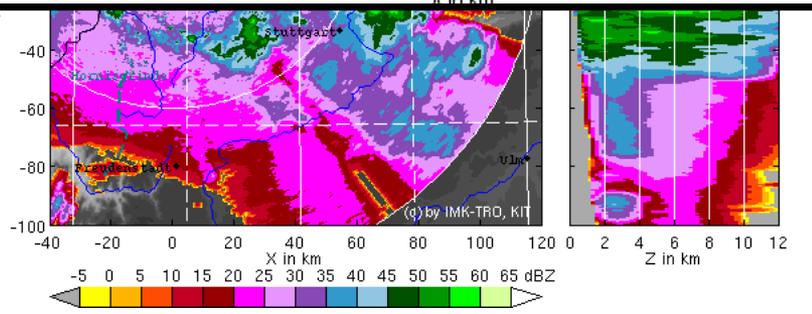
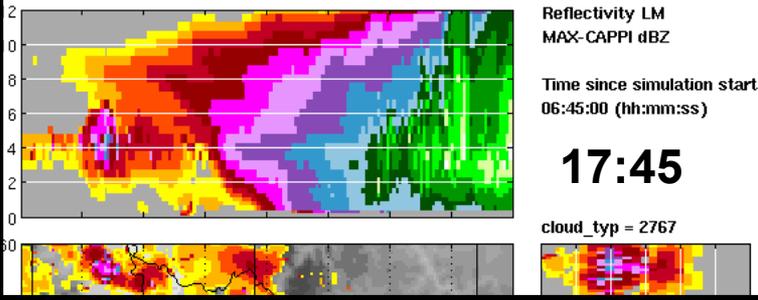
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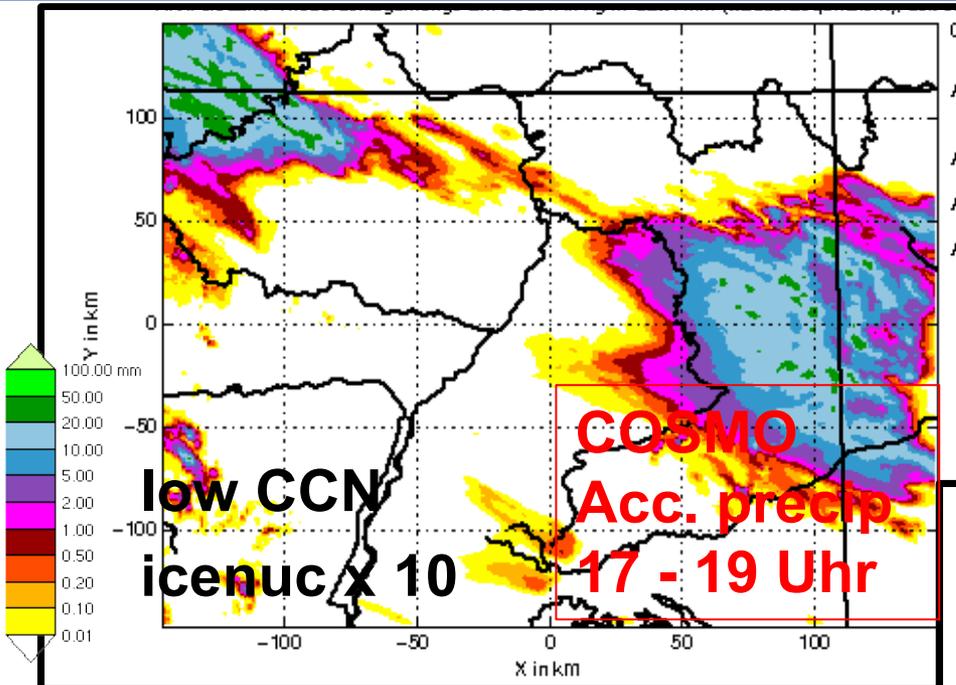
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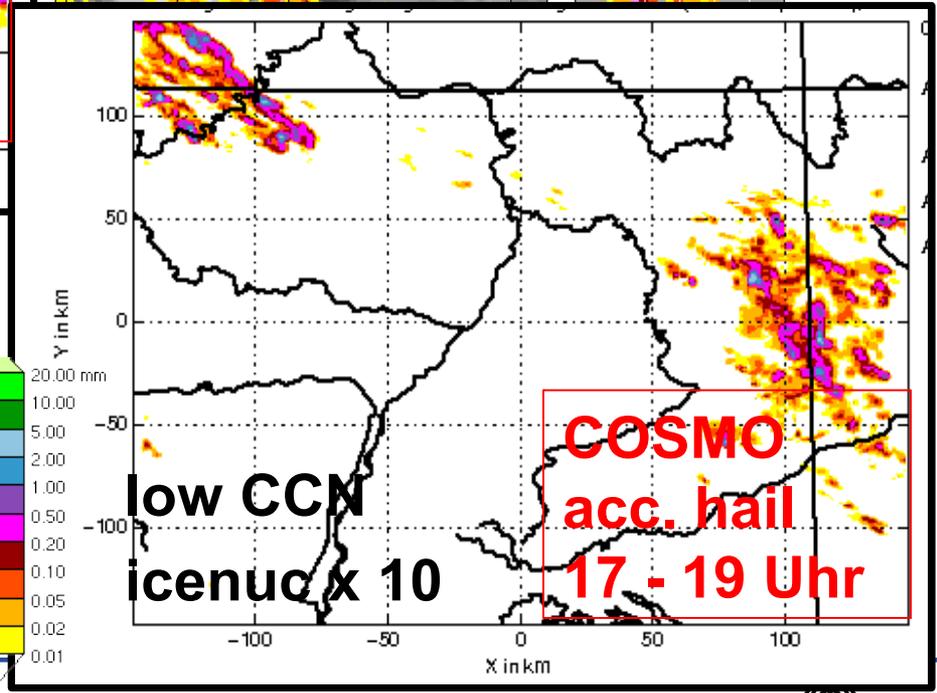
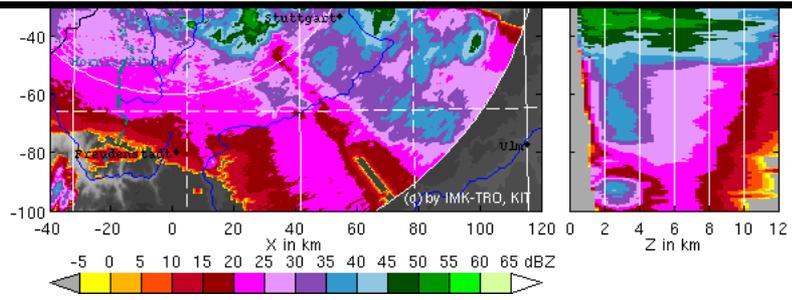
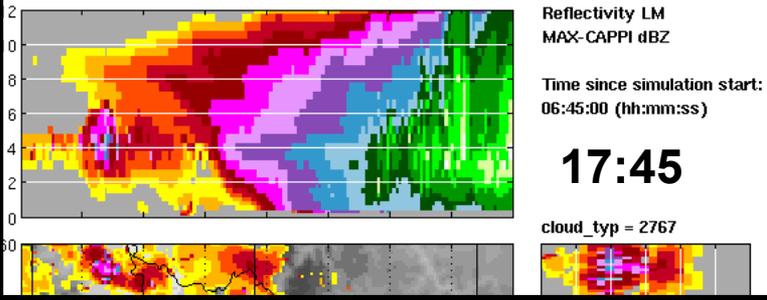
(dx = 1 km, nested into COSMO-DE, LES)



