Incorporating a 3D Subgrid Scale Turbulence Scheme into the 2.8km-Version of the LM

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1 Introduction

The incorporation of a 3D subgrid scale (SGS) turbulence model into the LM code is underway in order to be used in the envisaged model version with 2.8 km horizontal grid spacing. Compared with the grid resolution of the current operational LM (7 km) the intended refinement of the horizontal grid should have also consequences to choose a more adequate turbulence parameterization scheme. Such a scheme is expected to give up the common boundary layer approximation, and it takes into account a completely three-dimensional SGS model instead. In order to come to a reasonable result with not too much effort, we are taking advantage of an existing SGS-model already introduced into and applied with the LLM, which is an LES-like model closely related to the LM, with a horizontal grid distance of 96.5 m. For the sake of working economy this three-dimensional scheme is taken and is introduced into the LM in line with what has been prearranged in the Work Package List for Working Group 3 (Physical Aspects). Although proceeding in a straightforward manner, we have in mind the fact that there is no universal scheme independent of changing the grid resolution as we here meet with. Nevertheless, we think the work is necessary but not yet sufficient. Further work of adaptation toward the LM grid resolution remains to be done.

The SGS model we decided to use is a prognostic turbulent kinetic energy (TKE) scheme taking into account all three space directions. This will here be described in brief. For a more detailed presentation we refer to Herzog et al. (2002a) in connection with an LLM documentation (see also Herzog et al., 2002b). A thorough description of the SGS model is planned to be prepared including also the numerical technique for the necessary explanation of corresponding program code parts.

2 Physical conception of the SGS model

In a first step the prognostic equation set has been extended by taking into account completely the three-dimensionality of the turbulent flux terms instead of flux terms only in vertical direction in the given LM scheme. In the case of the momentum equations the divergence of a flux tensor with its 6 independent components is considered, and, accordingly, a vector flux is used for the heat and water components. The flux components are then specified by use of a first-order closure assumption. This means that the fluxes are set proportional to the local strain tensor (local gradient) for momentum (potential temperature and water components) multiplied by a local turbulence coefficient. Here a distinction is drawn as usual between a coefficient for momentum, K_m, and heat, K_h, respectively. For this specification we refer, for instance, to Mason and Brown (1999).

The present approach specifies the turbulence coefficients after Prandtl and Kolmogorov:

$$K_m = \Phi_m \Lambda \sqrt{e} \tag{1}$$

$$K_h = \Phi_h \Lambda \sqrt{e} \tag{2}$$

Apart from having still undetermined factors Φ_m, Φ_h as functions of stability and turbulence length scale Λ, the $K_{m,h}$ - specification needs the knowledge of turbulent kinetic energy $\overline{e}$. Thus, the prognostic TKE-equation is invoked (e.g. Stull, 1988), but used with a common parameterization of its terms (Stevens et al. 2000). In that way the following equation is found to be useful for our purpose:

$$\frac{\partial \overline{e}}{\partial t} = -\left[\frac{1}{a\cos\varphi}\left(u\frac{\partial \overline{e}}{\partial \lambda} + v\cos\varphi\frac{\partial \overline{e}}{\partial \varphi}\right)\right] - \dot{\zeta}\frac{\partial \overline{e}}{\partial \zeta} - K_h N^2 + K_m S^2 - c_\epsilon \frac{\overline{e}^{3/2}}{\Lambda} + $$
$$+\frac{1}{a\cos\varphi}\frac{\partial}{\partial\lambda}\left(2K_m\frac{\partial \overline{e}}{a\cos\varphi\partial\lambda}\right) + \frac{1}{a}\frac{\partial}{\partial\varphi}\left(2K_m\frac{\partial \overline{e}}{a\partial\varphi}\right) + \frac{g\rho_0}{\sqrt{\gamma}}\frac{\partial}{\partial\zeta}\left(\frac{g\rho_0}{\sqrt{\gamma}}2K_m\frac{\partial \overline{e}}{\partial\zeta}\right) \quad , \qquad (3)$$

