Formulation of the LM's Dynamical Lower Boundary Condition

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1 Motivation

What do you expect, if you perform idealized 2D LM-experiments with the following parameters and initial conditions?

- No physics parameterization (pure dynamic core)
- High gaussian mountain (1500m)
- Resting atmosphere (no winds, no horizontal gradients)
- Stable stratification (constant N)
- No Coriolis force
- $\Delta x = 7km, \Delta t = 40s$, 40 predefined unequally spaced vertical levels
- Standard horizontal diffusion

Physically, no change should happen in any of your prognostic variables !
But actually, you will find disastrous errors, $2\Delta z$-structures and large absolute values of the vertical wind (cf. Figure 2). In this study, the reason for that behaviour is investigated and a way to cure the problem is outlined.

2 First small time step error

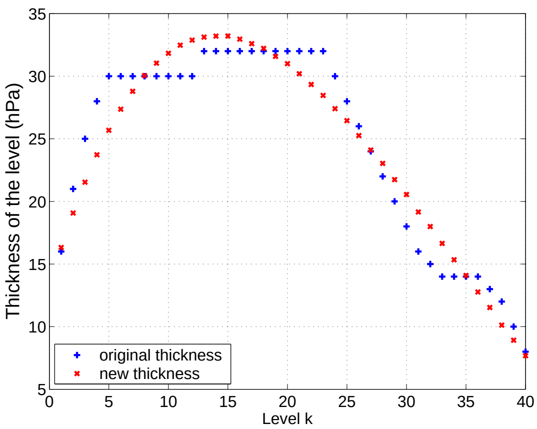
The LM employs a Klemp-Wilhelmson time splitting method. While stepping through the small time steps and computing the fast-waves contributions large time step advective tendencies are kept constant. In the presented experiment, the first small time step yields only contributions from the fast-waves for advection tendencies vanish. The following equation is integrated for the horizontal wind component

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho a \cos\varphi}\left(\frac{\partial p'}{\partial \lambda} - \frac{1}{\sqrt{\gamma}}\frac{\partial p_0}{\partial \lambda}\frac{\partial p'}{\partial \zeta}\right). \tag{1}$$

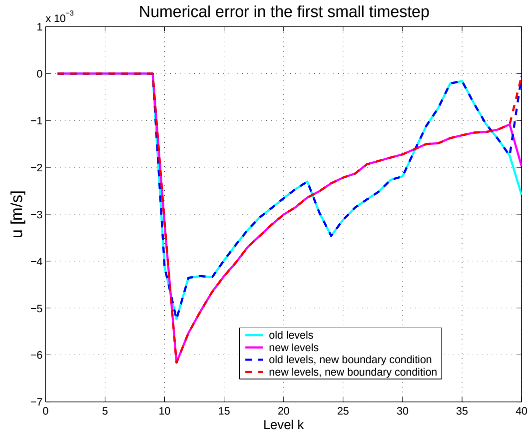
The metrical terms (second term in parantheses) become important near steep slopes. Prognostic variables are staggered on a C-grid. Generally, all differentials are approximated by centered finite differences. Vertical differences are one-sided at the lower boundary and no free slip boundary condition is considered at this stage explicitely.

Using the discontinous level distribution indicated by the blue dots in in Figure 1(a), the horizontal wind at the left slope of the mountain is obtained as shown in Figure 1(b) (light blue line) for the first small time step. Expected to be exactly zero, the deviations of u from zero signify the numerical error.

(a)



(b)



(c)

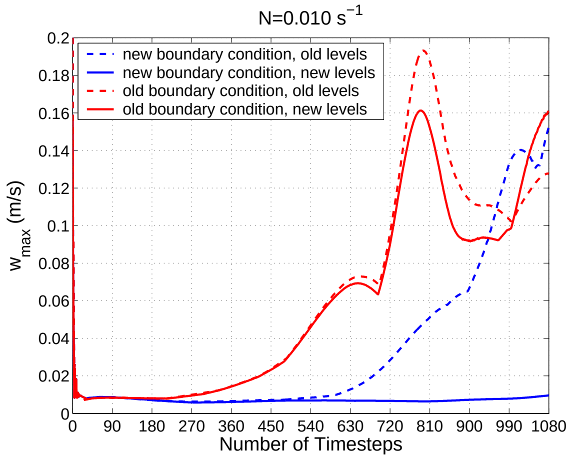


Figure 1:
(a): Vertical level thicknesses for the discontinuous and the continuous level distribution.
(b): Numerical error of the first small time step computation for u for the four different experiments.
(c): Temporal evolution of the domain's maximum absolute value of the vertical velocity.

The numerical error turns out to be quite large at the lowest model level (k=40). But also the numerical error in other levels is very erratic. The upper flat levels are free of errors per definition. In further time steps, $2\Delta z$ noise is induced by the erratic vertical gradients. These occur not only at the lower boundary, but also at higher levels. A deeper inspection reveals that the discontinuous distribution of the level thickness is responsible for a large part of the error-growing.