# Case study: 30.5.2008

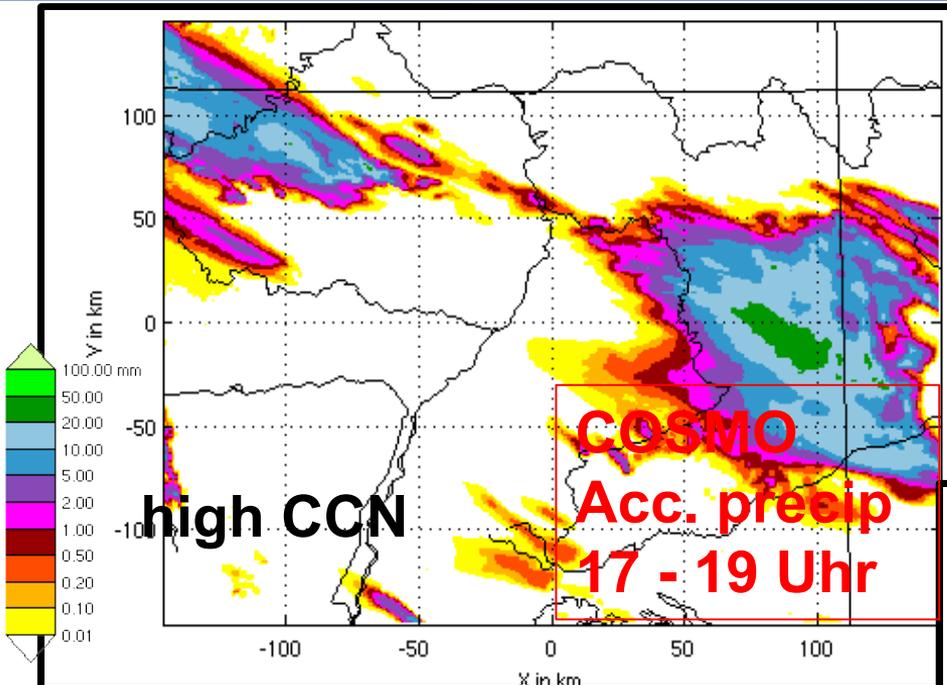


## COSMO - 2MOM

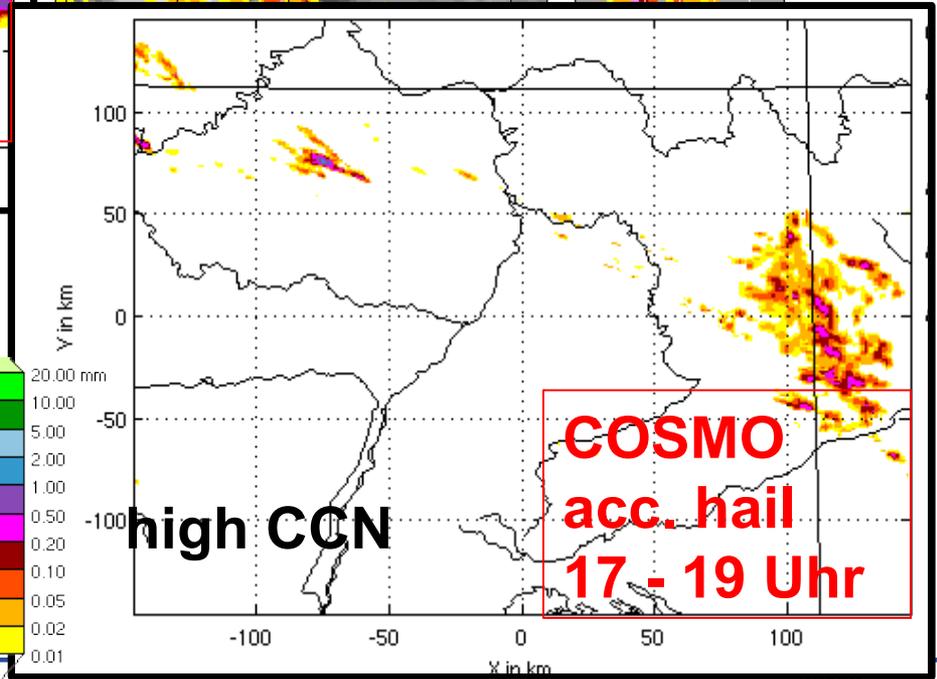
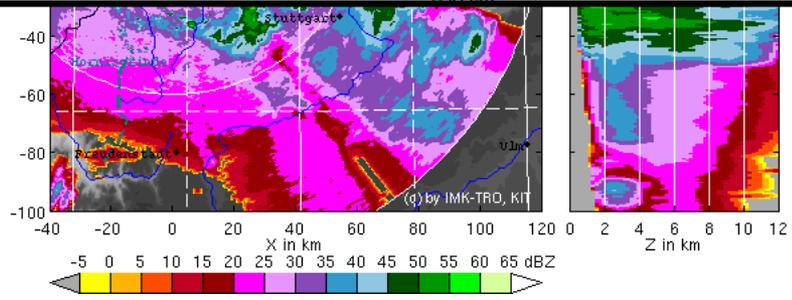
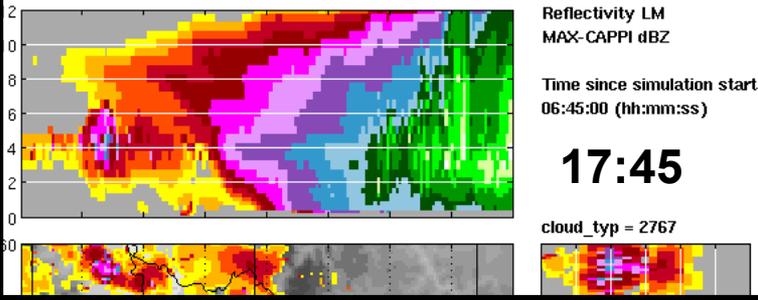


(dx = 1 km, nested into COSMO-DE, LES)

# Case study: 30.5.2008



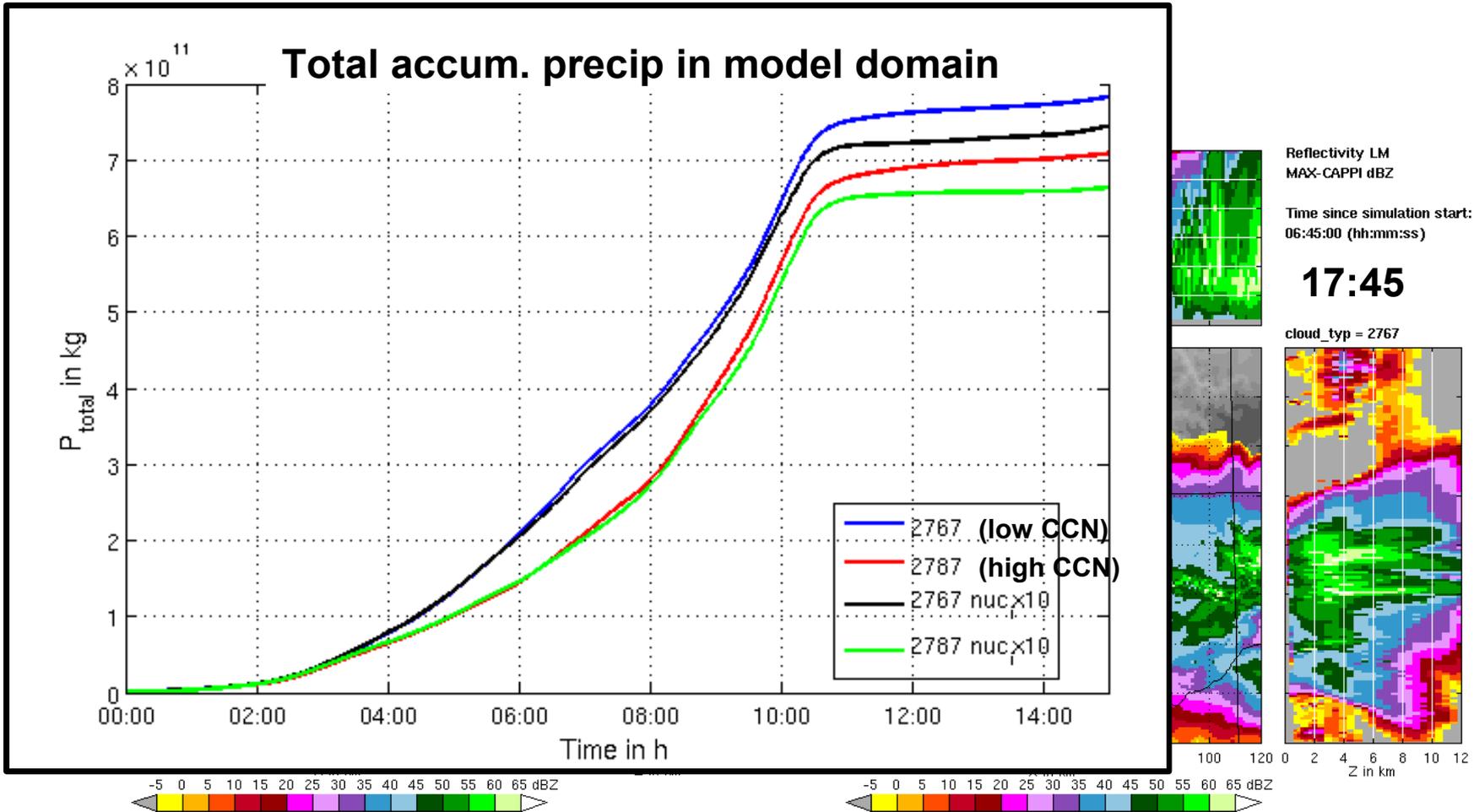
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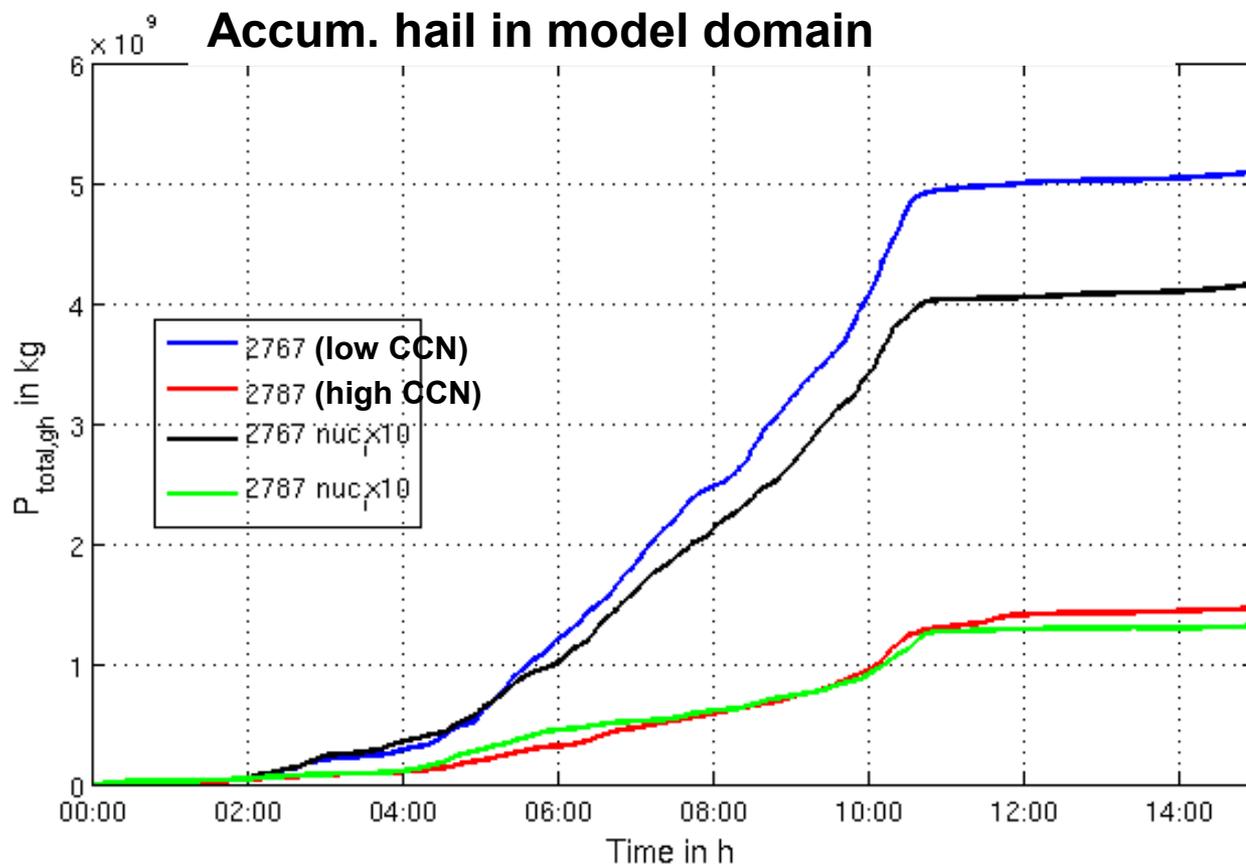
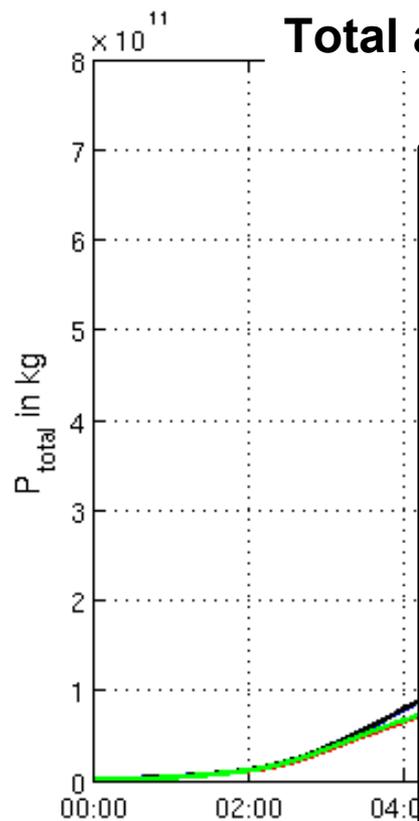
# Case study: 30.5.2008



