Horizontal nonlinear Smagorinsky diffusion

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1 Introduction

The COSMO model uses several diffusion and damping mechanisms to stabilize the dynamical core. One of these is an artificial 4th order hyper-diffusion acting in the horizontal direction. For example in the COSMO-DE (i.e. the COSMO model setup covering mainly Germany together with the most part of the Alpine region with a horizontal grid mesh size of 2.8 km) it is used to smooth the wind velocity components u, v, and w. The strength of the constant hyper-diffusion coefficient lies at about 5% of its possible maximum value (which is defined by the stability constraint). Only in a boundary zone it has a higher value and additionally the other dynamic variables pressure p' and temperature T' (i.e. their deviations from a reference state) are diffused.

In very rare events this artificial hyper-diffusion is not strong enough to prevent the model from a crash by horizontal shear instabilities. This shows that an additional, more physically based diffusion mechanism in the horizontal is needed.

2 Horizontal Smagorinsky Diffusion

The nonlinear diffusion proposed by Smagorinsky (1963) is formally a purely horizontally acting ('harmonic') diffusion. In Cartesian coordinates it may be written as

$$\frac{\partial u}{\partial t} + \vec{v} \cdot \nabla u = \ldots + K_{smag} \Delta u, \tag{1}$$

$$\frac{\partial v}{\partial t} + \vec{v} \cdot \nabla v = \ldots + K_{smag} \Delta v, \tag{2}$$

$$\frac{\partial w}{\partial t} + \vec{v} \cdot \nabla w = \ldots + 0, \tag{3}$$

with the diffusion coefficient

$$K_{smag} = l_s^2 \cdot \sqrt{T^2 + S^2}, \tag{4}$$

$$T = \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}, \tag{5}$$

$$S = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}. \tag{6}$$

This means that K_{smag} contains both parts of the horizontal tension strain T and of the horizontal shearing strain S. Smagorinsky (1993) pointed out that on the sphere additional metric correction terms must be considered in T and S. But those can be neglected for smaller scale model applications and with the main intention to prevent from model crashes by shear instabilities.

The length scale l_s (a sort of a mixing length) can be determined by the following argument: in any case a stability criterion

$$K_{smag} \cdot \Delta t \left(\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2} \right) \leq \frac{1}{2} \tag{7}$$

must be fulfilled. One can approximately set (motivated by numerical efficiency)

$$l_s^2 = \frac{c}{\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2}} \tag{8}$$

with a yet arbitrary 'Smagorinsky-constant' c. Then from stability constraint it follows, that the *dimensionless* diffusion coefficient

$$k_{smag} := c \cdot \Delta t \cdot \sqrt{T^2 + S^2} \tag{9}$$

must fulfill

$$k_{smag} \le \frac{1}{2}. \tag{10}$$

For example for COSMO-DE with $\Delta x \approx \Delta y \approx 2800$ m and $\Delta t \approx 25$ s and for a shear of $\Delta u = 28$ m/s per grid box this results in $k_{smag} \approx c \cdot \Delta t \cdot \Delta u / \Delta y \approx c \cdot 0.25$.
To get the dimensional value K_{smag} one has to multiply by $\left(\Delta t \cdot \left(\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2} \right) \right)^{-1} \approx 1.6 \cdot 10^5 \ m^2/s$. Smagorinsky (1963) proposes a value of about $c \approx 0.1$. If one uses this diffusion only as a mechanism to reduce shear instabilities without influencing too much the overall model behavior, a value of $c \approx 0.03$ was found as appropriate for COSMO-DE simulations.

To discretize S and T by centered differences in a symmetric way, T is discretized to the scalar position (i, j) whereas S is discretized to the position $(i+1/2, j+1/2)$ ('uv'-position). Afterward they are firstly squared (to prevent from annihilation by negative values) and secondly they are averaged to the u- and v-positions. To avoid a double counting, i.e. that small wavelengths are diffused too strong by both Smagorinsky- and the 4th order (hyper)-diffusion, the dimensionless hyper-diffusion coefficient is subtracted with a weight of 0.5.

This horizontal Smagorinsky diffusion is implemented in the official version COSMO 4.21. There is only one namelist switch (`l_diff_Smag`) to enable or disable it. The two above mentioned parameters are only internal parameters (with the intention that only experienced users should modify them): the Smagorinsky constant `c_smag = 0.03` and the weighting to avoid double counting `weight_K_4th = 0.5`.

