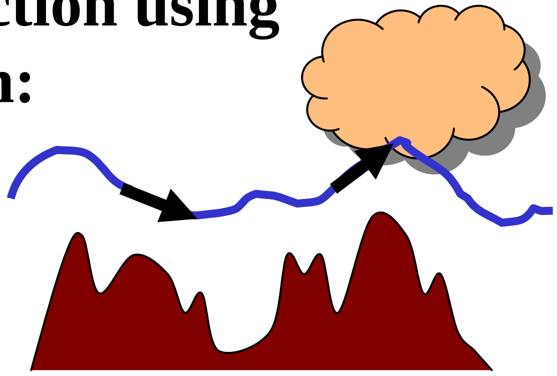
Eddy Dissipation Rate

Improvement of EDR forecast by consideration of horizontal shear and SSO wake production using the concept of scale separation:



DWD-Project:

Improvement of turbulence forecast for civil aviation

Matthias Raschendorfer, FE 14

Motivation:

Turbulence has major impacts on aviation:

NCAR News Release

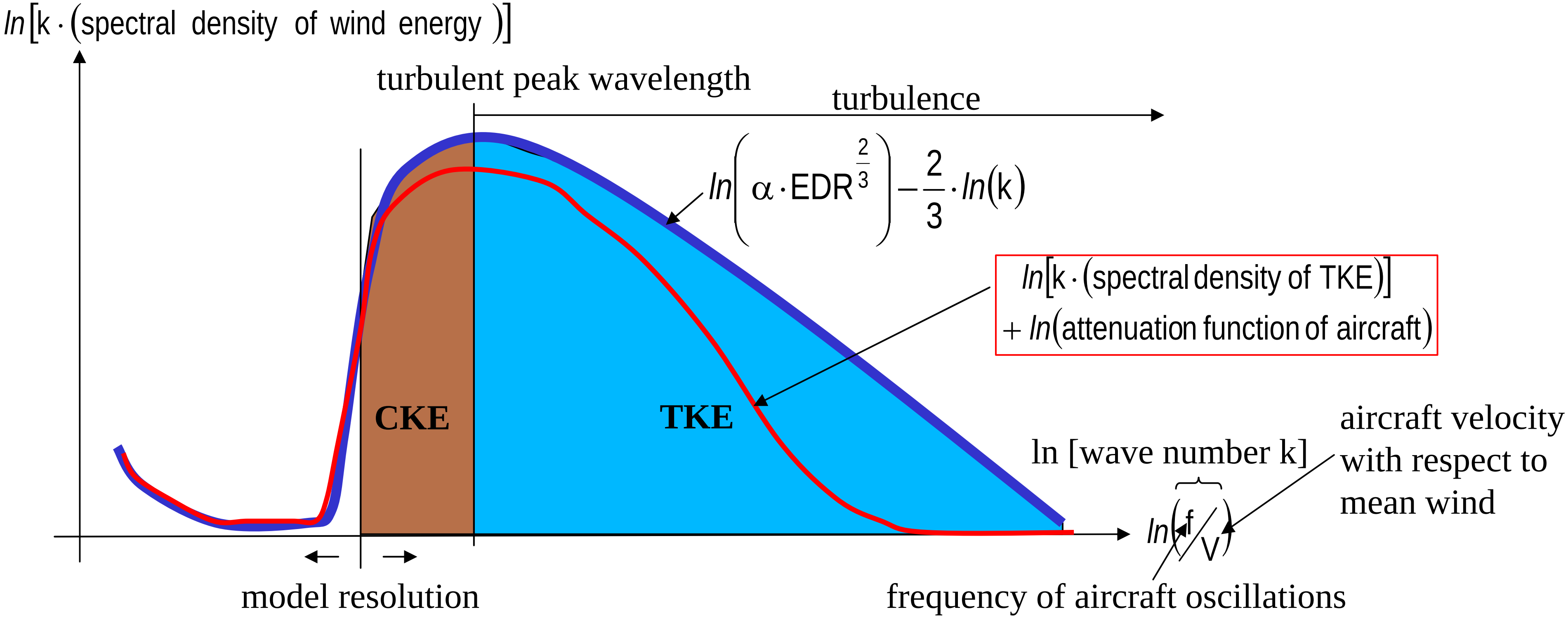
NCAR Teams with United Airlines to Pinpoint Turbulence in Clouds; Research Can Help Reduce Delays, Injuries, Costs

September 6, 2007

Impacts of turbulence

Turbulence has **major impacts on aviation**. According to a review of National Traffic Safety Board data from 1992 to 2001 by the National Aviation Safety Data Analysis Center, turbulence was a factor in at least **509 accidents** in the **United States**, including **251 deaths** (mostly in the general aviation sector). Additionally, the FAA Joint Safety Analysis Team estimated that there are **more than 1,000 minor turbulence-related injuries** on commercial aircraft annually. Airlines **lose millions of dollars every year due to turbulence** because of injury claims, delays, extra fuel costs, and aircraft damage.

Aircraft measurements of EDR:



spectrum of vertical oscillations

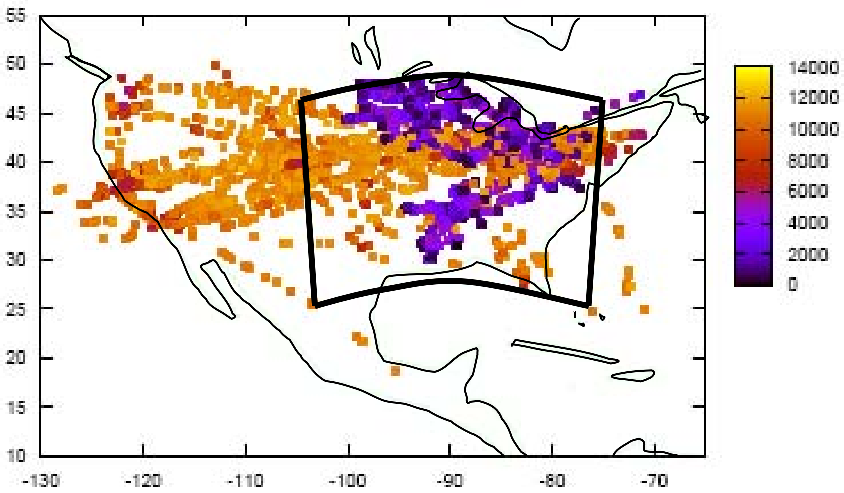
attenuation function
and velocity
of the aircraft

inertial sub range spectrum of atmosphere

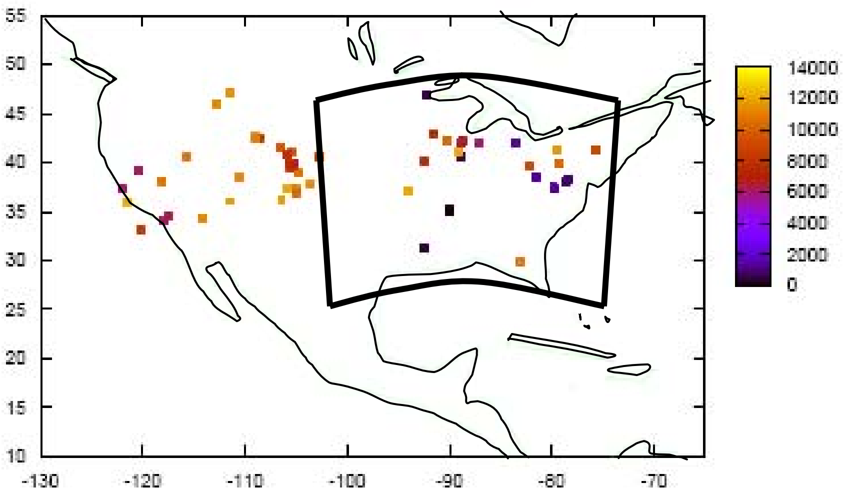
EDR by regression of the Kolmogorov spectrum