(dx = 1 km, nested into COSMO-DE, LES turbulence scheme, „low CCN“)



# Case study: 30.5.2008



(dx = 1 km, nested



# Case study: 28.6.2006

## „Villingen-Schwenningen-Zelle“

Deutscher Wetterdienst  
Wetter und Klima aus einer Hand

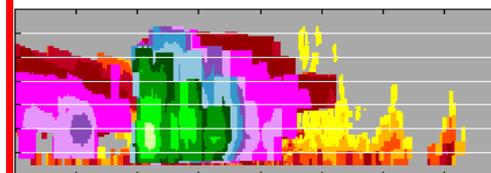


- ANTISTORM-project (Antropogenic Aerosols Invigorating Convective Storms)
- Golden case: Hail event in Villingen-Schwenningen on 28.6.2006
- Supercell developing over Black Forest, splitting; right-mover is longlived and brings a lot of big hail over the City
- Simulations with HUCM in 2D (Khain et al.) and COSMO in 2D and 3D
- COSMO 3D results published in Atmospheric Research:

Simulations of a hailstorm and the impact of CCN using an advanced two-moment cloud microphysical scheme

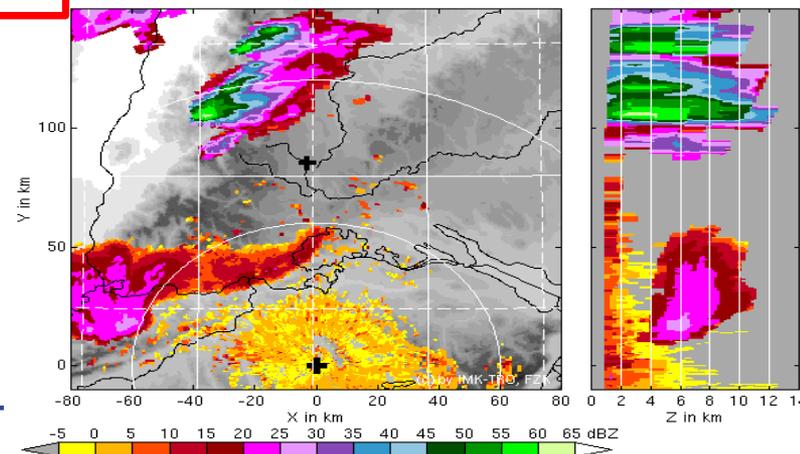
Heike Noppel<sup>a,\*</sup>, Ulrich Blahak<sup>a</sup>, Axel Seifert<sup>b</sup>, Klaus D. Beheng<sup>a</sup>

<sup>a</sup> Institut für Meteorologie und Klimaforschung, Universität/Forschungszentrum Karlsruhe, P.O. Box 3640, D-76021 Karlsruhe, Germany  
<sup>b</sup> German Weather Service, Frankfurter Str. 135, D-63067 Offenbach, Germany



Radar Albis  
MAX-CAPPI Z

28-Jun-2006 16:05:00  
UTC



# Case study: 28.6.2006

## „Villingen-Schwenningen-Zelle“

Deutscher Wetterdienst  
Meteo und Klima aus einer Hand

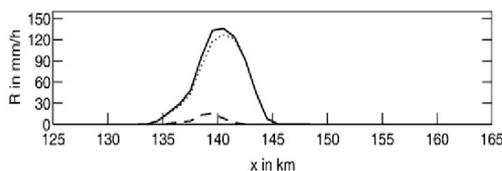
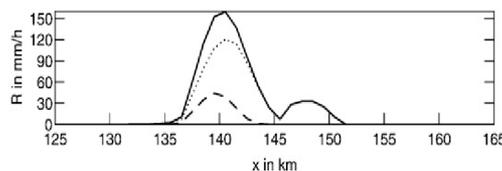
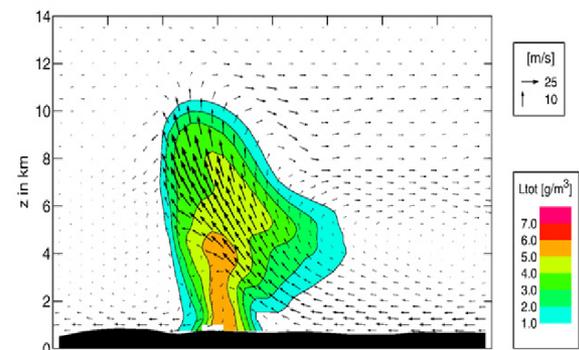
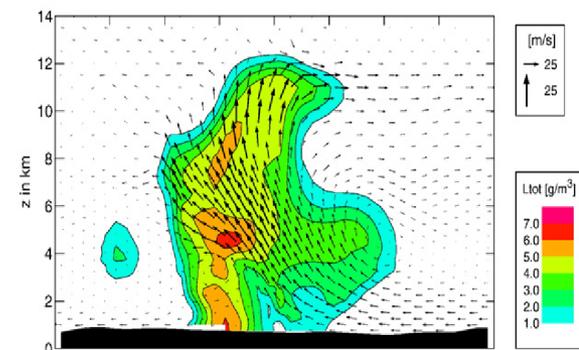
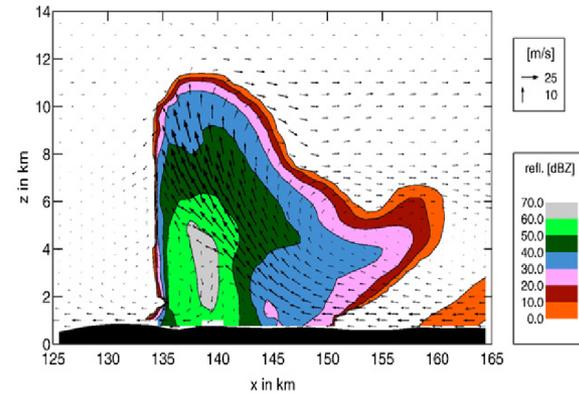
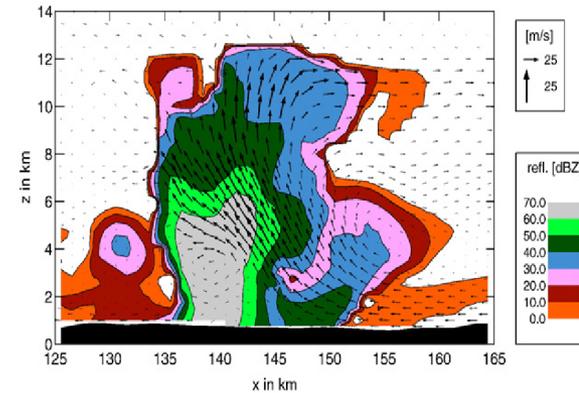
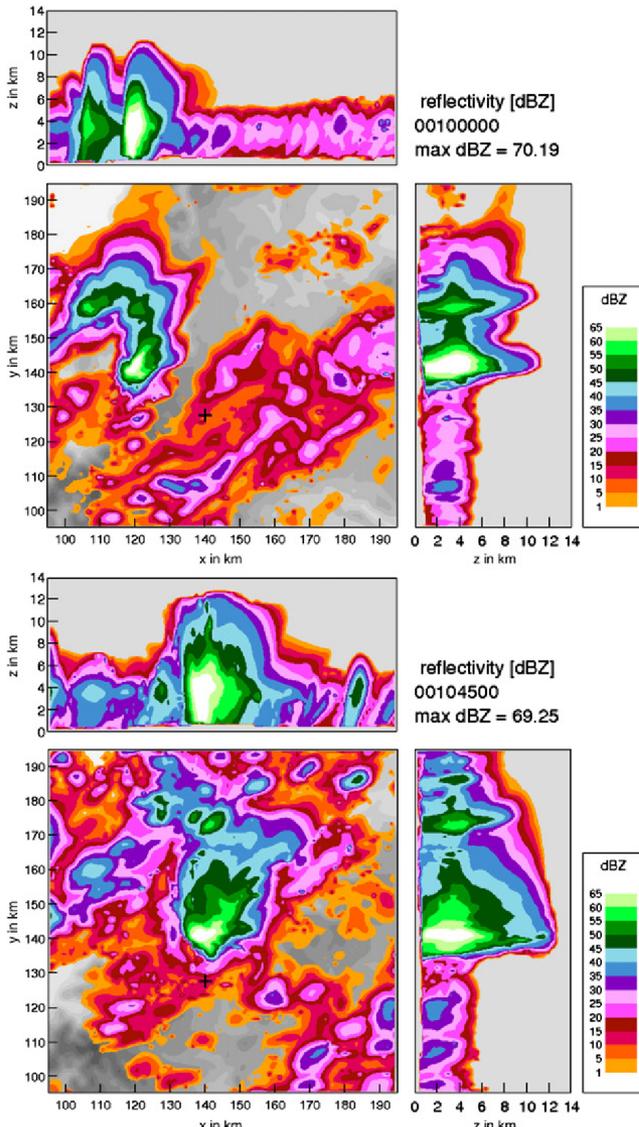


