A new version of the Spatio-Temporal Stochastic Pattern Generator

Michael Tsyrulnikov and Dmitry Gayfulin

HydroMetCentre of Russia

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Introduction

- Model errors are an important source of uncertainty in NWP and thus need to be simulated in ensemble prediction and ensemble data assimilation.
- Multi-model vs. stochastic approaches.
- Stochastic approach. Model errors are postulated to satisfy a stochastic spatio-temporal model error model (which can be additive, multiplicative, or other).
- The model error model requires a spatio-temporal random field as a stochastic input.
- The generation of pseudo-random 3-D and 2-D spatio-temporal fields with realistic and tunable structure is the purpose of the SPG we have developed.

Motivation

Existing approaches

1. Generate independent random numbers for large spatio-temporal boxes ("stochastic physics", Buizza et al., 1999).

2. Perform a spectral-space expansion and specify a temporal auto-regression for each spectral coefficient (Li et al., 2008; Berner et al., 2009; Bowler et al., Palmer et al., 2009; Bouttier et al., 2012). Use the same temporal length scale for all spatial wavenumbers.

Separability of spatio-temporal correlations

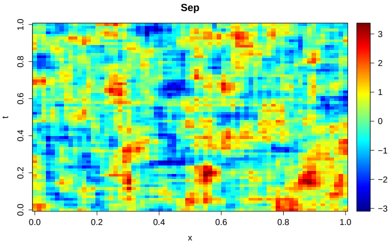
The existing approaches all produce *separable* space-time correlations:

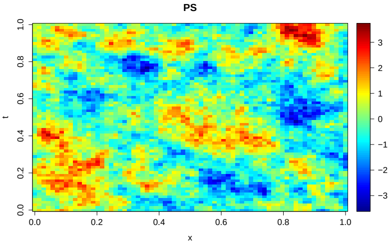
$$C(t, \mathbf{s}) = C_t(t) \cdot C_\mathbf{s}(\mathbf{s})$$

But: no space-time interactions.

In reality, longer spatial scales 'live longer' than shorter spatial scales, which 'die out' quicker. This "proportionality of scales" is widespread in geophysical fields (Tsyroulnikov QJRMS 2001) and other media.

Separable *vs.* non-separable correlation models





Our approach: Linear evolutionary stochastic partial differential equations

1. Flexibility (local inhomogeneity, non-stationarity, non-Gaussianity).
2. Sparse matrices $\Rightarrow$ fast computations.

The SPG design

First-order evolutionary model for SPG

$$\frac{\partial \xi(t, \mathbf{s})}{\partial t} + A\,\xi(t, \mathbf{s}) = \sigma\,\alpha(t, \mathbf{s})$$

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$$P(-\Delta) = \mu(1-\lambda^2\Delta)^q$$

$$\boxed{\frac{\partial \xi(t,\mathbf{s})}{\partial t} + \mu(1-\lambda^2\Delta)^q\,\xi(t,\mathbf{s}) = \sigma\,\alpha(t,\mathbf{s})}$$

“Proportionality of scales” implies that $q = \frac{1}{2}$

PS:

$$\tau_{\mathbf{k}} \sim \frac{1}{|\mathbf{k}|}$$

$$\boxed{\frac{\partial \xi(t, \mathbf{s})}{\partial t} + \mu \sqrt{1 - \lambda^2 \Delta} \cdot \xi(t, \mathbf{s}) = \sigma\, \alpha(t, \mathbf{s})}$$

But:

The spectra appear to decay too slowly for the variance $\mathrm{Var}\, \xi$ to be finite..