Heights (m) of TURBIDX 1,4,5 events

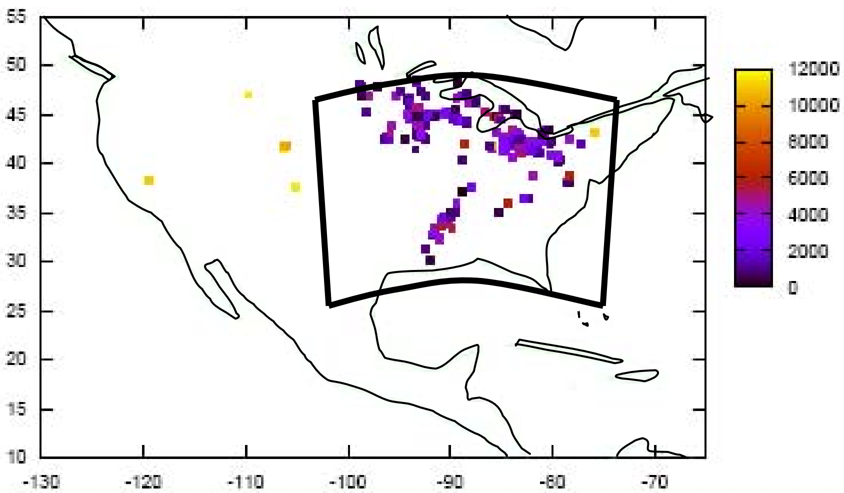
Turbulence index = 1 (light)



Turbulence index = 4 (moderate)



Turbulence index = 5 (severe)



Colours for measurement height in [m]

AMDAR January, 2006

TURBIDX	# Events mo1	# Events wk1
0	1348713	304761
1	9250	2191
2	1070	261
3	188	48
4	66	18
5	336	99
6	27	8
Total	1359649	307386

EDR as part of the budget for sub grid scale kinetic energy SKE:

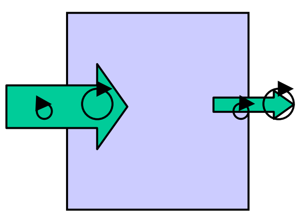
$q^2 := \sum_{i=1,3} \overline{v_i'^2}$ = summation of the velocity variances for all directions = 2 SKE

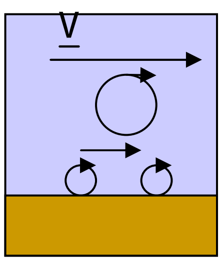
$$\overline{w'\theta_v'}\Big|_{turb} \approx \overline{w'\theta_v'} \quad + \overline{w'\theta_v'}\Big|_{circ} \;\; \text{Circulation production } (P_C) \; \geq 0$$

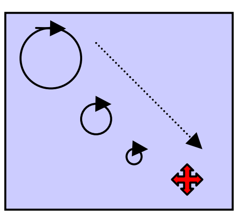
$$\partial_t\left(\frac{1}{2}q^2\right) \approx -\nabla\cdot\left[\frac{1}{2}\left(q^2\underline{v}+\sum_{i=1,3}\overline{v_i'^2\underline{v}'}\right)\right] - \sum_{i=1,3}\overline{v_i'\underline{v}'}\cdot\nabla\overline{v}_i - \upsilon\sum_{i=1,3}\overline{\left|\nabla v_i'\right|^2} + \frac{g}{\overline{\theta}_v}\overline{w'\theta_v'} - \overline{\underline{v}_h'\cdot\left(\nabla_h p\right)'}$$

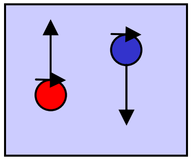
SKE — SKE flux density

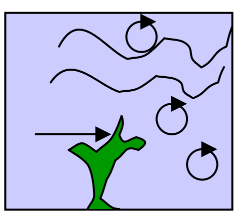
Time tendency of SKE = Convergence of SKE + Shear-Production (P_S) ≥ 0 + Eddy Dissipation Rate (**EDR**) < 0 + Buoyancy-Production (P_A) (unstable: > 0; neutral: $= 0$; stable: < 0) + Sub gr. Scale Orography Production (P_{SSO}) ≥ 0











Turbulence closure on level 2.5 (according to MY):

- We solve the system of **reduced second order equations** (after substituting the turbulent stress tensor by its **traceless** form)
  using special closure assumptions in order to express the unknown statistical moments in those equations
- They are in accordance only with a **special class** of sub grid scale structures that we call turbulence (stochastic isotropic)

