

# Parallel-in-time integration - Status Report

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# Progress

- Addressing applicability of Parareal for gravity wave propagation.
- Extending code to hybrid space-time parallelization.
- **Big open question:** "Enhancing" Parareal for explicit integrators for nonlinear problems.
- Addressing applicability of  
*"Parallel Full Approximation Scheme in Space and Time" (PFASST)*  
developed by M. Emmett, M. Minion (UNC) for Boussinesq system.
- Write up of recent results in [1].



*Explicit Parallel-in-time Integration of a Linear Acoustic-Advection System*, ICS-Preprint 2011-05.

# More Complex Equations

## Linear Compressible Boussinesq System

$$u_t + U u_x + W u_z + c_s p_x = 0$$

$$w_t + U w_x + W w_z + c_s p_z = b$$

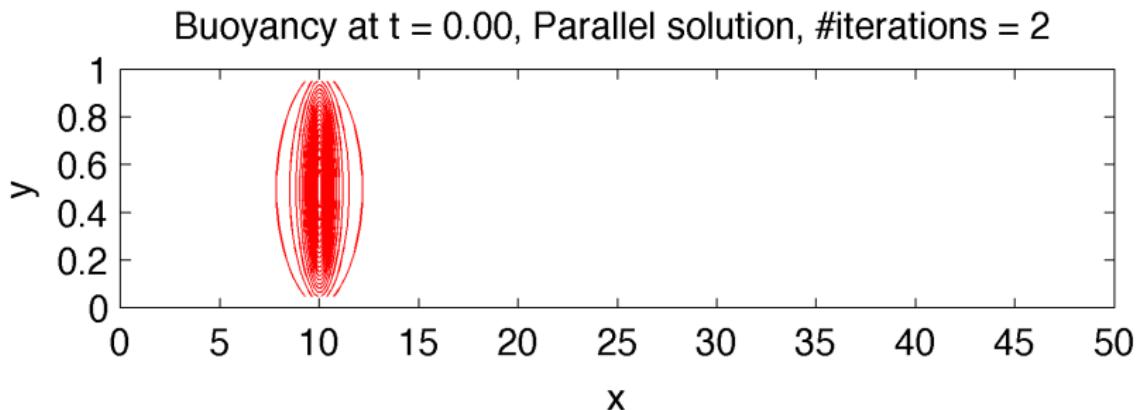
$$b_t + U b_x + W b_z + N^2 w = 0$$

$$p_t + U p_x + W p_z + c_s (u_x + w_z) = 0$$

## Test Problem

Gravity wave in a pipe, [Skamarock and Klemp, MWR 1994].

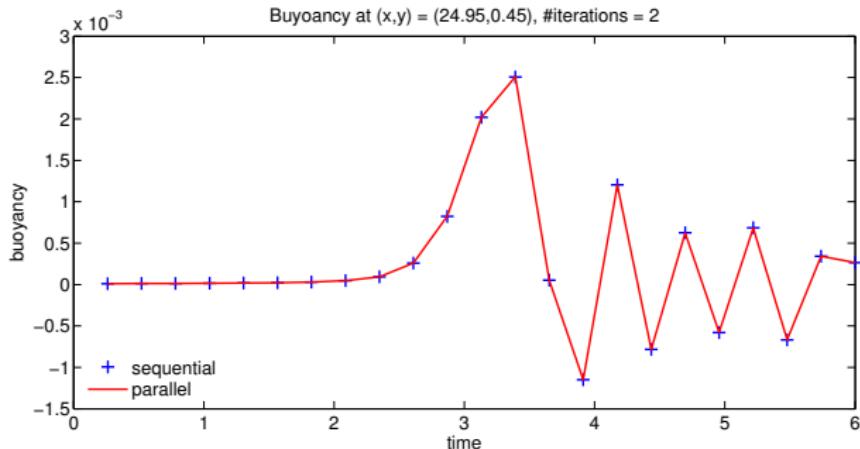
# Parallel Solution



## Speedup

Speedup  $s = \frac{\text{Runtime sequential solution}}{\text{Runtime parallel solution}} = 1.5$  on 8 cores.

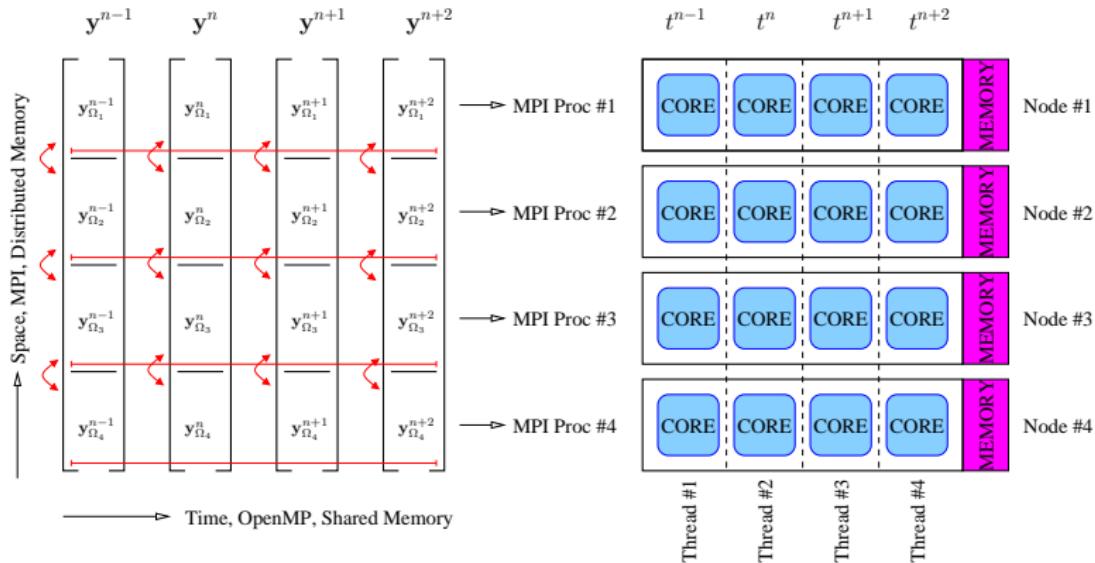
# Buoyancy over Time



## Relative Error

$$\text{Relative error } \frac{\|\mathbf{b}_{\text{par}} - \mathbf{b}_{\text{seq}}\|}{\|\mathbf{b}_{\text{seq}}\|} \approx 5 \cdot 10^{-2}$$

# Next Step: Hybrid Space-Time Parallelization



Goal:

Demonstrate that hybrid approach reduces run time after spatial parallelization has saturated.