The solution: a higher-order in time SPG model

Starting point:

$$\frac{\partial \xi(t, \mathbf{s})}{\partial t} + \mu \sqrt{1 - \lambda^2 \Delta} \cdot \xi(t, \mathbf{s}) = \sigma \, \alpha(t, \mathbf{s})$$

Higher-order in time SPG model

Starting point:

$$\frac{\partial \xi(t, \mathbf{s})}{\partial t} + \mu\sqrt{1 - \lambda^2 \Delta} \cdot \xi(t, \mathbf{s}) = \sigma\, \alpha(t, \mathbf{s})$$

Rewrite as

$$\left(\frac{\partial}{\partial t} + \mu\sqrt{1 - \lambda^2 \Delta}\right) \xi(t, \mathbf{s}) = \sigma\, \alpha(t, \mathbf{s})$$

Higher-order in time SPG model

Starting point:

$$\frac{\partial \xi(t,\mathbf{s})}{\partial t} + \mu\sqrt{1-\lambda^2\Delta}\cdot\xi(t,\mathbf{s}) = \sigma\,\alpha(t,\mathbf{s})$$

Rewrite as

$$\left(\frac{\partial}{\partial t} + \mu\sqrt{1-\lambda^2\Delta}\right)\xi(t,\mathbf{s}) = \sigma\,\alpha(t,\mathbf{s})$$

Generalize to a higher temporal order:

$$\boxed{\left(\frac{\partial}{\partial t} + \mu\sqrt{1-\lambda^2\Delta}\right)^p \xi(t,\mathbf{s}) = \sigma\,\alpha(t,\mathbf{s})}$$

The minimal order p that solves the problem in both 2D and 3D is $p = 3$.

The final formulation of the SPG model

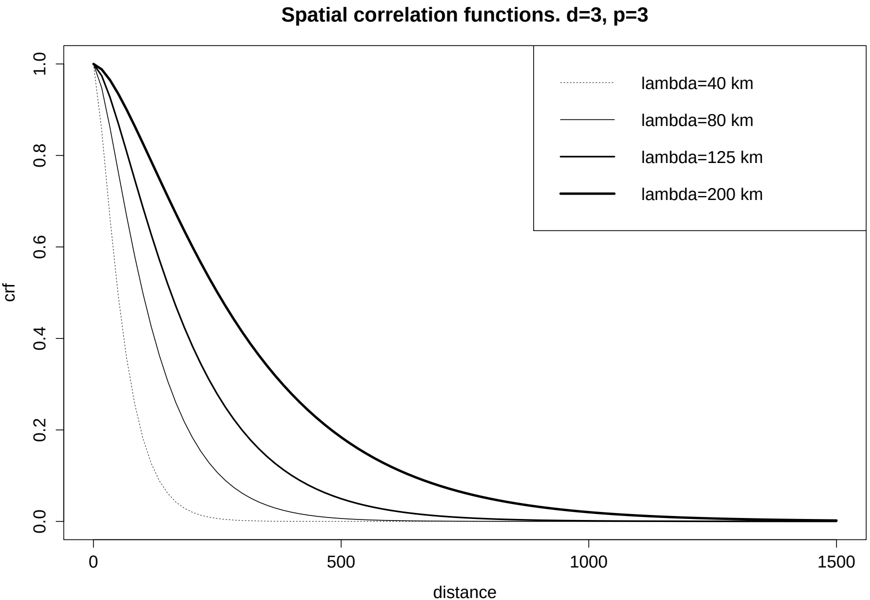
The resulting SPG model is

$$\boxed{\boxed{\left(\frac{\partial}{\partial t} + \frac{U}{\lambda}\sqrt{1 - \lambda^2 \Delta}\right)^3 \xi(t, \mathbf{s}) = \sigma\, \alpha(t, \mathbf{s})}} \tag{1}$$

