A two timelevel integration scheme for the LM

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The current three timelevel integration scheme for the LM has several disadvantages and should be improved or even replaced by an other scheme. The main drawbacks are its low order of approximation for advection, its need for a relatively large number of small time steps for the fast waves integration, its need of the Asselin time filter and its incompatibility to any positive definite advection scheme desired for the moisture variables.

By searching for an altenative one should turn the attention to a numerical consistent, stable and efficient scheme. In the framework of the split explicit time integration every two timelevel advection scheme should fit for these requirements. The philosophy behind the split explicit time integration scheme is to split slow and fast terms of the equations and to treat the fast sound and gravity wave terms with a shorter timestep than the others. During this short time step integration, the advective or slow tendency remains a constant. Besides the known three time level scheme of Klemp and Wilhelmson (1978) a 2 timelevel scheme was presented by Wicker and Skamarock (1998). But this scheme has the disadvantage that it only works in connection with a Runge Kutta scheme of second order in time and its stability properties are not excellent.

A new scheme works only a bit different than that of Wicker and Skamarock (1998). First, it integrates only the fast wave terms until the center of the time step and one gets result values ϕ^*. Second, from this ϕ^*-values advective tendencies are computed. These might be calculated with any stable 2 timelevel scheme. In a short notation they read like

$$\frac{\partial \phi}{\partial t} = \frac{\phi^{n+1} - \phi^n}{\Delta t} = F(\phi^*).$$

Here $F(\phi^*)$ denotes the advective tendency calculation. In the third step, short time steps are calculated from the beginning of the large time step till its end by retaining the advective tendencies constant. A stability analysis of this scheme shows eigenvalues smaller than one almost everywhere. That means the scheme is stable and reliable. The stability analysis was performed for several advection schemes (Runge Kutta scheme of second order in time, Lax Wendroff scheme, semi implicit scheme). They all exhibit good stability properties.

The new scheme needs only 6 short time steps in our configuration (instead of 7 in the current scheme). In tests the Runge Kutta scheme of second order in time is chosen for advection in the horizontal direction, whereas the semi implicit scheme stays for the vertical advection. Moisture variables are treated with a positive definit van Leer advection. The spatial order of approximation of the dynamic variables is third order upstream. This should also be taken into account for the metric terms appearing in the calculation of the contravariant vertical velocity $\dot{\zeta}$ and of the lower boundary condition for w.

First results suggest that the scheme is working, but the predicted fields of the vertical velocity and precipitation have still some deficiencies (see figures 1 and 2). The vertical velocity seems to have a too large amplitude and is too rough, especially on higher model levels. Despite the filtering of orography we get again dry valleys and wet tops of the mountains in alpine regions. There is still work to be done, but the two timelevel scheme is promising.