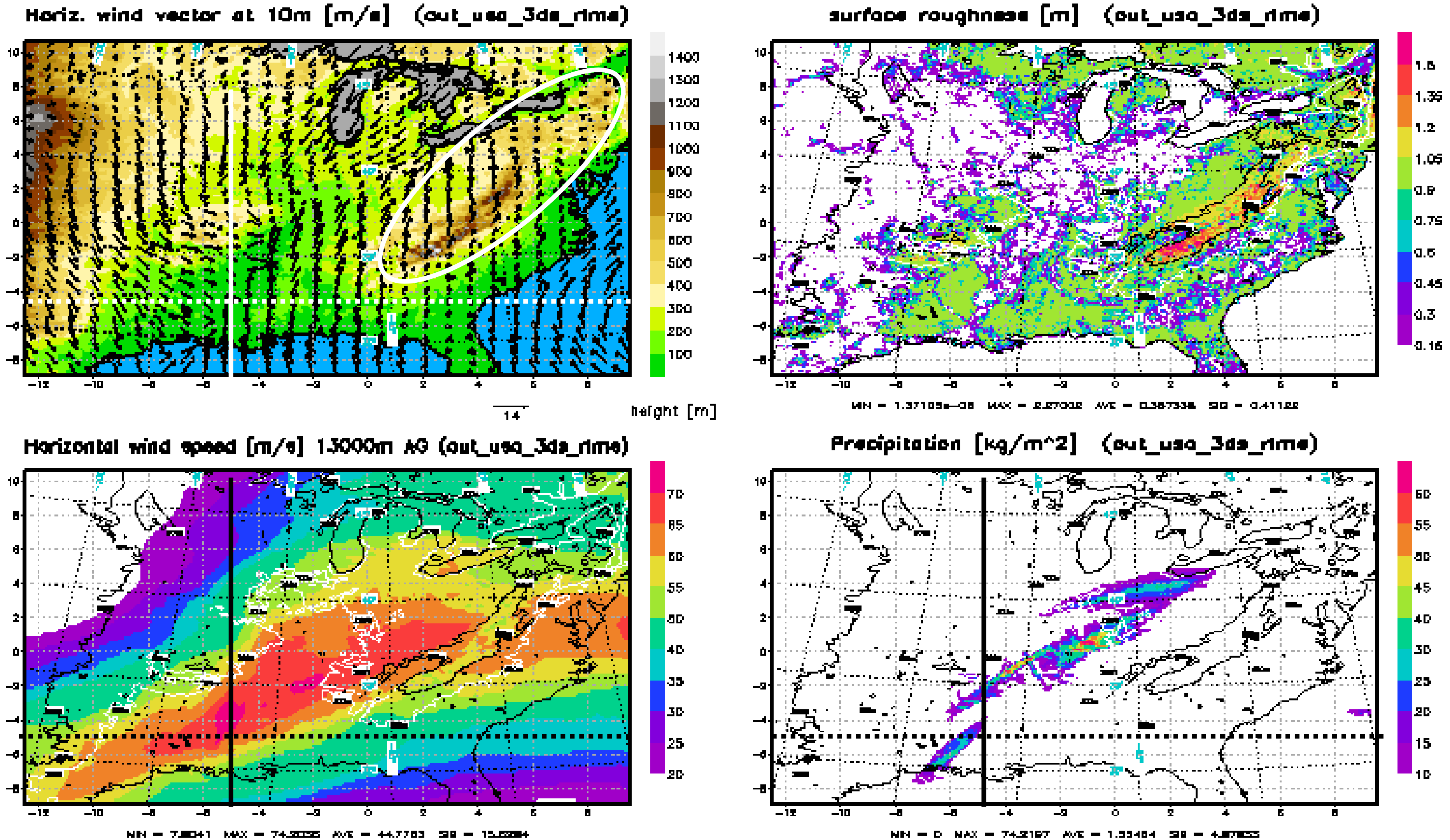
  - In particular we neglect all transport terms and time tendency terms
  - but we use a **prognostic equation for turbulent SKE (TKE)**
  - closure of **pressure correlation** and **dissipation** terms according to **Rotta** and **Kolmogorov** using a **turbulent length scale** ℓ according to **Blackadar**

  yielding in particular: $$EDR \approx -\frac{q^3}{\alpha_{EDR}\ell}$$

  ⇒ All reduced second order budgets degenerate to a **system of linear equations**

- **Neglect** of all **horizontal gradients** (BLA) in the second order budgets:

  ⇒ The only relevant **vertical flux** densities have got **flux gradient form**



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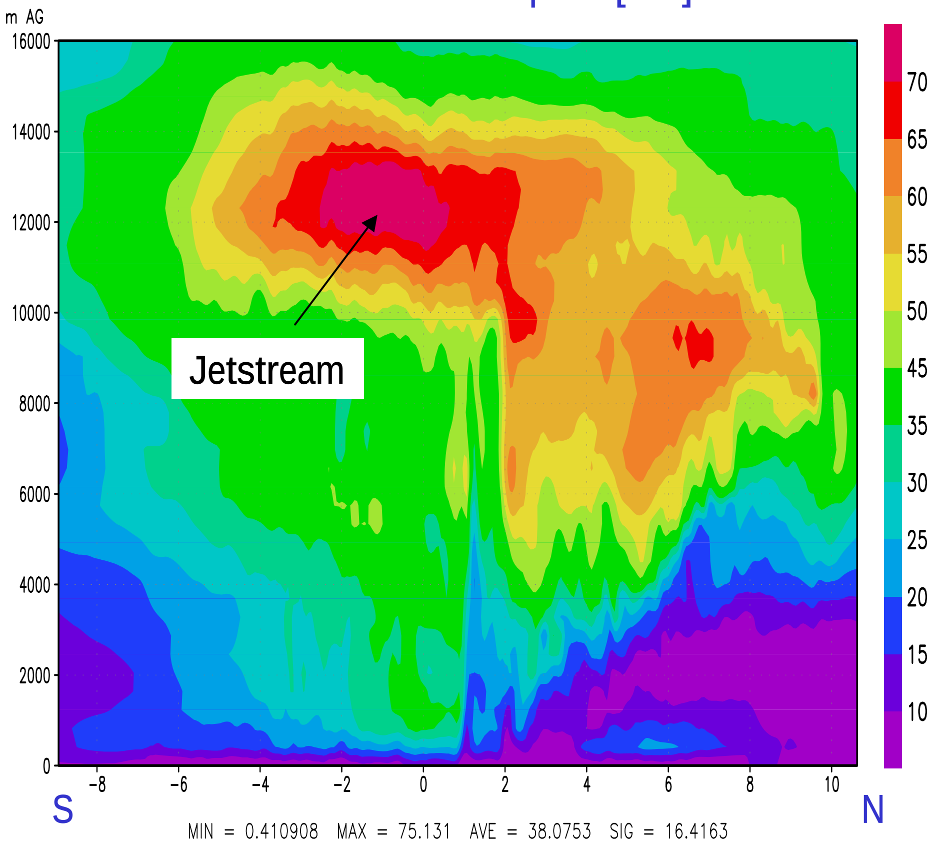
scale interaction by (positive definite **circulation term**) → **Physical based solution** also **above** the BL

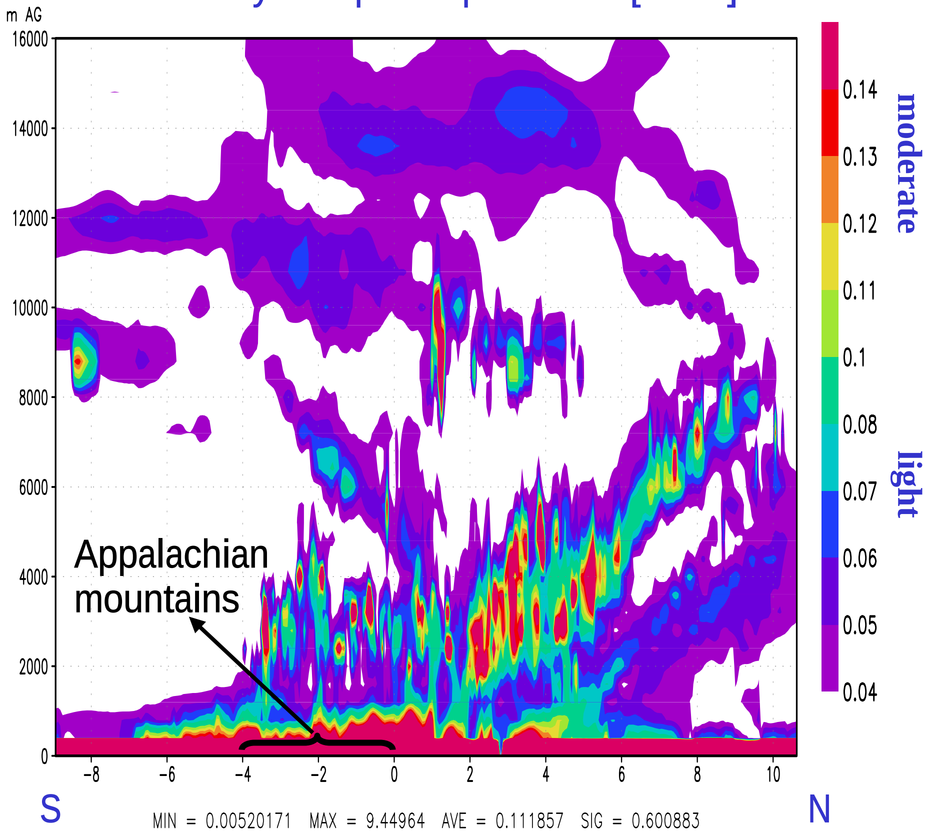
Cross section of model output over the chosen USA domain

from North to South cutting the jet stream

Horizontal wind speed [m/s]