3 Case Study

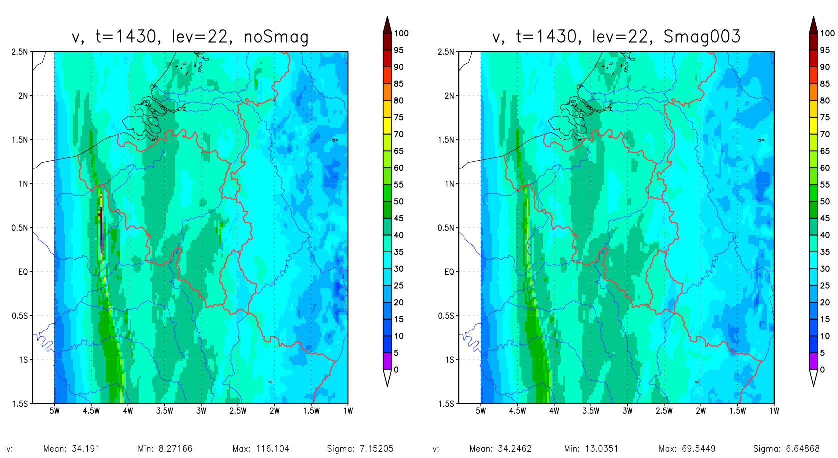

Figure 1: COSMO-DE simulation started at 26. Aug. 2011, 6 UTC: meridional component of wind velocity at level 22 at 20:30 UTC. Left: only linear horizontal hyper-diffusion, right: with additional Smagorinsky-diffusion.

An example which shows the necessity of a more physically based diffusion mechanism is the COSMO-DE at the 26. Aug. 2011. Here both the deterministic run and several runs of the DWD COSMO-DE ensemble prediction system crashed due to a shear instability which occurred in the vicinity of the westerly inflow boundary during the evening. The strong shear can be recognized in Fig. 1 (left) in the meridional wind component v. This strong shear occurred only in two levels but it was sufficient to cause a model abort shortly after this event due to CFL number violation. In contrast, in the simulation with Smagorinsky diffusion (Fig. 1, right) the wind shear is strongly reduced; consequently the model did not crash. Obviously, the Smagorinsky diffusion did not completely destroy the shear along the front. This is not unrealistic, e.g strong shear in the vicinity of so called 'narrow cold frontal rainbands' does occur in reality.

