Implementation of the 3D-Turbulence Metric Terms in LMK

Michael Baldauf

Deutscher Wetterdienst, Offenbach am Main, Germany

1 Introduction

At the Deutscher Wetterdienst the numerical weather forecast model LMK (LM-Kürzestfrist) is currently under development. It is based on the LM (Lokal-Modell) and will be used for very short range forecasts (up to 18 hours) and with a resolution on the meso-γ-scale (about 2.8 km). The development tasks cover the areas of data assimilation, numerics (e.g. Förstner et al., 2005), physical parameterisations (e.g. Theunert and Seifert (2005), and Reinhardt and Seifert (2005)), and new verification approaches (e.g. Lenz and Damrath, 2005).

In Baldauf (2005) the formulation of the turbulent fluxes and flux divergences in terrain following coordinates and for spherical base vectors was derived. The result for the scalar flux divergence is

$$\begin{aligned}\rho\frac{\partial s}{\partial t} = & \underbrace{-\frac{1}{r\cos\phi}\frac{\partial H^{*1}}{\partial\lambda}}_{(a)}\underbrace{-\frac{J_\lambda}{\sqrt{G}}\frac{1}{r\cos\phi}\frac{\partial H^{*1}}{\partial\zeta}}_{(b)}\underbrace{-\frac{1}{r}\frac{\partial H^{*2}}{\partial\phi}}_{(c)}\underbrace{-\frac{J_\phi}{\sqrt{G}}\frac{1}{r}\frac{\partial H^{*2}}{\partial\zeta}}_{(d)}\underbrace{+\frac{1}{\sqrt{G}}\frac{\partial H^{*3}}{\partial\zeta}}_{(e)} \\ & \underbrace{-\frac{2}{r}H^{*3}}_{(f)}\underbrace{+\frac{\tan\phi}{r}H^{*2}}_{(g)},\end{aligned} \tag{1}$$

and for scalar fluxes:

$$H^{*1} = -\rho K_s\frac{1}{r\cos\phi}\left(\frac{\partial s}{\partial\lambda}+\frac{J_\lambda}{\sqrt{G}}\frac{\partial s}{\partial\zeta}\right), \tag{2}$$

$$H^{*2} = -\rho K_s\frac{1}{r}\left(\frac{\partial s}{\partial\phi}+\frac{J_\phi}{\sqrt{G}}\frac{\partial s}{\partial\zeta}\right), \tag{3}$$

$$H^{*3} = +\rho K_s\frac{1}{\sqrt{G}}\frac{\partial s}{\partial\zeta}. \tag{4}$$

(Analogous expressions for the 'vectorial' diffusion of u, v and w). The terms (a), (c) and (e) in equation (1) describe the cartesian components of the flux divergence and are already contained in the 3D-turbulence scheme of the Litfass-LM (Herzog et al., 2002), which was implemented into the LMK code (Förstner et al., 2004). The metric terms (b) and (d) describe corrections due to the terrain following coordinate. As shown in Baldauf (2005), the terms (f) and (g) are due to the earth curvature and can be neglected with good approximation.

In this article, the implementation of the metric terms (b) and (d), (and also their counterparts in the scalar and vectorial fluxes and vectorial flux divergences) is described. Results of a test procedure are shown and in a first real case study the importance of the 3D-turbulence for meso-γ-models is examined.

2 Implementation and test of the metric terms

In subroutine `explicit_horizontal_diffusion`, the horizontal cartesian terms (a) and (c) are discretised explicitely whereas the vertical cartesian term (e) is in implicit form to consider the fact that the stability criterion can be violated in the case of small vertical level distance or large diffusion coefficients. The metric terms (b) and (d) are some sort of horizontal correction due to the terrain following coordinate system and therefore the first attempt for their discretisation is also in explicit form.