Using a continuous level distribution, indicated by the red dots in Figure 1(a), reduces the first-step numerical error discontinuities significantly (pink line in Figure 1(b)).

3 Lower boundary condition

The physical definition of the lower boundary is the free slip condition. In the LM, this condition is given by the satisfaction of the requirement

$$\dot{\zeta} = -\frac{1}{\sqrt{\gamma}}\left(\frac{u}{a\cos\varphi}\frac{\partial p_0}{\partial\lambda} + \frac{v}{a}\frac{\partial p_0}{\partial\varphi} + g\rho_0 w\right) = 0.$$

But as already mentioned, this condition is not considered when computing the vertical gradient of p' in equation (1), in fact, one-sided finite differences are used. Accordingly, we deal with an overspecified boundary condition. Omitting this overspecification is expected to improve the lower boundary problem and to prevent growing $2\Delta z$ noise. Thus, use the prognostic equation for the vertical velocity

$$\frac{\partial w}{\partial t} = \frac{g}{\sqrt{\gamma}} \frac{\rho_0}{\rho} \frac{\partial p'}{\partial \zeta} + g \frac{\rho_0}{\rho} \left(\frac{T - T_0}{T} - \frac{T_0 p'}{T p_0} \right)$$

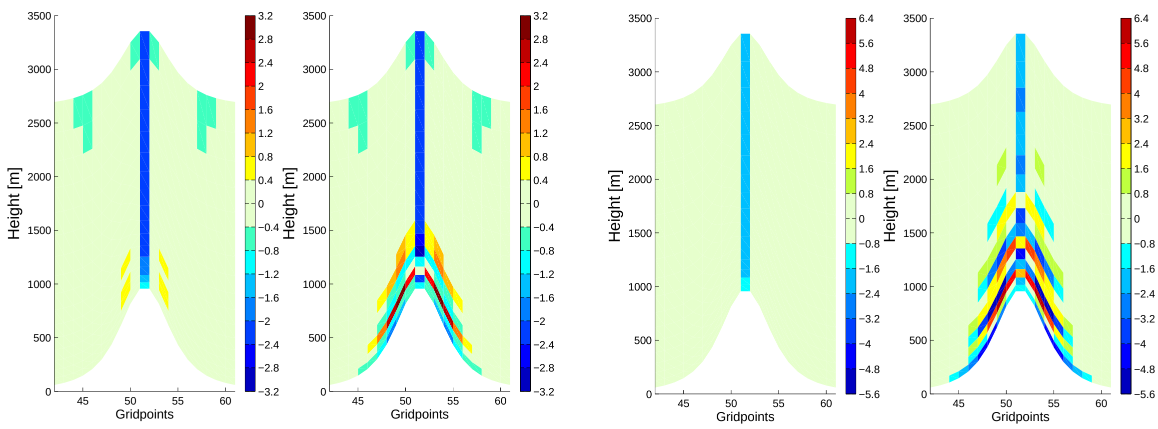
together with $\partial \dot{\zeta} / \partial t = 0$ to obtain the requested $\partial p' / \partial \zeta$ at the lower boundary and determine the horizontal wind. The first step error shrinks with this new condition, as indicated by the dashed lines in Figure 1(b).

The temporal evolution of the maximum absolute value of the vertical velocity in the domain over 12 hours is shown in Figure 1(c) for the four investigated cases that are different combinations of the lower boundary condition type and level distribution type.

Using the continuous level distribution and the new lower boundary implementation prevents the development of noise effectively. If the old boundary condition is applied, level distribution is of minor importance. But the new lower boundary conditioon should be used only together with the continous distribution.

Further experiments suggest that the error growth is the smaller the stronger the stability (Brunt-Vaisala-frequency N) is. The error growth is more or less independent of the ratio of grid spacing to time step and errors grow certainly at some time to a significant value. Thus, error growing might be suppressed but not circumvented by this method.

The vertical wind fields at different times are compared in Figures 2. Here, the continuous level distribution is used to compare the influence of the lower boundary condition alone. The errors with the new condition stay propotionally small, at least for the forecast time investigated. Very erroneous results with strong $2\Delta z$ structures occur for the old boundary formulation. An interesting feature is the negative vertical velocity that occurs exactly over the top of the mountain. It is assumed that it is caused by the divergence introduced by different algebraic signs of the numerical error for u at the two slopes.



6 h 12 h

Figure 2: Vertical wind field at 6 hours forecast (left two panels) and 12 hours forecast (right two panels). Results with the new condition are given at the left hand side and the reference result is shown at the right hand side, color scales are chosen to be comparable.

4 Relevance

A realistic experiment reveals no or very little influence of the lower boundary formulation. Physical processes and a non-vanishing wind field help to dissolve the gradients responsible for the error growth in the idealized experiments.

Nevertheless, a physically consistent boundary condition is numerically tidier. The same is true for the choice of the vertical levels, whose vertical distribution should be a continuous function. The experiments elucidate the importance of a consistent formulation of the model dynamics. In this connection it is worth to pursue these investigations as well as enforce the development of the z-coordinate (Steppeler) and inspect the question of grid-staggering (Herzog).