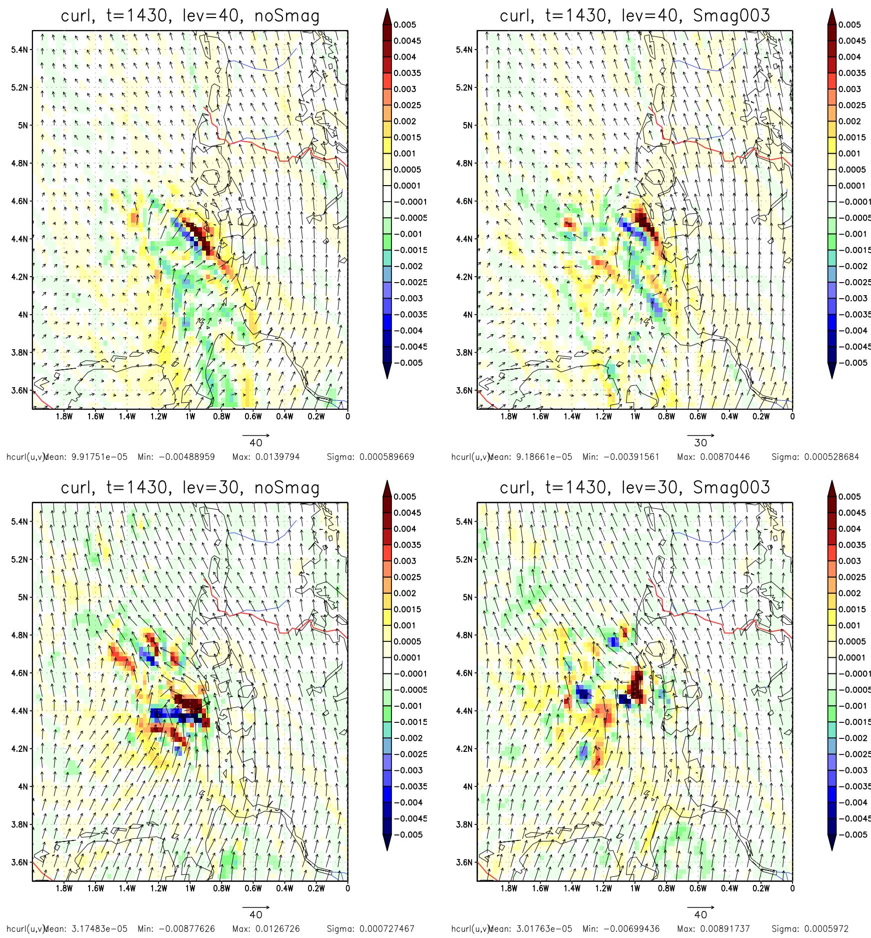

Figure 2: Simulation of 26. Aug. 2011, 6 UTC run, at 20:30 UTC: curl of the horizontal wind field (shaded) and wind velocity (arrows at every third grid position) at about $z \sim 900$ m (top row) and at about $z \sim 3.3$ km (bottom row) above ground. Only horizontal hyper-diffusion (left), and with additional Smagorinsky-diffusion (right).

At about the same time a mesocyclone occurred near the north western coast of Germany. This structure seemed to be realistic, therefore the Smagorinsky diffusion should not destroy such a phenomenon. This is demonstrated in Fig. 2: the runs with additional Smagorinsky diffusion (right) show the mesocyclone with about the same strength and position than the control run (left).

Fig. 3 displays the maximum and volume mean value of the dimensionless diffusion coefficient k_{smag}. The mean value $<k_{smag}> \approx 0.002$ corresponds to a mean value of about $<K_{smag}> \approx$ 300 m^2/s.

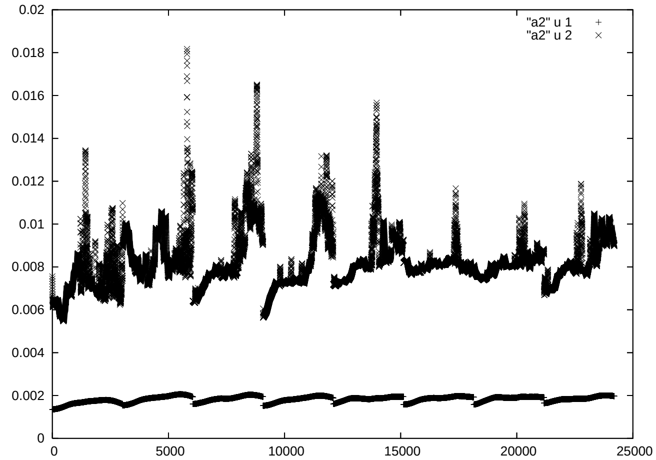

Figure 3: Temporal behavior of maximum (upper curve) and volume mean (lower curve) values of the dimensionless diffusion coefficient k_{smag} for the same simulation.

4 Summary and Outlook

The purpose of this implementation is the stabilization of the dynamical core (and the whole model) against horizontal shear instabilities. The reason for the occurrence of such effects is the lack of a horizontally acting turbulence scheme. The intention of this implementation was to keep the purely vertical turbulence scheme (mainly a Mellor-Yamada type, stage 2.5 scheme with some modifications) and to apply the Smagorinsky diffusion in addition to that. Therefore the 3-dimensional Smagorinsky-diffusion already implemented by Herzog et al. (2002a, 2002b) could not be used (see also the contribution of Langhans et al. (2012) in this COSMO-Newsletter). Moreover for such a stabilization purpose the emphasis lies more on efficiency than on physical accuracy. Therefore no metric terms of the terrain following coordinate were used. Such metric terms were derived for the fluxes and the flux divergence in Baldauf (2005) and can be used for the above mentioned 3D Smagorinsky diffusion (Baldauf, 2006) if the slope of the terrain is not too steep. Therefore, the additional computational costs are moderate with about 1% of the total run time (measured with COSMO-DE on 8 processors on the NEC SX9).

Until now only tests were made for the convection resolving model application COSMO-DE. The verification scores against synoptic observations and against upper air observations for two periods in winter (1.-28. Feb. 2011) and in summer (1.-30. Aug. 2011) were neutral compared to runs without the horizontal Smagorinsky diffusion. Despite this fact, a closer investigation of the influence of any horizontal diffusion to the initiation and development of resolved deep convection would be of interest. It should be further tested if this Smagorinsky diffusion should be applied for the larger-scale model applications, too (e.g. for COSMO-EU with 7 km horizontal grid mesh size).

References

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