Fig. 10. Same as Fig. 9 but for different parameters. Top: radar reflectivity; center: total hydrometeor mass content; bottom: precipitation rate at the ground by rain (dotted line), by hail (dashed) and in total (solid).



# Case study: 28.6.2006

## „Villingen-Schwenningen-Zelle“

Deutscher Wetterdienst  
Meteo und Klima aus einer Hand

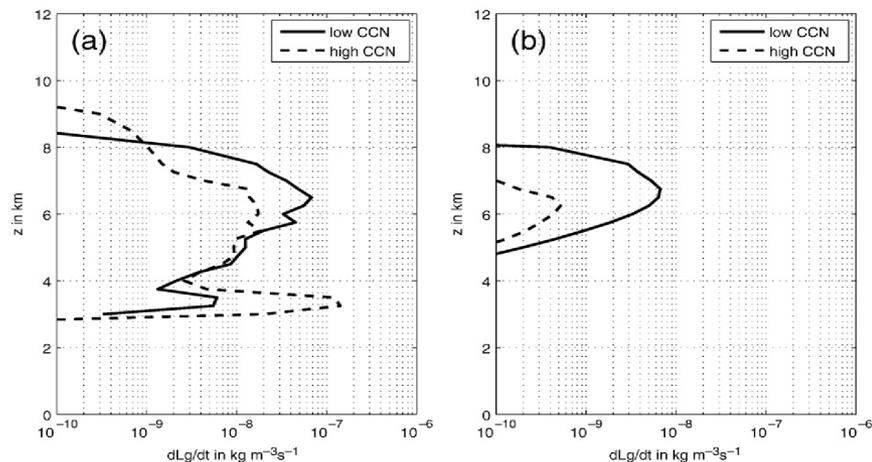


Fig. 16. Profiles of graupel production (a) by riming of ice and snow and (b) by freezing of drops at  $t = 10:45$  horizontally averaged over the region where total cloud mass exceeds  $10^{-2}\text{g m}^{-3}$ .

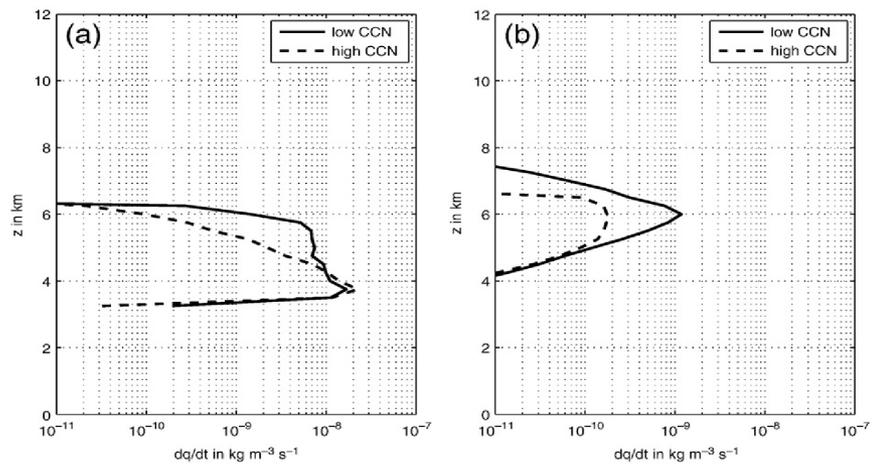


Fig. 17. Same as Fig. 16, but for the production of hail (a) by riming of graupel (wet growth) and (b) by freezing of drops.



# Case study: 28.6.2006

## „Villingen-Schwenningen-Zelle“

Deutscher Wetterdienst  
Wetter und Klima aus einer Hand



- **Main results (COSMO):**  
After some trying, good simulation of super cell (a little late, but in the right place). Simulated hail at the ground decreased with increasing CCN concentration. Cell life time and total precip was rather in-sensitive to CCNs.
- **COSMO 2D and HUCM 2D were in qualitative agreement, but did not produce the same sensitivities of hail at the ground to changes in CCN and 0-degree-level ...**
- **Topic still open ...**

**Table 2**

Maximum accumulated total precipitation in mm, after 12 h simulation time for different model runs, taking into account only the right-moving cell.

CCN concentr.	Cloud droplet size distribution		
	(a) Broad	(b) n-small	(c) n-large
1 Low	51	50	35
2 Intermediate	44	40	41
3 High	39	43	38
4 Very high	37	37	45

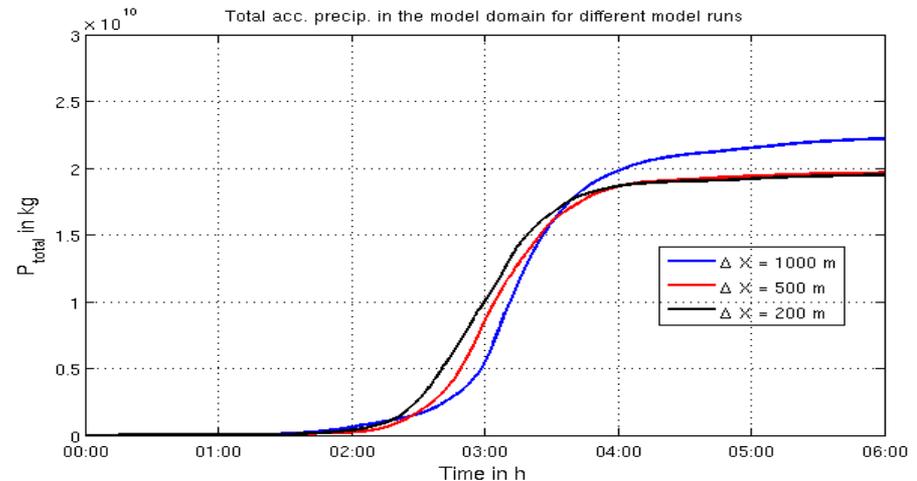
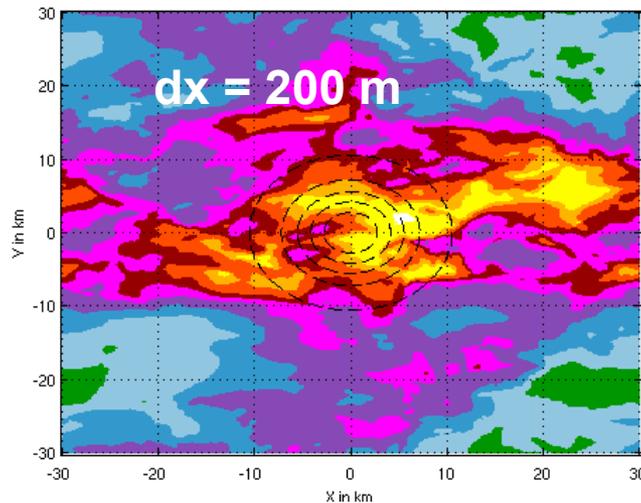
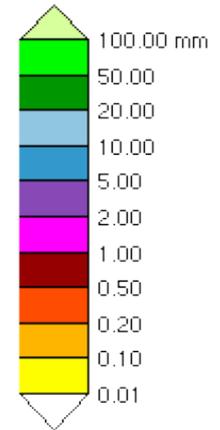
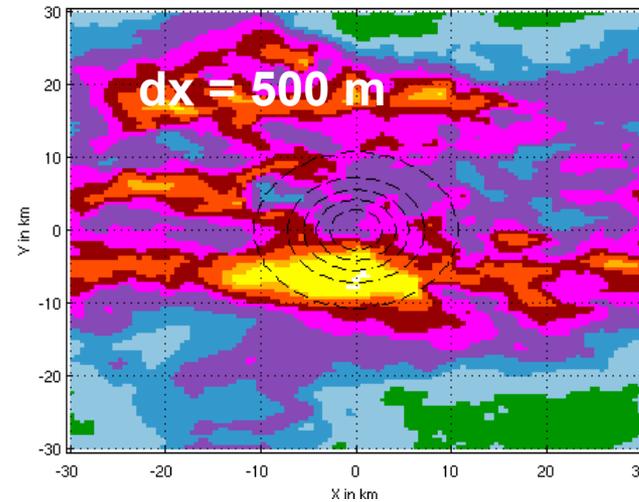
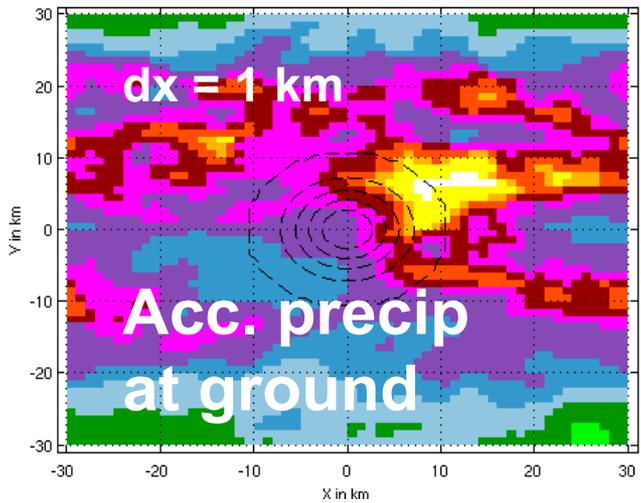
**Table 3**