In Fig. 1 the positions of different sorts of variables in the staggered grid are shown. The scalar variables and the diagonal terms of the momentum fluxes are sitting at the center of the grid (an exception of this rule is the scalar variable 'turbulent kinetic energy', which is defined at the w-positions). The velocity variables and the fluxes of scalar variables have positions due to the Arakawa-C/Lorenz grid. The non-diagonal momentum fluxes are again staggered relative to their velocity components. A positive side effect of the Arakawa-C/Lorenz grid is that this sort of staggering simplifies the discretisation of the cartesian components of fluxes and flux divergences: simple centered differences of these terms are easily possible. In contrast to this, for the non-cartesian metric components (like the terms (b) and (d) above) additional averages to other positions in the staggered grid are needed. Surprisingly these additional calculations are not very costly: in the real case study presented below, the complete 3D-turbulence scheme needs about 8.5 % of the total calculation time. The calculation of the metric correction terms alone needs about 3.5 % of the total calculation time of the LMK run (activation of the metric terms will be possible by setting the new namelist-parameter `l3dturb_metr=.TRUE.` in one of the subsequent LM versions).

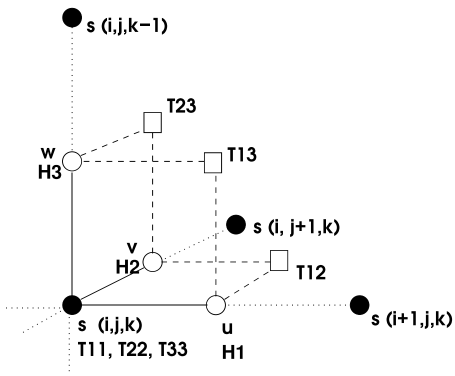

Figure 1: Positions of scalars s, vectors v^i or H^i and 2. rank tensor components T^{ij} in the staggered grid.

For the upper and lower boundaries ($k = 1$ and $k = ke$) all the centered differences were replaced by one-sided differences (for fluxes and flux divergences). In an idealised diffusion test, where the diffusion cloud arrives at the boundaries, no artefacts at the boundaries were observed.

The correct discretisation of the 3D-turbulence and especially the metric terms was tested with an isotropic diffusion problem. It is well known that the diffusion equation (3D) for a tracer ϕ

$$\frac{\partial \phi}{\partial t} = K \Delta \phi \tag{5}$$

with constant diffusion coefficient $K = const.$ and a Gaussian initial distribution

$$\phi(x, y, z, t = 0) = 1 \cdot e^{-r^2/a^2} \tag{6}$$

has also a Gaussian solution

$$\phi(x,y,z,t) = \frac{\Phi_0}{\sqrt{4\pi K(t+t_0)}^3}\, e^{-\frac{r^2}{4K(t+t_0)}}, \qquad r := \sqrt{x^2+y^2+z^2} \tag{7}$$

with

$$t_0 = \frac{a^2}{4K}, \qquad \frac{\Phi_0}{\sqrt{4\pi K t_0}^3} = 1 \tag{8}$$

For this test a resolution of $\Delta x = \Delta y = \Delta z = 50$ m was used[1] with $60 \times 60 \times 60$ gridpoints. For $K = 100$ m^2/s and a good resolution of the initial distribution ($a = 250$ m was chosen) it follows $t_0 = 156.25$ sec. The stability criterion for explicit horizontal discretisation needs a time step of at most $[2K\,((\Delta x)^{-2} + (\Delta y)^{-2})]^{-1} \approx 6$ sec. Here a time step of 3 sec. was chosen.

To test the metric terms an orography with a sinus shape in both horizontal directions $h(x,y) = h_0/2\;(1 + \sin(k_x x)\cdot \sin(k_y y))$, $h_0 = 250$ m, $k_x = 2\pi/(12\Delta x)$, $k_y = 2\pi/(12\Delta y)$, was chosen and shifted in a way that the initial distribution and the orography have no common reflection symmetry planes or points. This orography is plotted in the horizontal cross-section in Fig. 3 (right). Of course, the orography should not at all influence the diffusion process, if it is away enough from the tracer. But it induces a distortion of the numerical grid and this grid distortion has to be corrected by the metric terms.

Three tests were performed:

- case 1: vertical (i.e. 1D) diffusion,
- case 2: 3D-diffusion, without metric,
- case 3: 3D-diffusion, with metric.

In each of the cross-section plots in Figrs. 2 to 4, the analytic solution (7) is plotted with thin, black lines, whereas the numerical solution is plotted with thick coloured lines.

Case 1: Due to the missing horizontal diffusion terms, the 1D-diffusion scheme diffuses anisotropically only in vertical direction as shown in a vertical (x-z-)cross-section in Fig. 2.