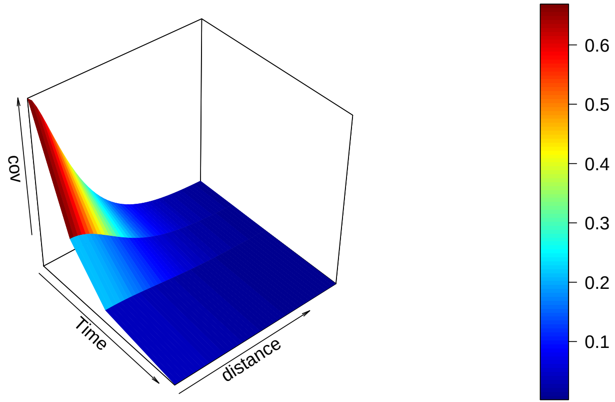
Spatio-temporal covariances

The Matérn class in space-time

Spatial/temporal correlation functions



Spatio−temporal covariances



Ranges: t=0...12 h, r=0...750 km

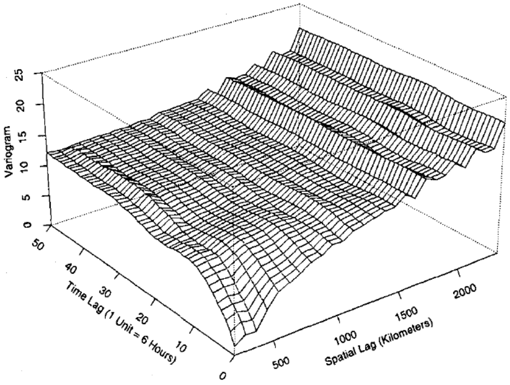


Figure 8. Empirical Spatio-Temporal Variogram Evaluated at Spatio-Temporal Lags $\{\mathbf{h}(1), \ldots, \mathbf{h}(27)\} \times \{0, 1, \ldots, 50\}$.

Fig.8 from (Cressie and Huang 1999): empirical spatio-temporal wind-speed variogram.

NB: the variogram is proportional to 1-C(t,x), where C is the correlations function.

Numerical scheme

The basic spectral in space and finite-difference in time scheme

Continuous OSDE equation for each spectral coefficient:

$$\boxed{\left(\frac{\mathrm{d}}{\mathrm{d}t} + \mu\sqrt{1+\lambda^2 k^2}\right)^3 \tilde{\xi}_{\mathbf{k}}(t) = \sigma\, \tilde{\alpha}_{\mathbf{k}}(t)}$$

The solver: an implicit time differencing scheme for each wavevector.

Two techniques to improve the performance of the time integration scheme

1. Correction of spectral variances

   Knowing the stationary spectral variances for both exact continuous and approximate time-discrete scheme, apply correction coefficients to $\hat{\xi}_{\mathbf{k}}(i)$.