Same as [Table 2](#) but for precipitation by hail only.

CCN concentr.	Cloud droplet size distribution		
	(a) Broad	(b) n-small	(c) n-large
1 Low	14.9	12.1	9.1
2 Intermediate	9.9	8.5	11.5
3 High	5.6	6.5	2.0
4 Very high	1.5	3.5	2.1

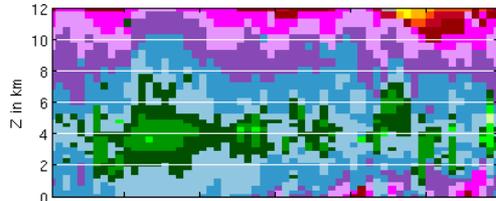


# Is $\Delta x = 1 \text{ km}$ „enough“?



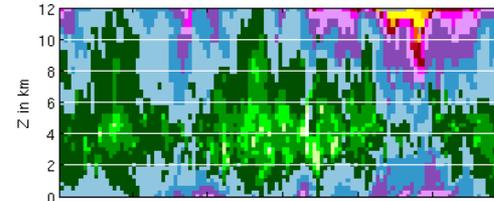
(Idealized, periodic BCs, pre-specified  $H_0$  und  $E_0$ , LES-turbulence closure, initial profiles after Weisman and Klemp (1982) with  $CAPE = 2800 \text{ J/kg}$ ,  $U_0 = 10 \text{ m/s}$ )



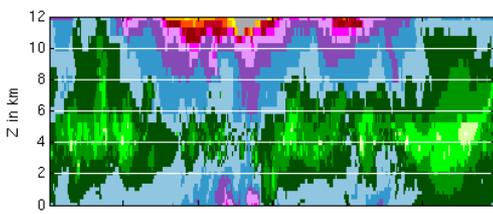
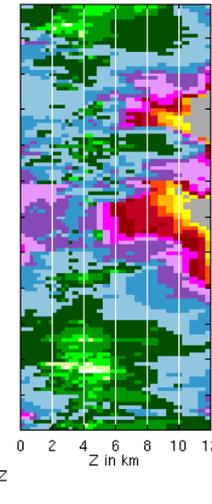
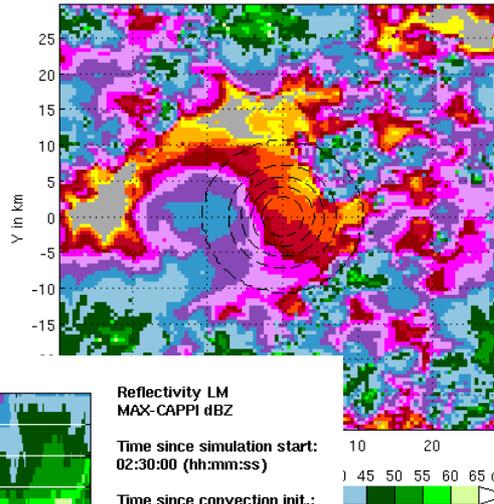
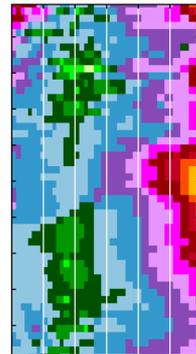
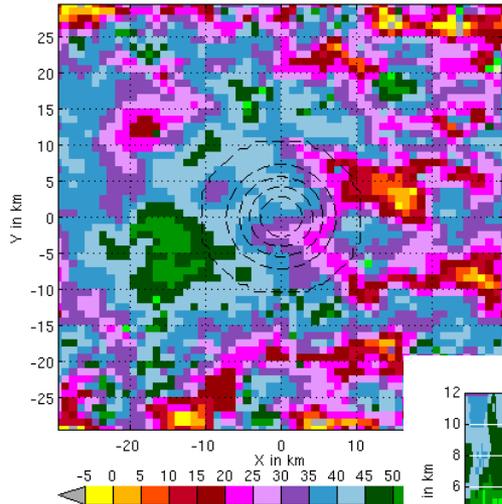


Reflectivity LM  
MAX-CAPPI dBZ  
Time since simulation start:  
02:30:00 (hh:mm:ss)  
Time since convection init.:  
02:30:00  
cloud\_typ = 2787

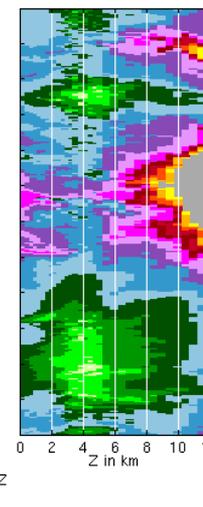
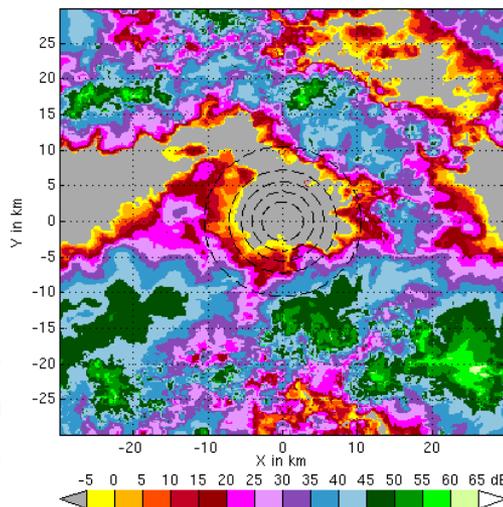
““



Reflectivity LM  
MAX-CAPPI dBZ  
Time since simulation start:  
02:30:00 (hh:mm:ss)  
Time since convection init.:  
02:30:00  
cloud\_typ = 2787