Case 2: In the 3D-diffusion scheme without metric terms a horizontal diffusion takes place but the distorted grid also distorts the solution. Comparing this with Case 1, one could be astonished that the 1D-turbulence has no problems with orography. This is indeed the case, because each column of the 1D turbulence scheme works correctly. The false influence of the distorted grid in Case 2 comes purely from the horizontal diffusion terms.

Case 3: In contrast to Case 2 the 3D-diffusion scheme with metric terms is able to reproduce almost correctly the analytic solution. Figure 4 shows vertical and horizontal (i.e. x-z-, y-z-, x-y-) cross-sections and also the temporal behaviour of the numerical solution. As can be seen, the metric terms are able to correct the deformation of the purely cartesian diffusion almost completely. Any mistake in the discretisation would lead to a deviation at least in one spatial direction. Therefore this test should prove the correct discretisation of each single term.

It should be remarked here, that this test is designed only for the scalar variables. It does not work exactly for the velocity components ('vectorial' diffusion). Nevertheless a similar test for the u-component performed rather well, too.

[1] The reason for such a fine resolution is that LM uses a base state with a variation of the density with height. Therefore also the diffusion coefficient (which is multiplied by the density) would no longer be constant. To reduce this artefact in this special test the model area was chosen to have a relatively small vertical extension. Otherwise more parts of LM would have had to be reprogrammed for this test.

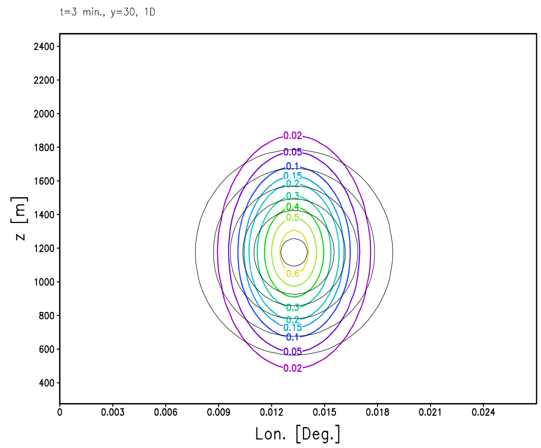

Figure 2: Case 1: 1D diffusion; vertical (x-z-)cross-section after 3 Min., analytic solution (thin, black lines) and numeric solution (thick, coloured lines).

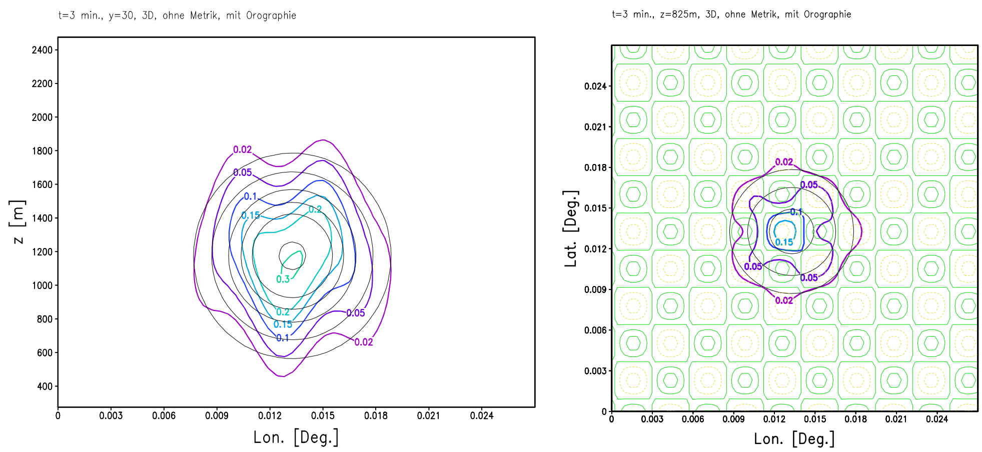

Figure 3: Case 2: 3D diffusion without metric terms; vertical (x-z-)cross-section (left), horizontal (x-y-)cross-section (right).