Eddy dissipation parameter [$m^{2/3}/s$]



06.02.2008 00UTC + 12h -90 E

Aim of the DWD-Project:

EDR from turbulence model

- **improved by consideration of <u>horizontal shear</u> and <u>related length scales</u> as an additional TKE production**
- **Improved by considering the interactions between turbulence and <u>mountain blocking</u>**
- Improvement by considering the interactions between turbulence and <u>convection</u>

+

Other proper turbulence indicators, like
- Ri-number, vertical gradients of temperature und wind
- Horizontal shear
- large scale surface roughness
- …

regression model

EDR from aircraft measurements

$$f_{a_1,\dots,a_n}\left(EDR|_{mod}, Ri\_Zahl, Scherung, \dots\right) \approx EDR|_{mes}$$

3D-shear production for turbulent-isotropic structures:

$$\overline{v_i' v_j'} \approx -K^M \cdot \left(\partial_j \bar{v}_i + \partial_i \bar{v}_j\right)$$

isotropic-turbulent contribution to the stress tensor

$$K^M = q \cdot S^M \cdot \ell$$

isotropic-turbulent diffusion coefficient

Isotropic-turbulent 3D shear term:

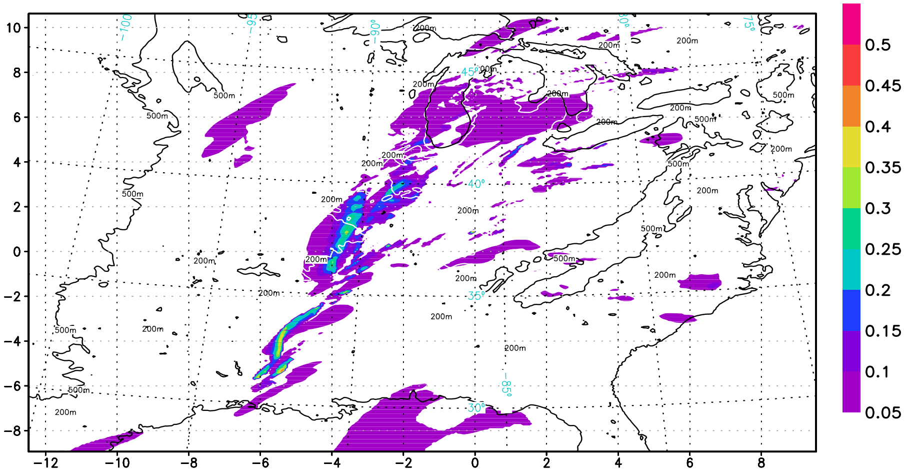
$$P_S := -\sum_{i=1}^{3} \overline{v_i' \underline{v'}} \cdot \nabla \bar{v}_i \approx K^M \cdot \underbrace{\sum_{i,j=1}^{3} \left(\partial_j \bar{v}_i + \partial_i \bar{v}_j\right) \cdot \partial_i \bar{v}_j}$$

$$\left(\partial_1 \bar{v}_2 + \partial_2 \bar{v}_1\right)^2 + \left(\partial_1 \bar{v}_3 + \partial_3 \bar{v}_1\right)^2 + \left(\partial_2 \bar{v}_3 + \partial_3 \bar{v}_2\right)^2$$
$$+ 2\left[\left(\partial_1 \bar{v}_1\right)^2 + \left(\partial_2 \bar{v}_2\right)^2 + \left(\partial_3 \bar{v}_3\right)^2\right]$$

$$\xrightarrow{\text{HBA}} \left(\partial_z \bar{v}_1\right)^2 + \left(\partial_z \bar{v}_2\right)^2$$

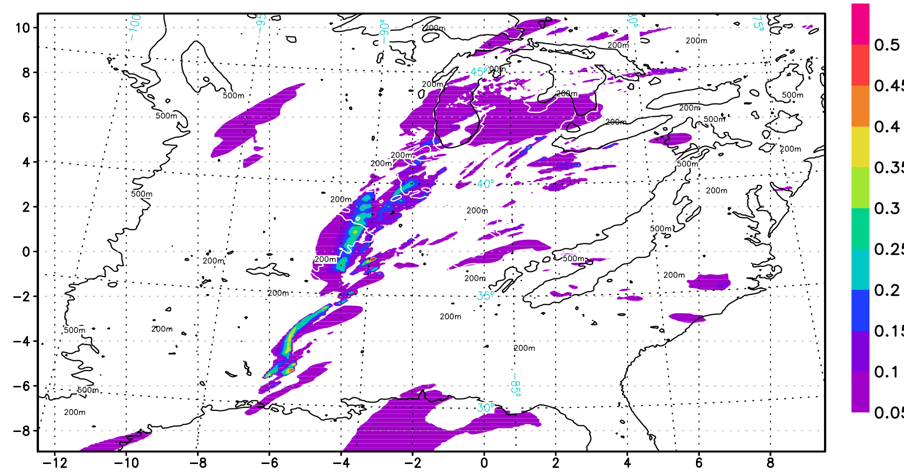
eddy dissipation parameter [m^(2/3)/s]

out_usa_3ds_rlme



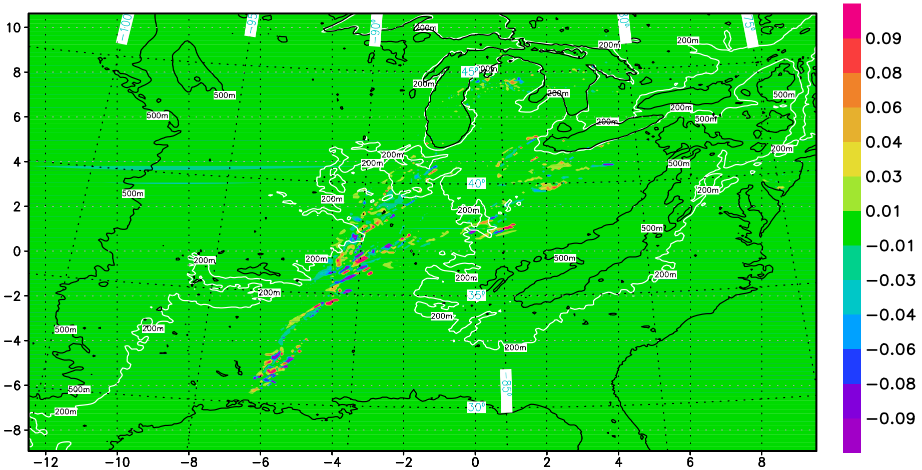
MIN = 0.00294708 MAX = 0.536413 AVE = 0.0371303 SIG = 0.022919