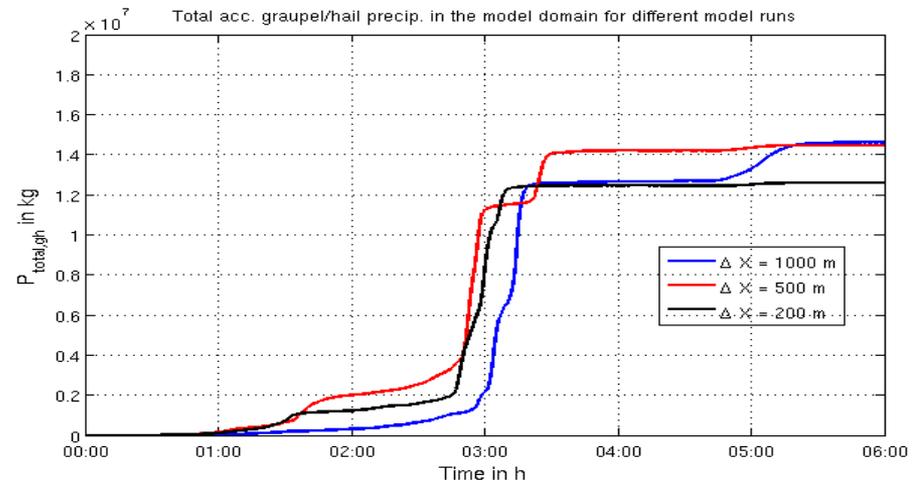
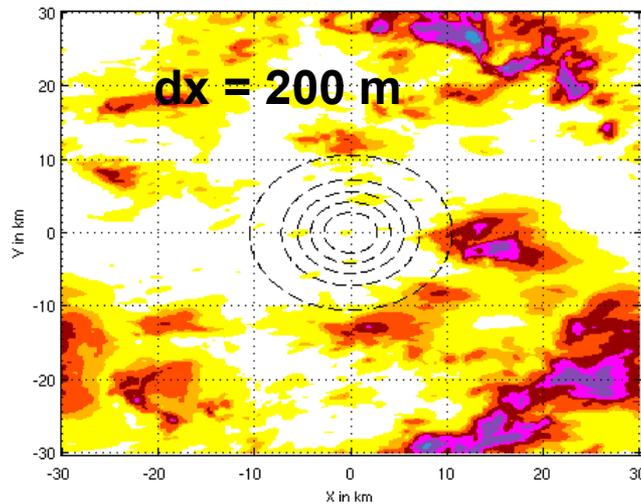
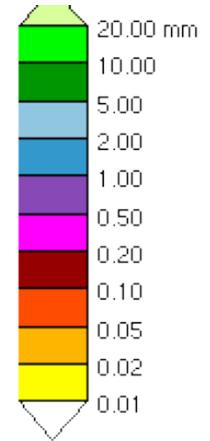
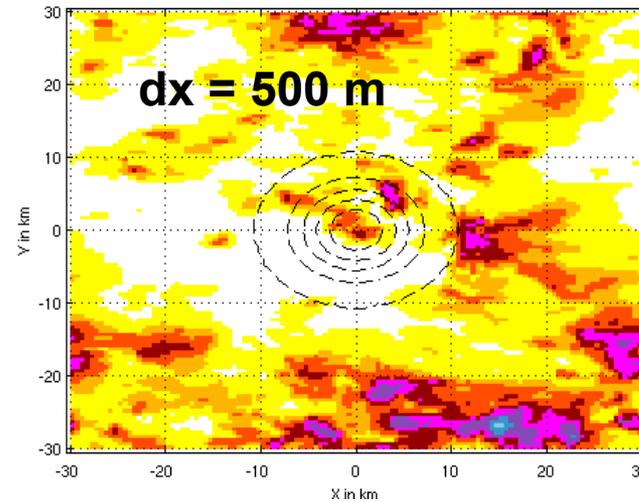
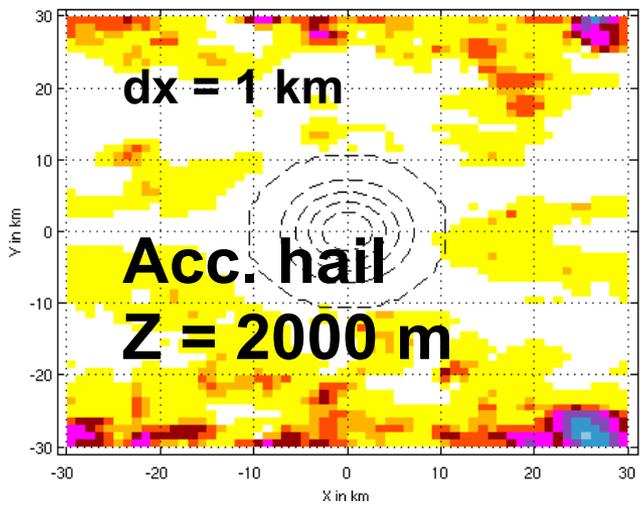
Reflectivity LM  
MAX-CAPPI dBZ  
Time since simulation start:  
02:30:00 (hh:mm:ss)  
Time since convection init.:  
02:30:00  
cloud\_typ = 2787



pressure, initial profiles after

(Idealized, periodic BCs, pre-Weisman and Klemp (1982))

# Is $dx = 1$ km „enough“?



(Idealized, periodic BCs, pre-specified  $H_0$  und  $E_0$ , LES-turbulence closure, initial profiles after Weisman and Klemp (1982) with  $CAPE = 2800$  J/kg,  $U_0 = 10$  m/s)





- 2-moment-scheme extended by a „hail“ class and „tuned“.
- Most of the relevant processes are covered, but melting is probably overestimated.
- Compared to 3D radar data, obtain realistic simulation of convective cells and „reflectivity cores“ (although anvils sometimes too small).
- Allows for formation of large hail, but interpretation has to be done with care, in light of the parameterization assumptions!
- Mesh sizes of 1 km **may** sometimes be „enough“ for the simulation of hail processes, at least for the COSMO-Model with two-moment scheme in some cases and regarding spacial hail statistics. But certainly more experience needed here!





## ■ Outlook:

Introduction of additional variables, which describe the internal water content of partially melted ice particles.

With this, the parameterization of melting will be enhanced and shedding and wet growth of hail may be taken into account explicitly.

## ■ Outlook 2:

Splitting of the outcome of collision terms for solid-to-rain collisions according to size ranges, based on partitioning of the integral and use of incomplete gamma functions.

**Development has been done, now case studies are necessary.**





## ■ Gleichung für die spektrale Anzahldichteverteilungsfunktion:

$$\frac{\partial f_j(\vec{r}, t, x)}{\partial t} + \nabla \cdot (\vec{v}(\vec{r}, t) f_j(\vec{r}, t, x)) + \frac{\partial}{\partial z} (v_{sj}(x) f_j(\vec{r}, t, x)) + \frac{\partial}{\partial x} (\dot{x} f_j(\vec{r}, t, x)) = \sum_{k=1}^N \sigma_{jk} \quad j = 1 \dots N$$

$$\sigma_{jk} \neq \sigma_{kj}$$

## ■ Kollisionsterme ( $\sigma_{jk}$ ): Beispiel Regen (j=2) – Graupel (k=5)

$$\sigma_{25} = \left. \frac{\partial f_r(x_r)}{\partial t} \right|_{coll,gr} = - \int_0^\infty f_r(x_r) f_g(x_g) K_{gr}(x_r, x_g) dx_g$$

$$\sigma_{52} = \left. \frac{\partial f_g(x_g)}{\partial t} \right|_{coll,gr} = + \int_0^{x_g} f_g(x_g - x_r) f_r(x_r) K_{gr}(x_g - x_r, x_r) dx_r - \int_0^\infty f_g(x_g) f_r(x_r) K_{gr}(x_g, x_r) dx_r$$

„Geometrische“ Koagulationsfunktion:

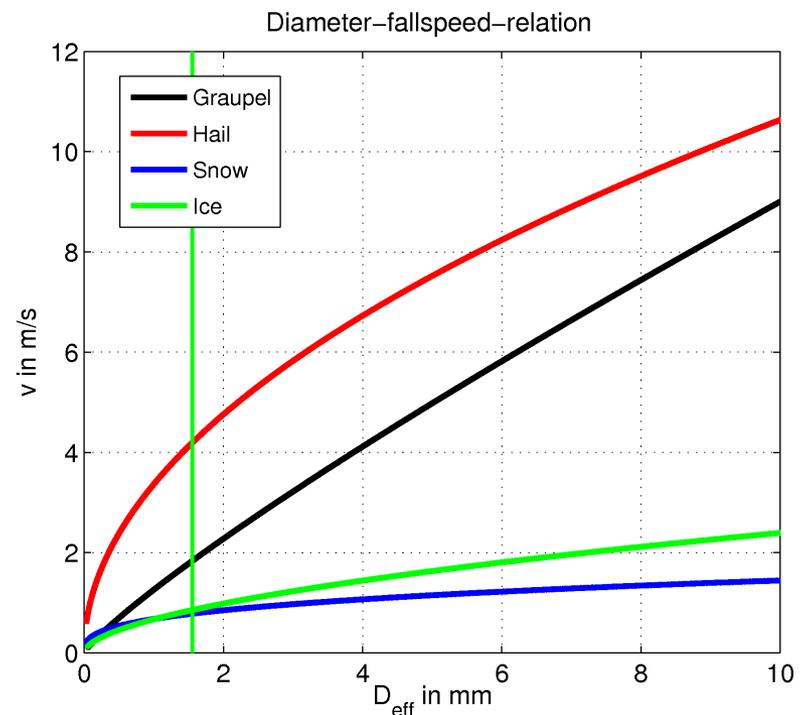
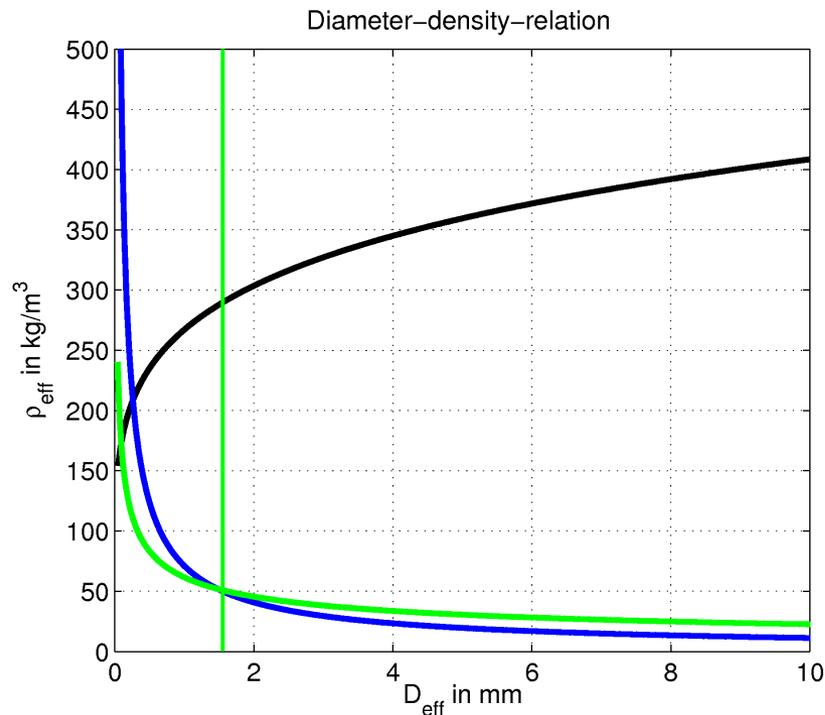
$$K_{gr}(x_g, x_r) = \frac{\pi}{4} (D_g^2(x_g) + D_r^2(x_r)) |v_g(x_g) - v_r(x_r)| E_{gr}(x_g, x_r)$$



# Microphysics parameterization

1) Aufteilung in Hydrometeorarten (Wahl)

2) Festlegung von Durchmesser-Masse- und Durchmesser-Fallgeschwindigkeits-Beziehungen (Wahl)



3) ...