3 A real case study

To inspect the importance of 3D-turbulence and especially the metric terms a first real case study was performed. The simulation was started at the 12. August 2004, 12 UTC, and lasted 18 h. This was a strong convective situation with the development of a squall line. Again the three cases (1) only 1D (vertical) turbulence, (2) 3D-diffusion without metric terms, and (3) 3D-diffusion with metric terms, were carried out. The simulations were performed with the 'standard' LMK horizontal resolution of 2.8 km and a time step of 30 sec. As in the idealized simulations, the explicit discretisation of the metric terms did not generate any stability problems.

The precipitation sum after 18 h simulation time is plotted in Fig. 5 (upper part). The left plot shows the whole simulation area, whereas the right plot zooms into the part of Southern Germany with a wider distribution of mid-level mountains. This area was chosen, because during the simulation time the precipitation event travelled completely over this area and lied outside of it at the end of the simulation. Therefore in the following difference plots, we can be sure, that the differences do not alter because of new precipitation events.

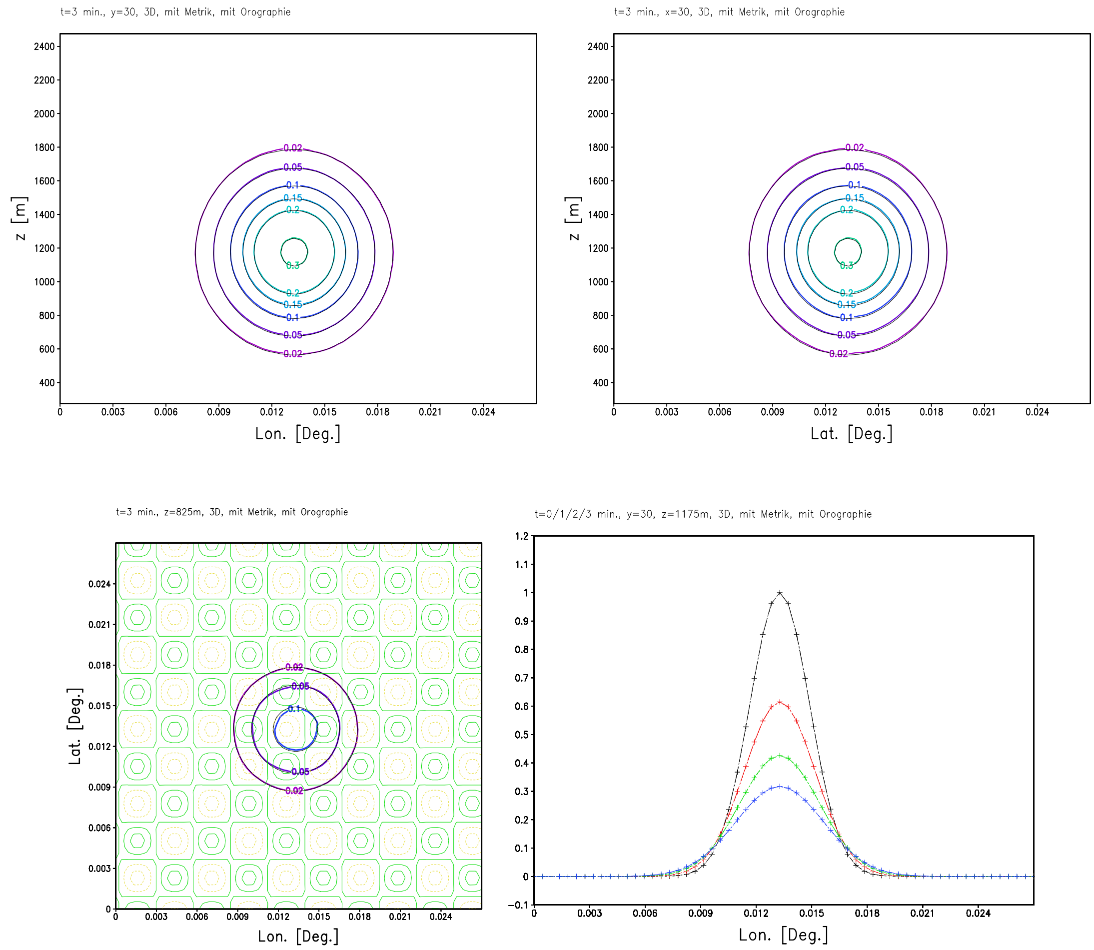

Figure 4: Case 3: 3D-diffusion with metric terms; x-z-cross-section (upper left), y-z-cross-section (upper right), x-y-cross-section (lower left), x-cross-section for different times (lower right).