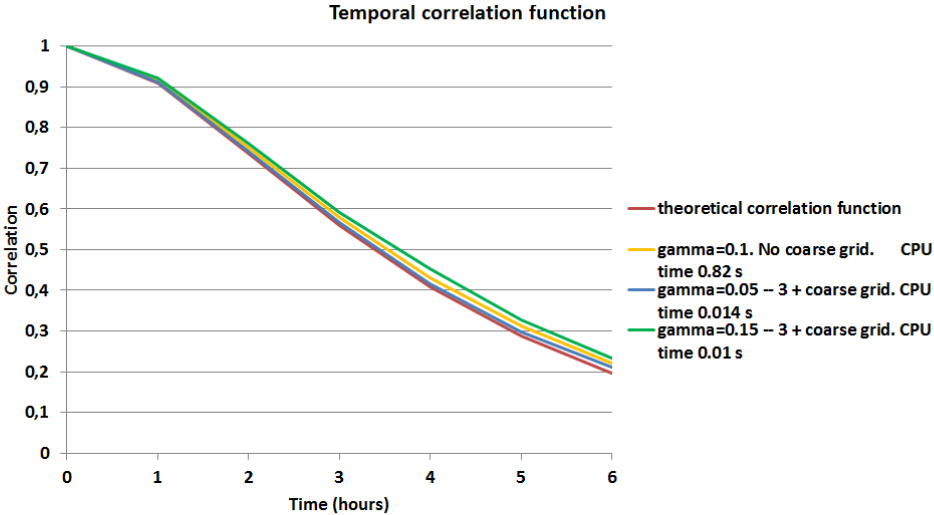
2. Warm start

   Ensure stationarity from the beginning of the time integration. We sample the 3 initial conditions $\boldsymbol{\xi}^{ini} := (\hat{\xi}_{\mathbf{k}}(1), \hat{\xi}_{\mathbf{k}}(2), \hat{\xi}_{\mathbf{k}}(3))^\top$ from the stationary distribution.

Two techniques to speed up the computations

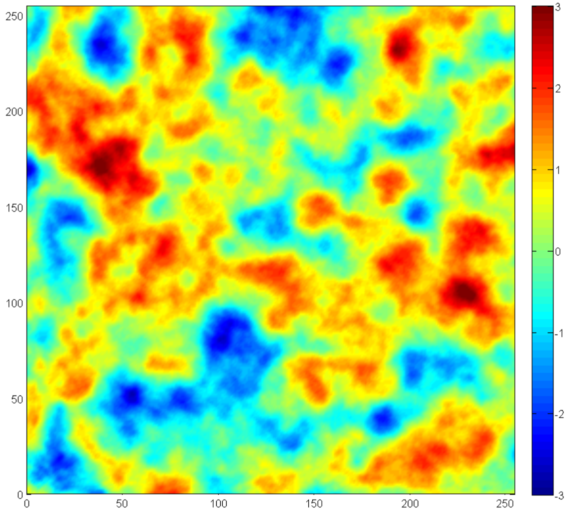
1. Make the time step dependent on the spatial total wavenumber k
2. Introduction of a coarse grid in spectral space
   - Perform time integration for $\mathbf{k}$ on the coarse grid.
   - Interpolate the spectral coefficients $\hat{\xi}_{\mathbf{k}}(t)$ from the coarse grid to the full grid in spectral space.
   - Remove the undesirable spectral-space correlations by multiplying $\hat{\xi}_{\mathbf{k}}(t)$ by $\mathrm{e}^{\mathrm{i}\theta_{\mathbf{k}}}$ (*random phases*).

Numerical acceleration: impact on temporal correlations and CPU times (NB: the *spatial* correlations are always perfect). $\gamma = \Delta t/\tau_k$. 2-D grid: 300×300. FFT every hour.

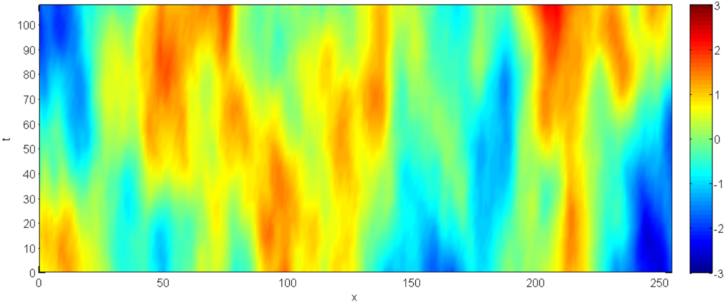


Simulated fields

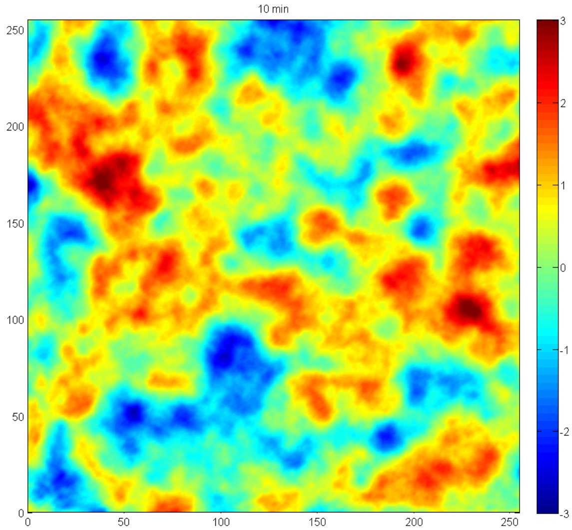
Spatial field (xy)