out_usa_3ds_rlme_3dsher



MIN = 0.00293883 MAX = 0.626111 AVE = 0.03717 SIG = 0.0230347

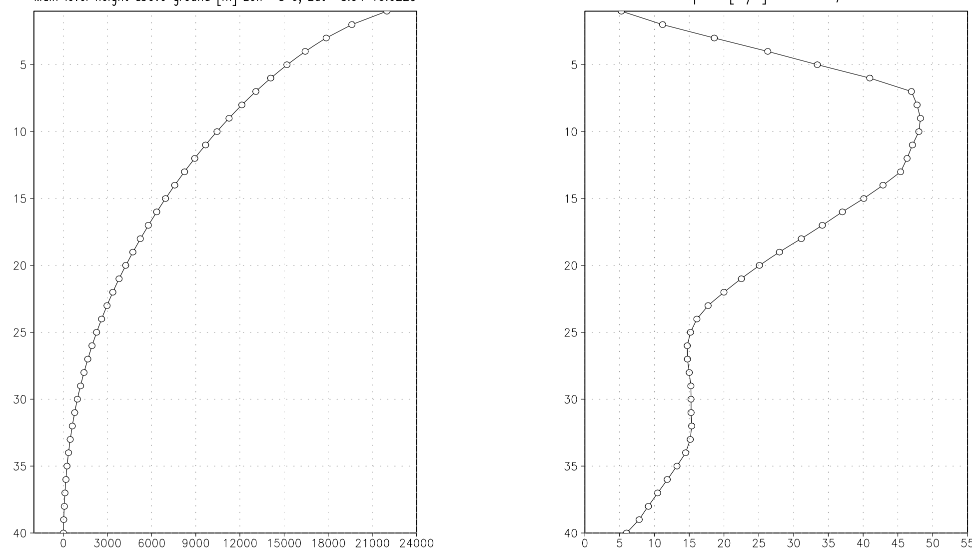
out_usa_3ds_rlme_3dsher – out_usa_3ds_rlme



MIN = –0.498748 MAX = 0.49603 AVE = 3.9615e-05 SIG = 0.0088193

Main level height above ground [m] Lon –8 0, Lat –8.94 10.6225

Horizontal wind speed [m/s] Lon –8 0, Lat –8.94 10.6225

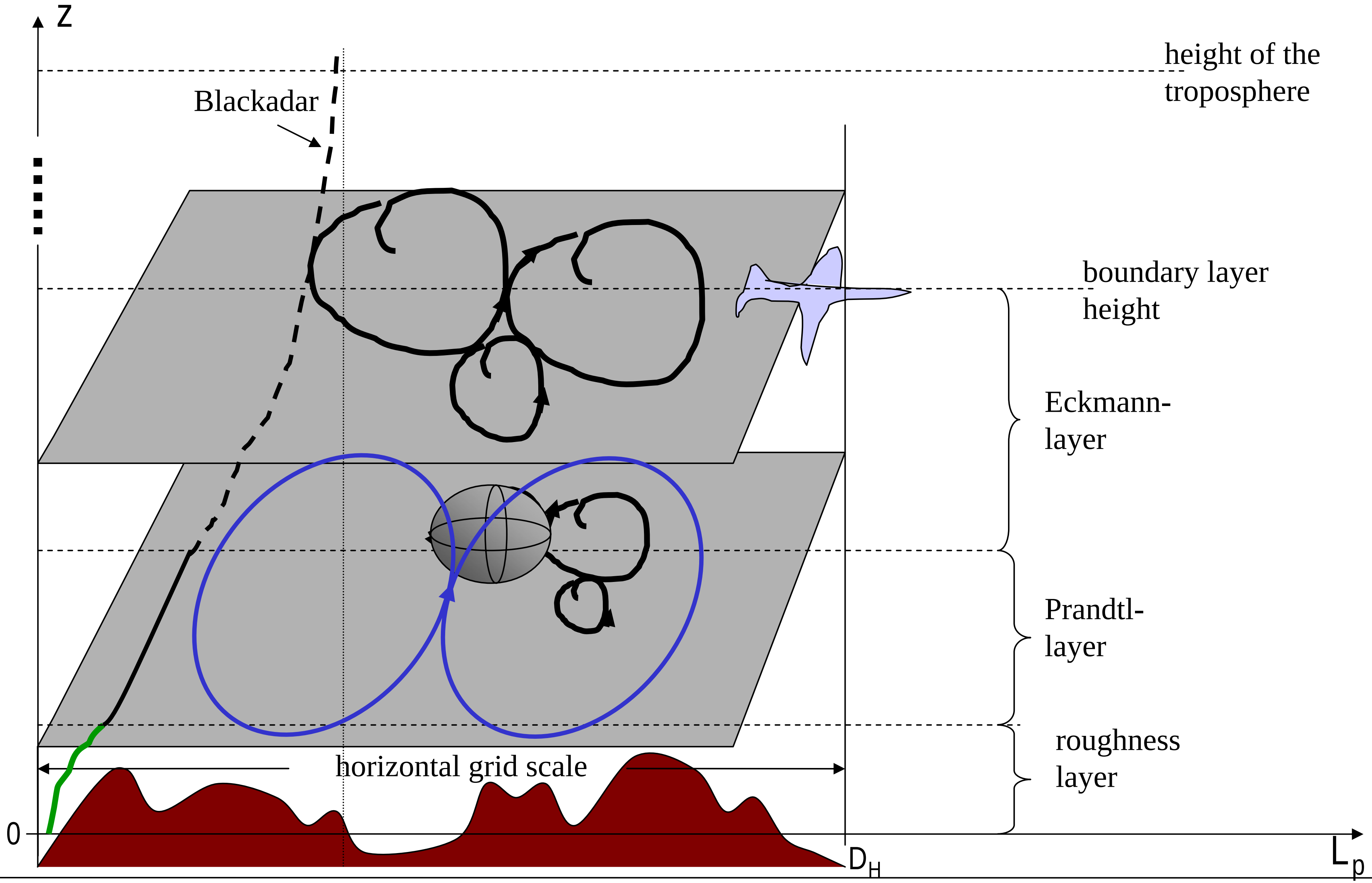


out usa 3ds rlme

out usa 3ds rlme

pr_time=06Z06FEB2008 pr_hour=6hr Lev 12 ≈9000m

Separated horizontal shear mode:



Parameterization of the separated horizontal shear mode:

Separated horizontal shear term:

$$P_{SH} := q_H \cdot \underbrace{\beta_H D_H}_{} \cdot \underbrace{\left[(\partial_1 \bar{v}_2 + \partial_2 \bar{v}_1)^2 + 2(\partial_1 \bar{v}_1)^2 + (\partial_2 \bar{v}_2)^2 \right]}_{=: F_H^M}$$

$\beta_H D_H$: effective mixing length of diffusion by separated horizontal shear eddies: $\beta_H < 1$ related scaling parameter

q_H: velocity scale of separated horizontal shear eddies

Equilibrium with scale transfer towards the turbulence mode:

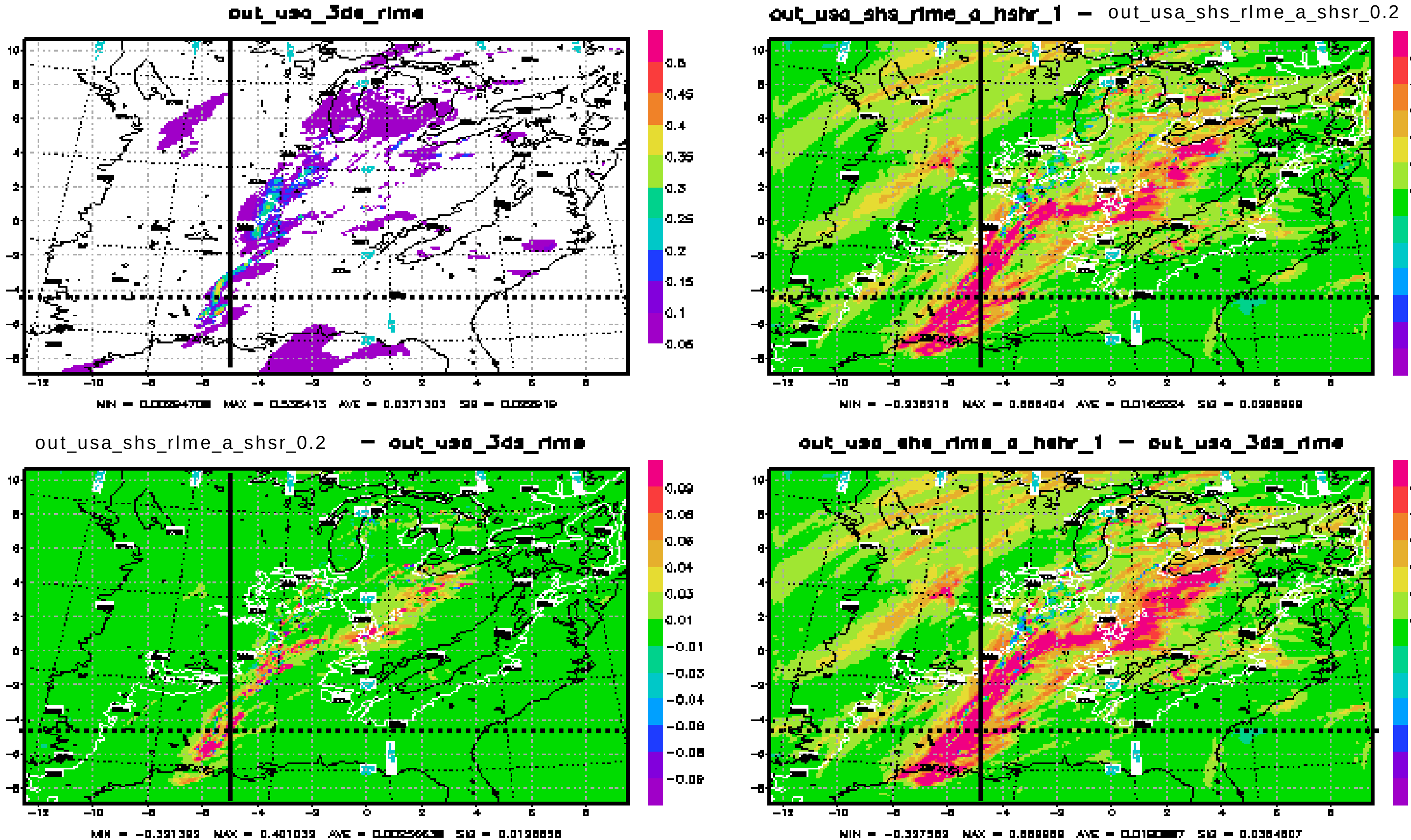
$$q_H \cdot \beta_H D_H \cdot F_H^M = \frac{q_H^3}{\alpha_H D_H}$$

$\alpha_H < 1$ scaling parameter similar to α_{EDR}

$$\Rightarrow \quad P_{SH} = q_H \beta_H D_H \cdot F_H^M = \underbrace{\alpha_H^{\frac{1}{2}} \beta_H^{\frac{3}{2}}}_{=: \alpha_S^2} D_H^2 \cdot F_H^{M\,\frac{3}{2}}$$

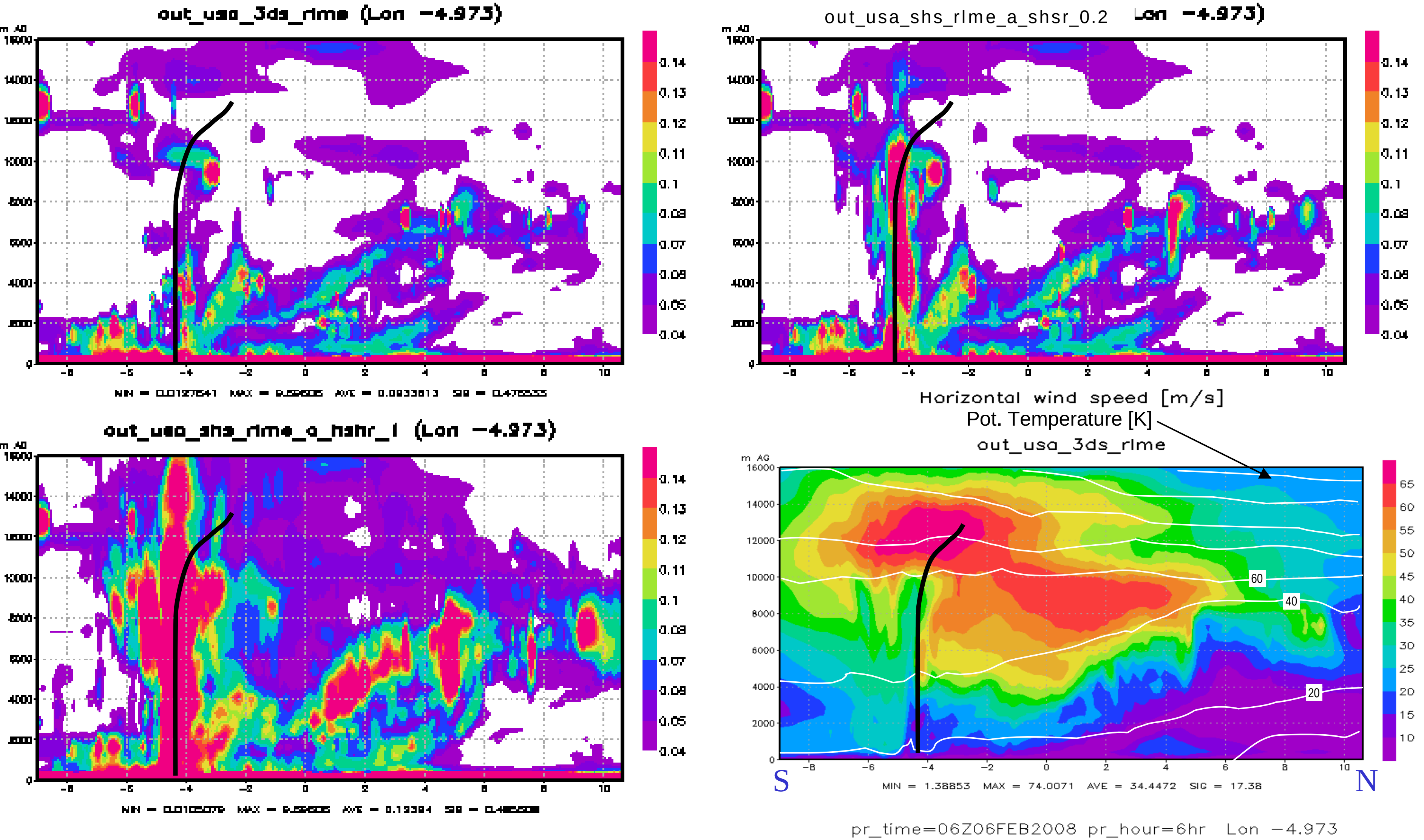
$=: \alpha_S^2$ effective scaling parameter

eddy dissipation parameter [m^(2/3)/s]



pr_time=06Z06FEB2008 pr_hour=6hr Lev 12 ≈9000m

eddy dissipation parameter [m^(2/3)/s]