Figure 5 (below, left) shows a difference plot between Case 3 (complete 3D-turbulence) and Case 1 (only vertical turbulence). The differences can reach maximum and minimum values nearly at the same order as the precipitation sum itself (see Fig. 5). But a look to the mean value shows, that the 3D-turbulence has almost no influence to the total amount of precipitation. This seems to be reasonable, as turbulence occurs mostly in the boundary layer, whereas the most part of precipitation is generated above. But the transport of precipitation (especially with the prognostic precipitation scheme, which is unconditionally necessary at this resolution) is heavily influenced by the boundary layer flow. The 18h-precipitation sum is a marker of all these integrated flow changes due to 3D-turbulence.

The effect of the metric terms themselves is shown in Fig. 5 (below, right), the difference between Case 3 (with metric terms) and Case 2 (without metric terms). The maxima and minima are slightly smaller, but obviously they cannot be neglected in comparison to Case 2 (this was already theoretically derived in Baldauf, 2005). The mean value of the difference 'Case 1 - Case 3' is even smaller as in the difference 'Case 2 - Case 3'. If one accepts the statement, that 3D-turbulence does not alter the total amount of precipitation, then the metric terms obviously have a positive impact on the conservation of total precipitation amount, too.

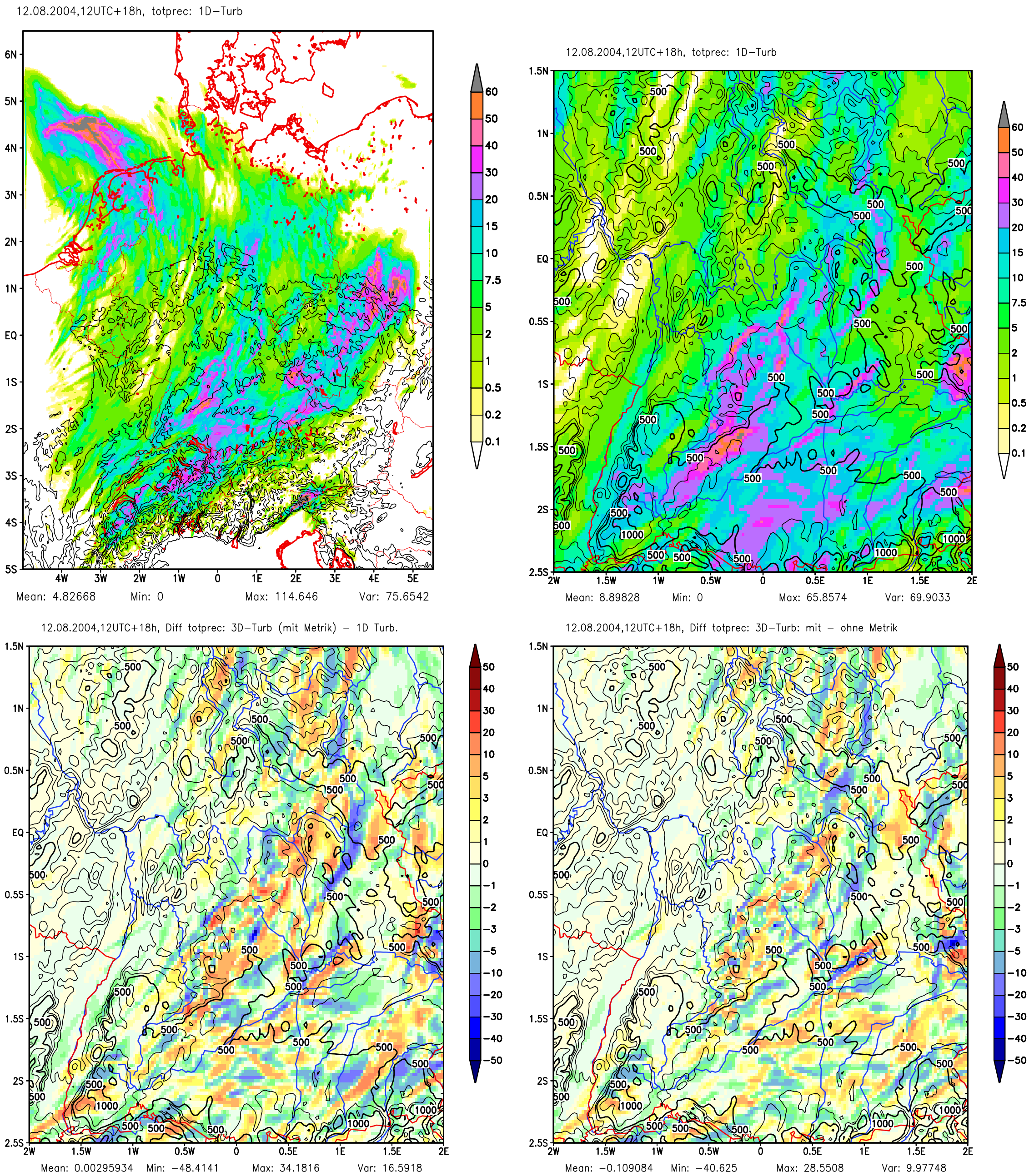

Figure 5: Above: Total precipitation sum over 18h for the 12.08.2004, 12 UTC run with the currently used 1D turbulence. Left: total simulation area, right: zooming into southern Germany.
Below: Differences of the 18h precipitation sum. Left: between 3D-turb. with metric terms and 1D-turb., right: between 3D-turb. with and without metric terms.

A remark should be made about the diffusion coefficient. In the Litfass-LM-turbulence scheme (Herzog et al., 2002) the ratio between horizontal and vertical diffusion coefficient was stretched with the grid length ratio $\Delta x/\Delta z$ with the consequence that near the bottom K_{hor} is about one order of magnitude bigger than K_{vert}, whereas above the boundary layer, K_{hor} is much smaller than K_{vert}. This stretching designed for LES-simulations seems not to be adequate in a statistical turbulence model. Here, as a first guess, the diffusion coefficient was assumed to be isotropic $K_{hor} \equiv K_{vert}$ (e.g. Klemp and Wilhelmson, 1978). This choice has the further advantage to circumvent the problem that the distinction between horizontal and vertical diffusion coefficients becomes much more difficult in steep terrain and should be set on a more profound theoretical basis.