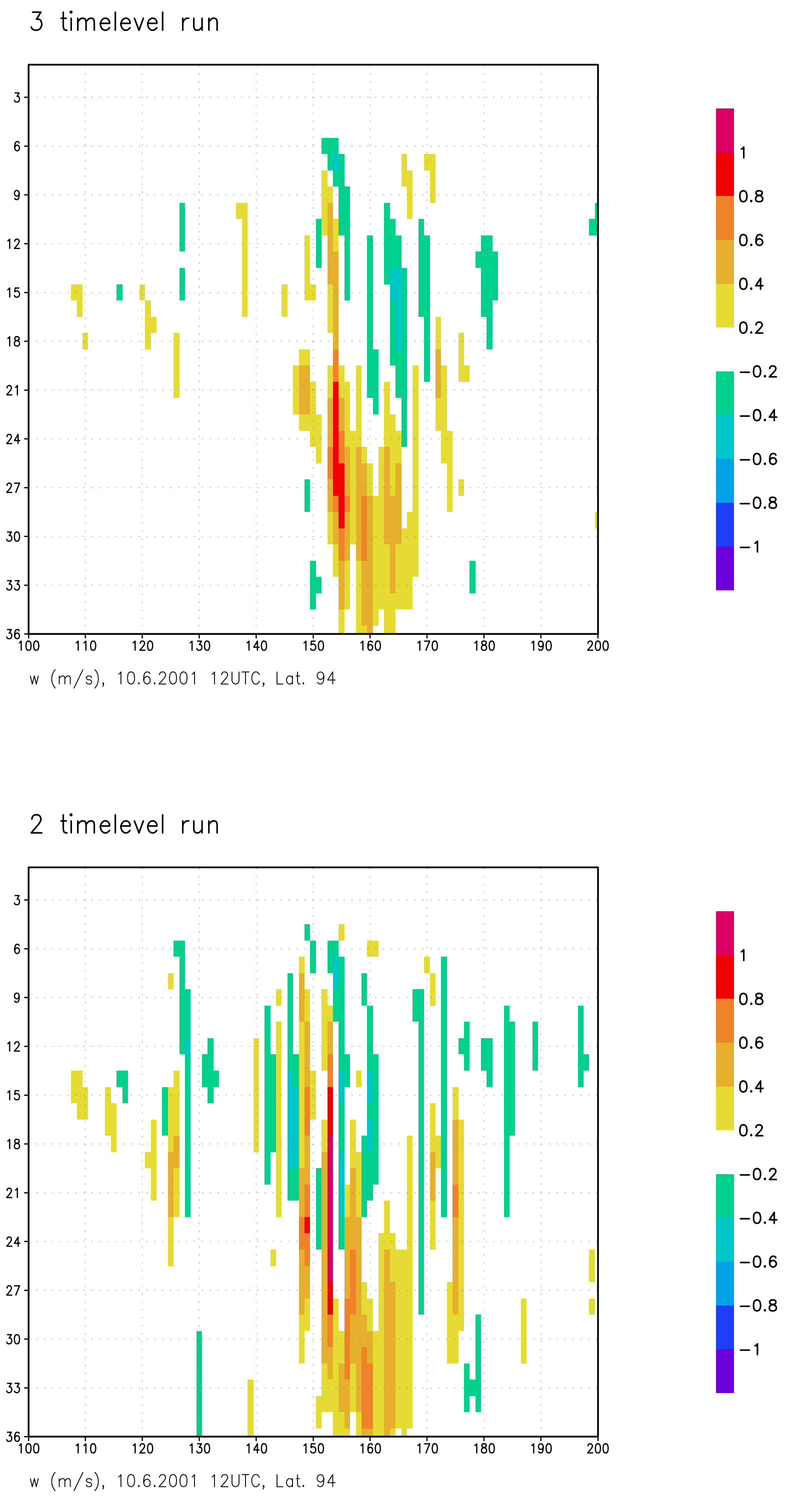

Figure 1: Vertical velocities (m/s) for a west-east vertical cross section in the Alps, upper panel: with the three timelevel scheme, lower panel: with the two timelevel scheme

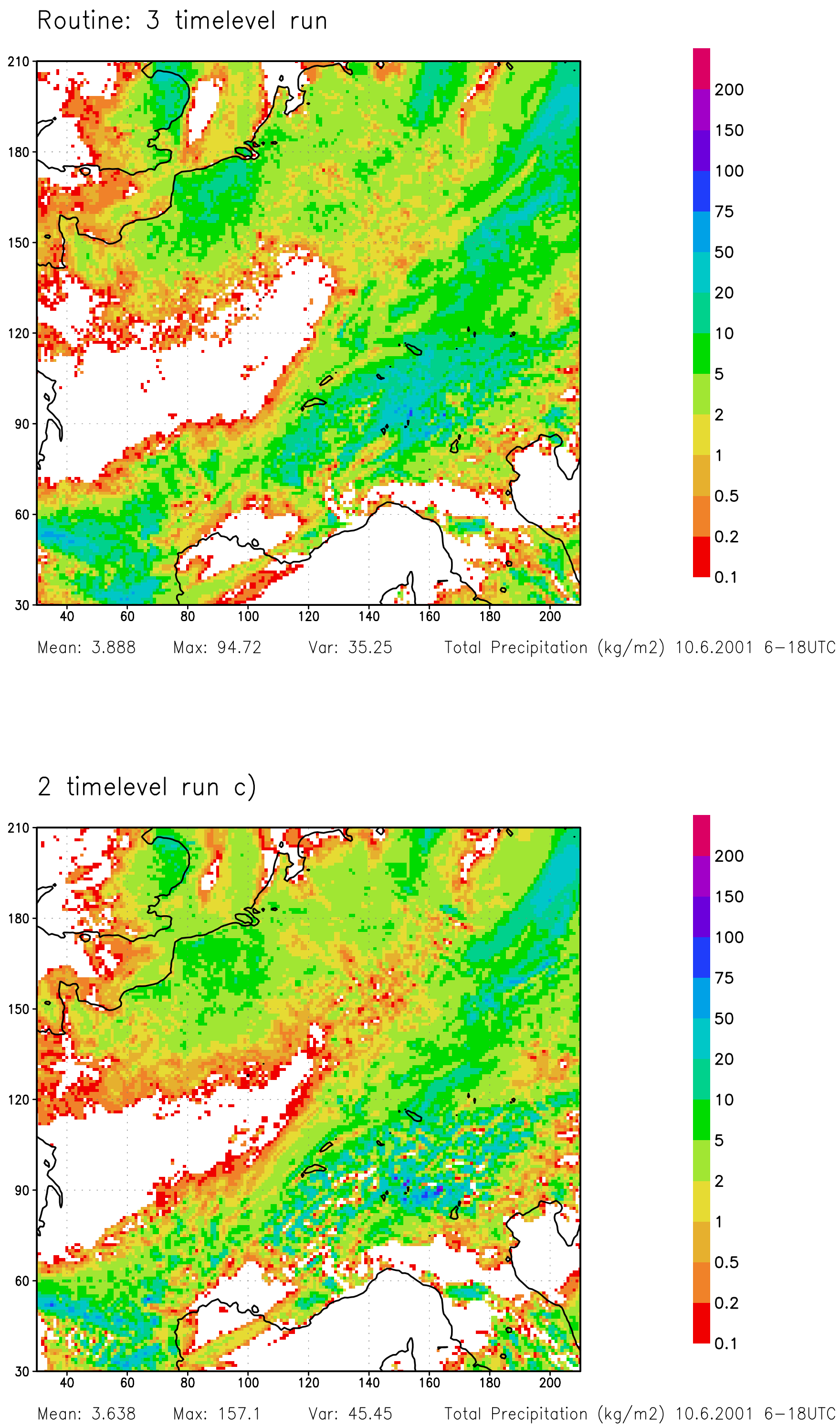

Figure 2: Predicted preciptiation field with the thre timelevel scheme (upper panel) and with the two timelevel scheme (lower panel)