where the deformation squared is

$$S^2 = 2\left[\left(\frac{\partial u}{a\cos\varphi\partial\lambda}\right)^2 + \left(\frac{\partial v}{a\partial\varphi}\right)^2 + \left(\frac{g\rho_0}{\sqrt{\gamma}}\frac{\partial w}{\partial\zeta}\right)^2\right] + \left(\frac{\partial u}{a\partial\varphi} + \frac{\partial v}{a\cos\varphi\partial\lambda}\right)^2 +$$
$$+\left(\frac{\partial w}{a\cos\varphi\partial\lambda} - \frac{g\rho_0}{\sqrt{\gamma}}\frac{\partial u}{\partial\zeta}\right)^2 + \left(\frac{\partial w}{a\partial\varphi} - \frac{g\rho_0}{\sqrt{\gamma}}\frac{\partial v}{\partial\zeta}\right)^2 \quad , \qquad (4)$$

and the Brunt-Väisälä frequency squared is

$$N^2 = \frac{g}{\Theta_v}\frac{\partial\Theta_v}{\partial z} \quad . \qquad (5)$$

Here, the TKE equation is written in spherical coordinates and with the LM-specific vertical coordinate. Metrical terms are neclected. To complete the determination of the turbulence coefficients in (1,2), the stability functions must be known. They form a stability-dependent Prandtl number. We have derived these functions from an extended Smagorinsky-typ SGS model (cf. Mason and Brown, 1999), which can be seen as an equilibrium limit case of the prognostic TKE equation (3). In that Smagorinsky model corresponding stability functions are based loosely on observations from the 'Kansas data'. Finally, the length scale Λ is related to grid spacing as a measure of the numerical resolution and is multiplied by a corrective factor in case of an anisotropic grid (Scotti et al., 1997), which is a more drastic 'pancake-like' grid for the LM compared to the LLM grid. For more details we refer to Herzog et al. (2002a).

3 Numerical treatment

The differencing in space and time is made by a strict adaptation to the LM numerics. All rules and definitions of differencing and averaging operators are applied as documented in Doms and Schättler (2002). In detail, the C-grid in the horizontal and the L-grid in the vertical is mandatory to be used. In a first step the incorporation of the three-dimensional divergence of turbulent flux terms has been realized, where metrical simplifications are implied. All horizontal diffusion terms (in the physical and not computational sense!) are treated by forward-in-time differences, and so those terms of vertical momentum flux arising from horizontal inhomogeneities. The vertical diffusion terms are treated partially implicit by a Crank-Nicolson scheme. This leads together with the vertical advection terms to a tridiagonal vertical structure equation to be solved by Gaussian elimination as provided from the LM code. In our case this method has been applied to the τ_{33}-term in the prognostic w-equation, too. For the TKE-equation a mixed time scheme is applied: horizontal advection is approximated by a leap-frog scheme, for vertical advection plus vertical diffusion of TKE a partially implicit Crank-Nicolson scheme is applied, and horizontal diffusion as well source/sink terms (shear production, buoyant production/consumption, dissipation) are treated by a forward scheme. In effect, a tridiagonal form for $\overline{e}$ can be found, to be solved with the Gaussian elimination algorithm in the slow-mode regime (as an extension in the subroutine slow_tendencies). The resulting $\overline{e}^{(n+1)}$ is then available to specify $K_{m,h}$.

4 Conclusions

We emphasize that the running and not yet terminated activity can be seen as a by-product from the DWD project LITFASS, for which the LLM as a real-world LES model has been developed. Further, the LLM development has begun from the LM of that time (version 1.10), where an adequate SGS model for very high grid resolution ($\sim$ 100 m) was not available. It is somewhat comfortable to be now in a position to take advantage of the given LLM for the present LM development toward a 2.8 km grid spacing, and the mutual concern should be to bring the LLM code in that way toward the up-to-day LM level in order to approach a unified LM/LLM code. Finally, we mention that in a first step the adoption of the 3D SGS scheme will not be available for the option of a two-time level scheme.

5 References

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