Space-time plot (xt)

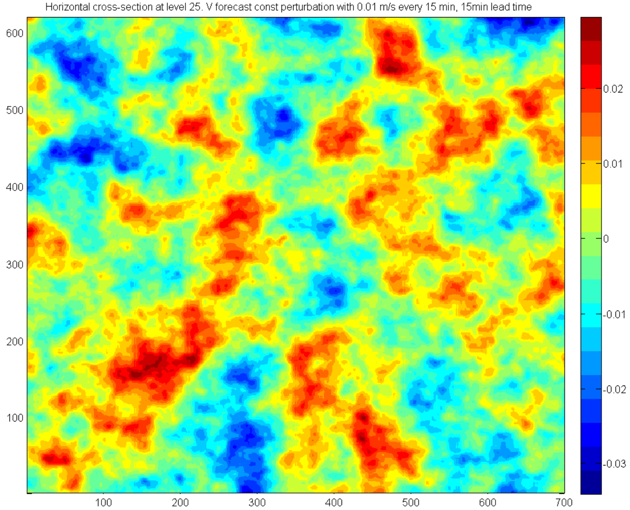


Spatio-temporal random field (animated)



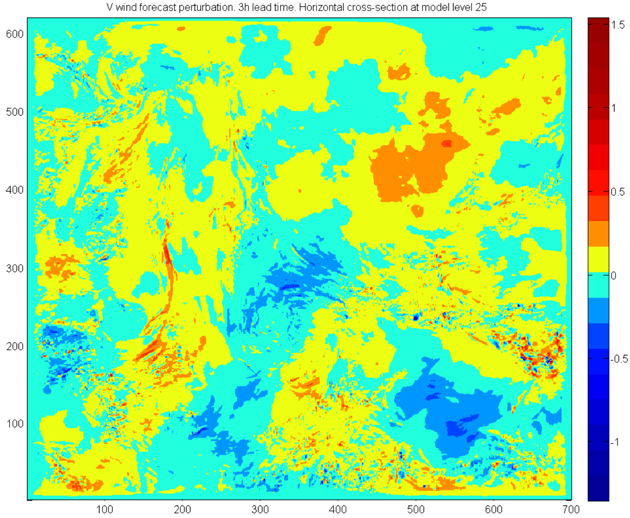
Application to the COSMO model.

First SPG V perturbation added to the model fields at $t = 15$ min.



Application with the COSMO model.

The forecast V perturbation at at $t = 3$ h.



References

- Tsyrulnikov M. and Gayfulin D. A Stochastic Pattern Generator for ensemble applications. – COSMO Tech. Rep. N29, 2016, 51 pp.
- Tsyrulnikov M. and Gayfulin D. A limited-area spatio-temporal stochastic pattern generator for ensemble prediction and ensemble data assimilation. – Meteorol. Zeitschrift, 2016 (under review).

- The Fortran code of the standalone SPG is freely available from github.com/gayfulin/SPG.

Conclusions

The SPG produces 2-D and 3-D Gaussian pseudo-random fields on a limited area domain with non-separable spatio-temporal correlations.

1. Advantages of the SPG
   - Realistic space-time interactions, proportionality of scales.
   - Easily tunable spatial and temporal length scales.
   - Beautiful spatio-temporal correlations.
   - Fast computations.
2. Implementation
   - The SPG is embedded into the COSMO code.
   - The SPG code is modified to comply with the COSMO coding standards.
   - The numerical scheme accelerators are not yet implemented.
3. Application areas
   - Model error perturbations.
   - Initial and boundary-condition perturbations.
   - Soil perturbations.