## ■ 3a) Bin-Schema:

**Gleichung für die spektrale Anzahldichteverteilungs-funktion wird durch numerische Diskretisierung in „bins“ gelöst. Je gewählter Hydrometeorart gibt es eine solche diskretisierte Gleichung pro „bin“!**

**(z.B. Khain, 1996; Geresdi, 1998)**

**Sehr aufwendig, deswegen bisher fast nur in 2D eingesetzt.**



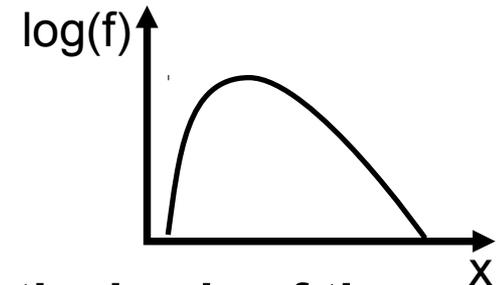
## 3b) 2-moment scheme



- Hydrometeor classes: cloud droplets, rain, cloud ice, snow, graupel, hail („simple“ extention, Noppel et al., 2006)

- For each class, budget equations for N and Q

$$N = \int_0^{\infty} f(x) dx \quad Q = \int_0^{\infty} x f(x) dx$$



- Parameterization of microphysical processes on the basis of the spectral budget equation(s) assuming a generalized gamma-distribution for f(x):

$$f(x) = N_0 x^{\nu} \exp(-\lambda x^{\mu})$$

$$\nu, \mu = \text{const.}, \quad \lambda = \left[ \frac{\Gamma(\frac{\nu+1}{\mu})}{\Gamma(\frac{\nu+2}{\mu})} \frac{Q}{N} \right]^{-\mu}, \quad N_0 = \frac{\mu N}{\Gamma(\frac{\nu+1}{\mu})} \lambda^{\frac{\nu+1}{\mu}}$$

- Coupling to the COSMO-Model



# 3b) 2-moment scheme



## ■ Parametrisierung von Kollisionen (Seifert und Beheng, 2006)

### Beispiel: Graupel + Regen = Graupel

$$\left. \frac{\partial N_g}{\partial t} \right|_{coll,gr} = 0$$

$$\left. \frac{\partial N_r}{\partial t} \right|_{coll,gr} = -\frac{\pi}{4} \bar{E}_{gr} \overline{\Delta v_{gr}^{(00)}} N_g N_r \left( \delta_{gg}^{(00)} D_g^2(\bar{x}_g) + \delta_{gr}^{(00)} D_g(\bar{x}_g) D_r(\bar{x}_r) + \delta_{rr}^{(00)} D_r^2(\bar{x}_r) \right)$$

$$\left. \frac{\partial L_g}{\partial t} \right|_{coll,gr} = \frac{\pi}{4} \bar{E}_{gr} \overline{\Delta v_{gr}^{(01)}} N_g L_r \left( \delta_{gg}^{(01)} D_g^2(\bar{x}_g) + \delta_{gr}^{(01)} D_g(\bar{x}_g) D_r(\bar{x}_r) + \delta_{rr}^{(01)} D_r^2(\bar{x}_r) \right)$$

$$\left. \frac{\partial L_r}{\partial t} \right|_{coll,gr} = - \left. \frac{\partial L_g}{\partial t} \right|_{coll,gr}$$

**Insgesamt 18 derartige Kollisionsprozesse im 2-Momenten-Schema**



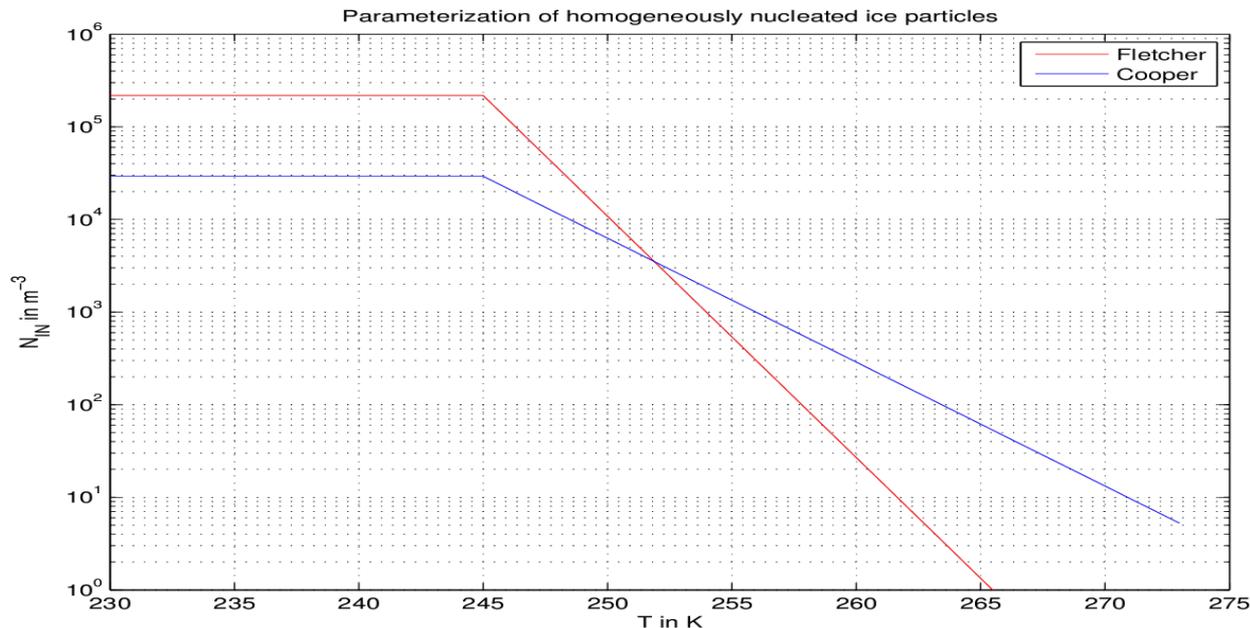
# 3b) 2-moment scheme



## Eisnukleation „traditionell“ (Fletcher, Cooper):

$$N_{ice,diag} = a \exp(b(T - T_0))$$

$$\frac{\partial N_i}{\partial t} = \begin{cases} \frac{N_{ice,diag} - N_i}{\Delta t} & \text{for } N_{ice,diag} > N_i \\ 0 & \text{otherwise} \end{cases}$$



## 3b) 2-moment scheme



### ■ Eisnukleation „traditionell“ Fletcher (T), Cooper (T), Meyers (S<sub>i</sub>)

$$N_{ice,diag} = a \exp(b(T - T_0))$$

$$\frac{\partial N_i}{\partial t} = \begin{cases} \frac{N_{ice,diag} - N_i}{\Delta t} & \text{for } N_{ice,diag} > N_i \\ 0 & \text{otherwise} \end{cases}$$

Parameterization of homogeneously nucleated ice particles

Inzwischen auch andere Möglichkeiten implementiert:

- 1) **Für Cirren / stratiforme Eiswolken:**  
basierend auf Kärcher und Lohmann (2002), Kärcher et al. (2006)
- 2) **Für Kopplung mit explizitem Aerosol-Modul (COSMO-ART):**  
Phillips et al. (2008)



# 3b) 2-moment scheme

## ■ Homog./heterog. freezing:

$$\frac{1}{f_w(x, t)} \frac{\partial f_w(x, t)}{\partial t} = x J(T)$$

### Heterog. Freezing:

$$J(T) = a \exp(b(T - T_0) - 1)$$

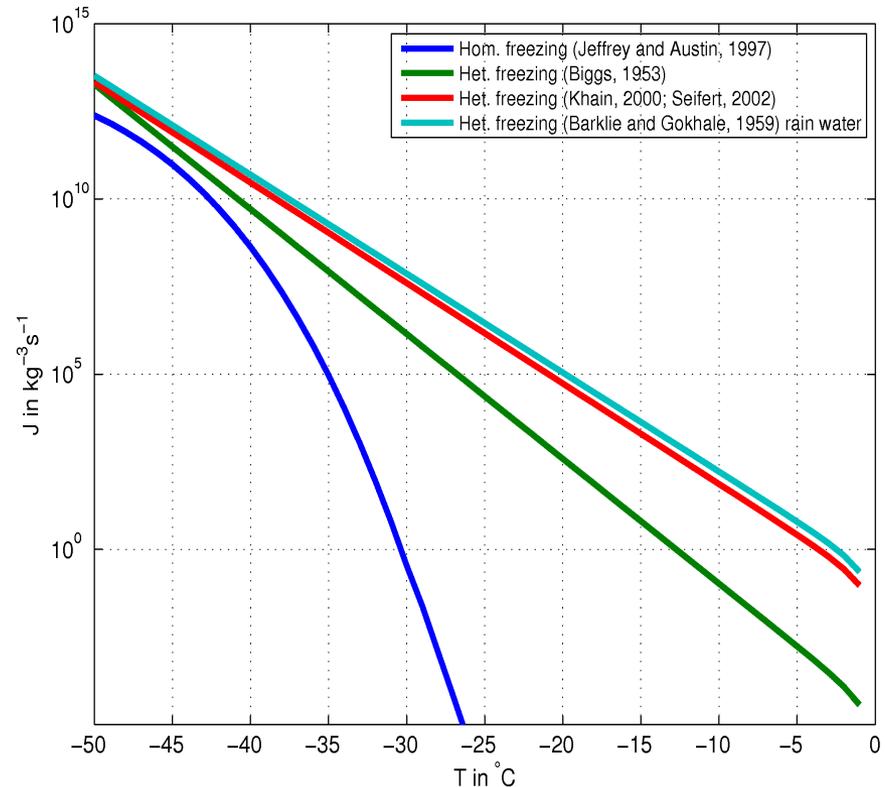
### Homog. Freezing:

$$J(T) \text{ after Cotton und Field (2001)}$$

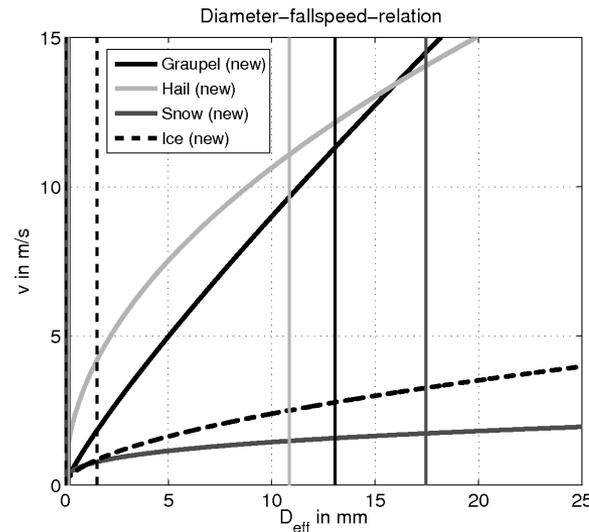
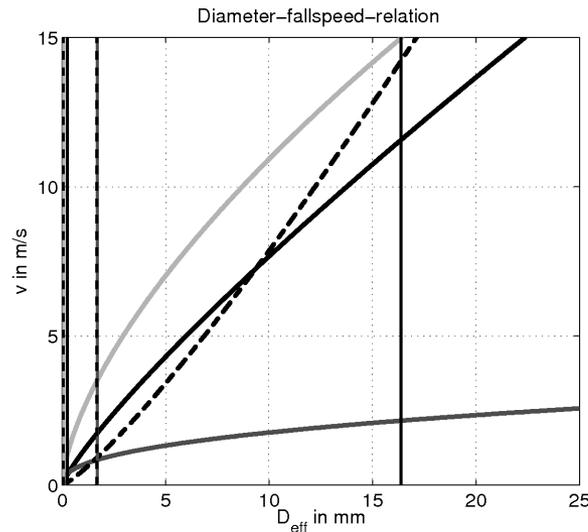
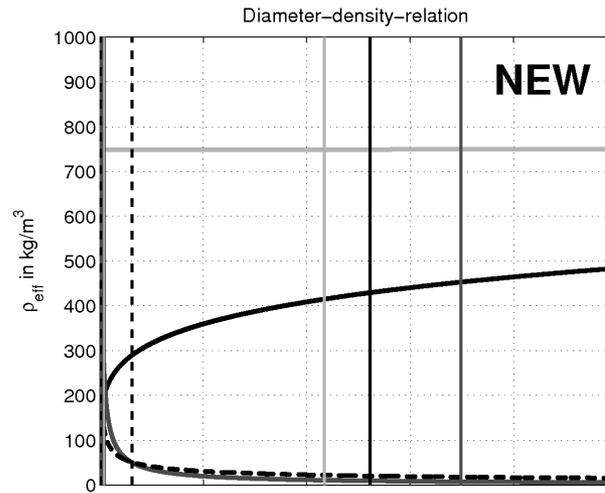
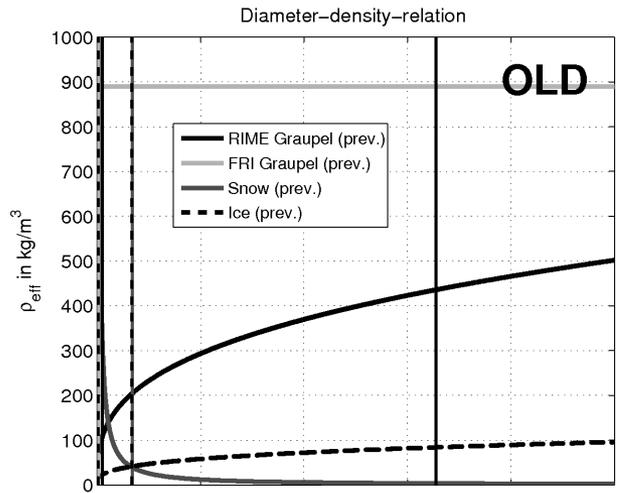
### Integration of $\partial_t f_w$ and $x \partial_t f_w$ over $x$ :

$$\Rightarrow \left. \frac{\partial N_w}{\partial t} \right|_{\text{freeze}} = -Q_w J(T)$$

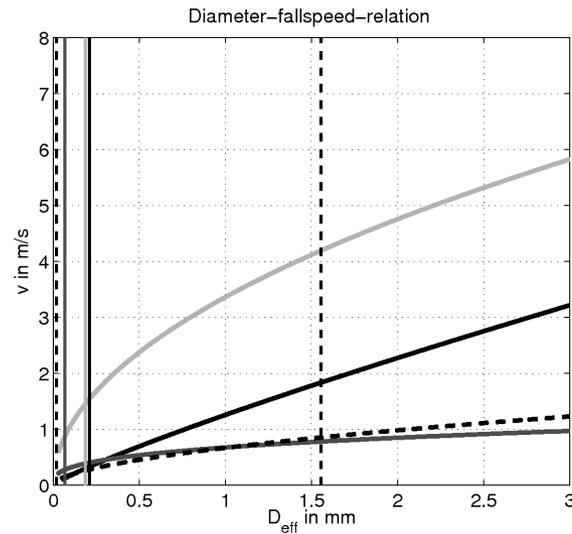
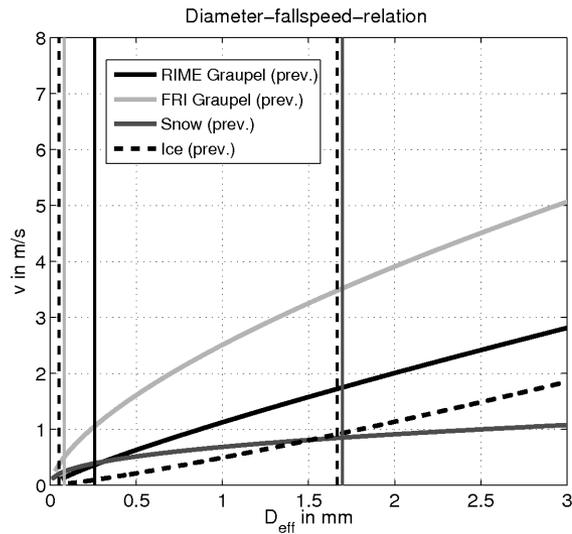
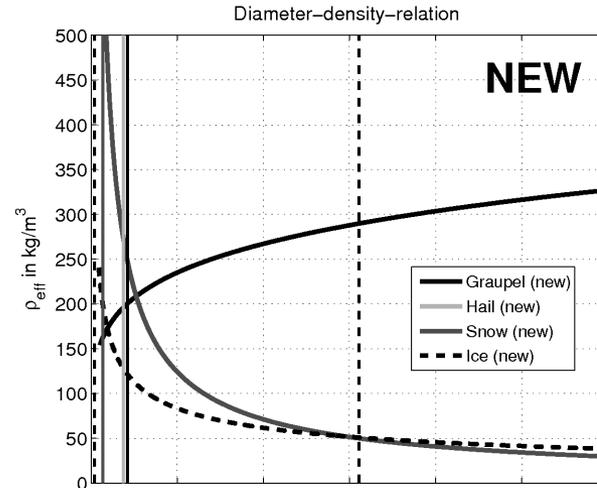
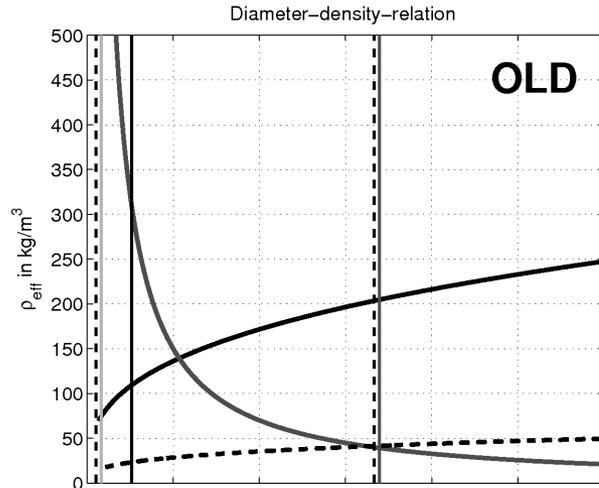
$$\Rightarrow \left. \frac{\partial Q_w}{\partial t} \right|_{\text{freeze}} = -M_w^{(2)} J(T)$$



# 3b) 2-moment scheme



# 3b) 2-moment scheme



# 3b) 2-moment scheme

Deutscher Wetterdienst  
*Wetter und Klima aus einer Hand*