4 Summary and outlook

The correct implementation of the 3D-diffusion terms (at least for the scalar variables) for terrain following coordinates into the LMK was shown with the analytically known case of isotropic Gaussian diffusion. The stabilitiy of the explicit discretisation of the metric terms was demonstrated in this idealised test and also in the real case study.

A certain influence of the 3D-turbulence on the precipitation pattern was found from the one real case study. The 3D-turbulence seems not to alter the total amount of precipitation but can shift the precipitation areas up to 30-40 km. This would be a non-neglectable effect if one has e.g. hydrological applications in mind. Of course further work has to be done to inspect if this influence can be seen also in other weather situations.

References

M. Baldauf, 2005: The Coordinate Transformations of the 3-dimensional Turbulent Diffusion in LMK, *COSMO-Newsletter*, No. 5, 132-140

Förstner, J., M. Baldauf and A. Seifert, 2005: Courant Number Independent Advection of the Moisture Quantities for the LMK, *COSMO-Newsletter*, No. 6

Förstner, J. and H.-J. Herzog and G. Vogel, 2004: Implementation of a 3D-Turbulence Parameterisation for the Very Short Range Forecast Model LMK, *WGNE Blue Book*, `http://www.cmc.ec.gc.ca/rpn/wgne/`

H.-J. Herzog and G. Vogel and U. Schubert, 2002: LLM - a nonhydrostatic model applied to high-resolving simulations of turbulent fluxes over heterogeneous terrain, *Theor. Appl. Climatol.*, 73, 67-86

J. B. Klemp and R. B. Wilhelmson, 1978: The simulation of three-dimensional convective storm dynamics, *J. Atmos. Sci.*, 35, 1070-1096

Lenz, C.-J. and U. Damrath, 2005: First Results on Verification of LMK Test Runs basing on Synop Data, *COSMO-Newsletter*, No. 6

Reinhardt, T. and A. Seifert, 2005: A Three-Category Ice Scheme for LMK, *COSMO-Newsletter*, No. 6

Theunert, F. and A. Seifert, 2005: Simulation Studies of Shallow Convection with the Convection-Resolving Version of the DWD Lokal-Modell *COSMO-Newsletter*, No. 6