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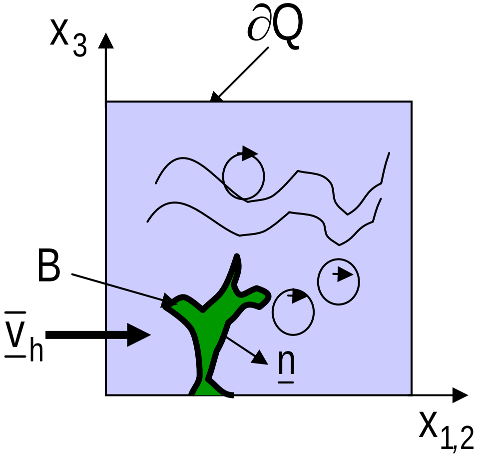
The wake production term:

Filtered momentum budget:

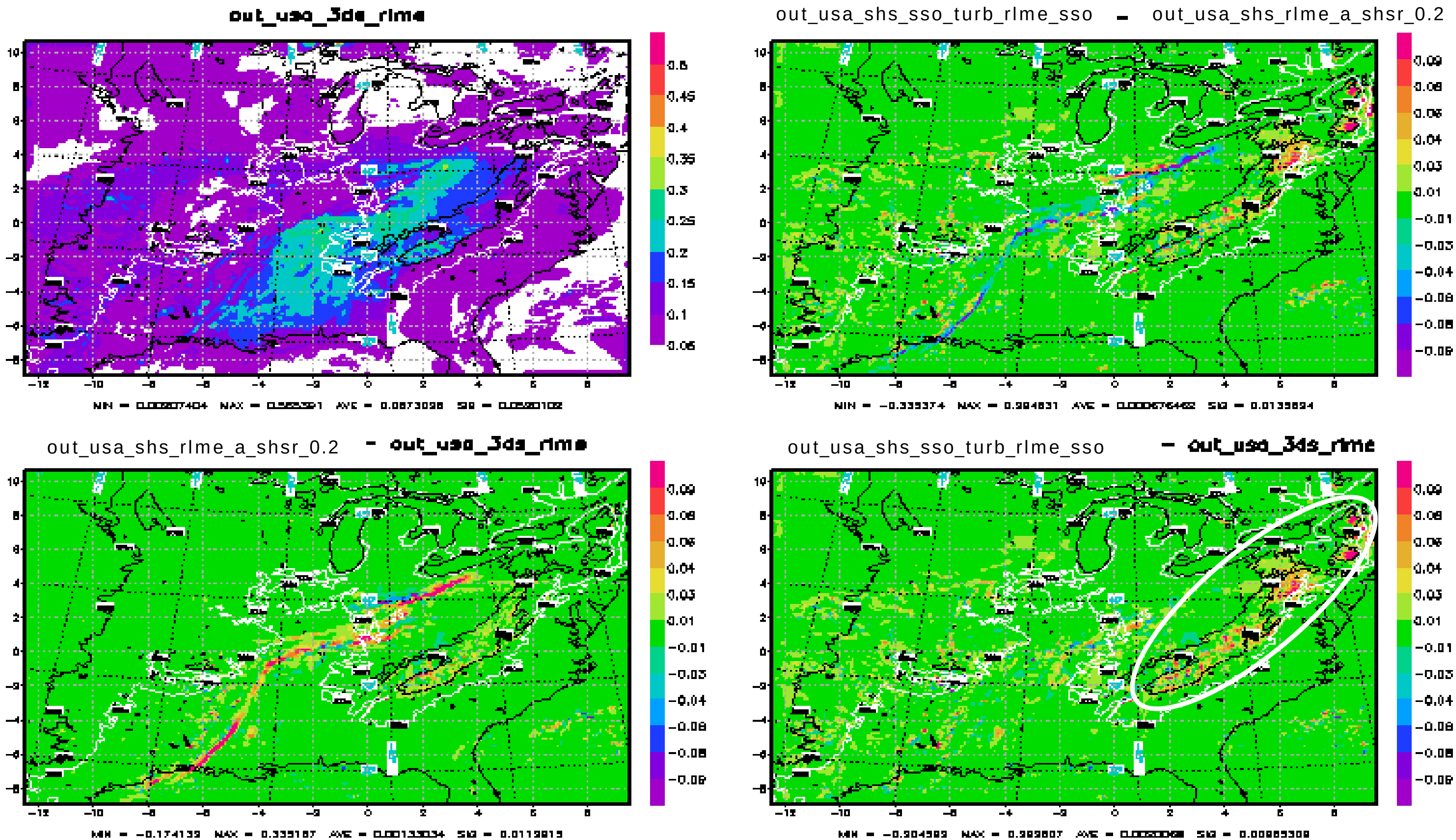
$$\partial_t(\overline{\rho}\,\overline{v}_i) = -\nabla\cdot\left[\overline{\rho}\left(\overline{v}_i\,\underline{\overline{v}} + \overline{v_i'\underline{v}'}\right) - \upsilon\nabla\overline{v}_i\right] - \overline{\rho}g - \partial_i\overline{p} - \underbrace{\frac{1}{|G|}\int_{s\in B} p'(\mathbf{s})\,n_i\,d^2s}_{Q_{SSO}^{v_i}\ \text{blocking}}$$

In the TKE-budget:

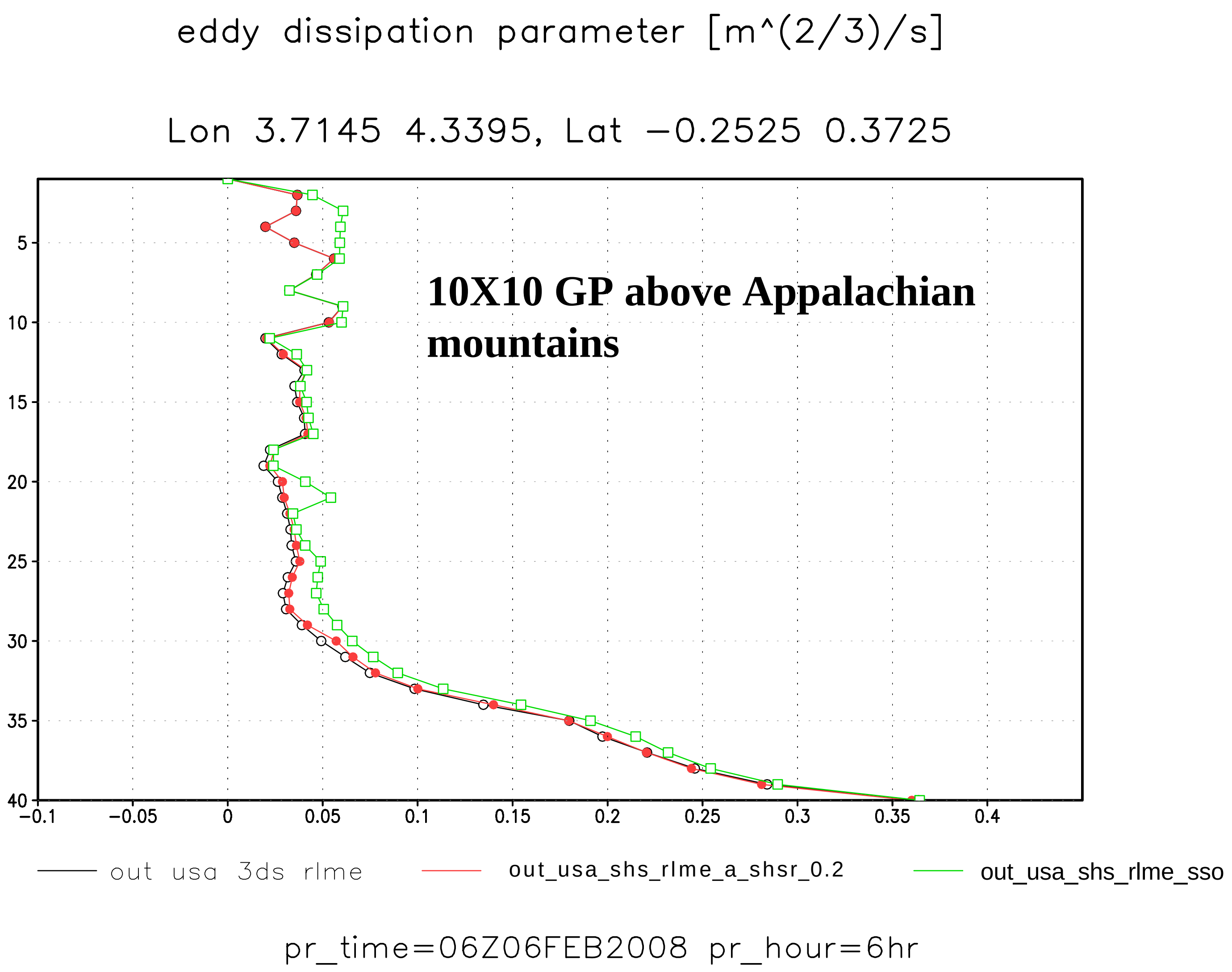
$$P_{SSO} \approx -\overline{\underline{v}'_h\cdot(\nabla_h p)'} \approx \frac{\underline{\overline{v}}_h}{|G|}\cdot\int_{s\in B} p'(\mathbf{s})\,\underline{n}_h\,d^2s = -\sum_{i=1}^{2} v_i\cdot Q_{SSO}^{v_i}$$



eddy dissipation parameter [m^(2/3)/s]



pr_time=06Z06FEB2008 pr_hour=6hr Lev 34 ≈ 350m


eddy dissipation parameter [m^(2/3)/s]
Lon 3.7145 4.3395, Lat −0.2525 0.3725
10X10 GP above Appalachian mountains
5
10
15
20
25
30
35
40
−0.1
−0.05
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
out usa 3ds rlme
out_usa_shs_rlme_a_shsr_0.2
out_usa_shs_rlme_sso
pr_time=06Z06FEB2008 pr_hour=6hr

Conclusion:

- **3D-shear terms** have got a significant effect **only**, when formulated as a **scale interaction term** producing TKE by shear of a **separated horizontal shear mode** with its **own length scale**

- **Wake production** of TKE by **blocking** can be formulated as a **scale interaction term** as well and can be described by **scalar multiplication** of the **horizontal wind vector** with its **SS0-tendencies** yielding some effect above mountainous terrain.

Prospect:

- We intent to derive a similar **scale interaction term** from the **convection** scheme as well.

- Next we plan to introduce a **vertically resolved roughness layer** including a more sophisticated **interaction between turbulence and topographic blocking**.

Thanks for your attention!

Second order closure for the unknown statistical moments:

2-nd order budgets in TA:

sub grid scale macroscopic transport

shear production

$$D_t\left(\overline{\rho\phi''\psi''}\right) := \partial_t\left(\overline{\rho\phi''\psi''}\right) + \overline{\nabla}\cdot\left(\overline{\rho\phi''\psi''}\,\hat{v} + \overline{\rho\phi''\psi''\tilde{v}''} + \overline{\psi''\tilde{e}^{\phi}} + \overline{\phi''\tilde{e}^{\psi}}\right) = -\left(\overline{\tilde{e}^{\phi}}\cdot\overline{\nabla}\hat{\psi} + \overline{\tilde{e}^{\psi}}\cdot\overline{\nabla}\hat{\phi}\right) - \left(\overline{\rho\phi''\tilde{v}''}\cdot\overline{\nabla}\hat{\psi} + \overline{\rho\psi''\tilde{v}''}\cdot\overline{\nabla}\hat{\phi}\right)$$

$$+\left(\overline{e^{\phi}\cdot\nabla\psi} + \overline{e^{\psi}\cdot\nabla\phi}\right)$$ → molecular dissipation

$$+\left(\overline{\psi''Q^{\phi}} + \overline{\phi''Q^{\psi}}\right) + Q^{\phi\psi}_{prs}$$ → phase change production

molecular flux density $e^{\phi} := -a^{\phi}\nabla\phi$

neglected outside the laminar layer

$$Q^{\phi\psi}_{prs} = \begin{cases} 0 & , \ \phi \in H \\ -\overline{\phi''\partial_i p} & , \ \phi = v_i \end{cases} + \begin{cases} 0 & , \ \psi \in H \\ -\overline{\psi''\partial_j p} & , \ \psi = v_j \end{cases}$$

$$-\overline{\phi''\partial_i p} = -\overline{\partial}_i\overline{\phi''p'} \quad \text{(pressure transport)}$$

$$\frac{g}{\hat{\theta}_v}\overline{\rho\phi''\theta_v''} \approx -\overline{\phi''}\,\overline{\partial}_i\overline{p} + \overline{\phi''\left(\partial_i z|_{\sigma}\right)'}\,\partial_z\overline{p} \quad \text{(buoyancy source)}$$

$$+\overline{p'\partial_i\phi''} + \overline{p'\left(\partial_i z|_{\sigma}\right)'}\,\partial_z\hat{\phi} \quad \text{(pressure destruction)}$$

pressure transport

buoyancy source

pressure destruction

Solution in horizontal boundary layer approximation:

- **Neglect** of all **horizontal gradients** in the second order budgets:

⇒ The only relevant **vertical flux** densities have got **flux gradient form**:

$$\overline{\phi' w'} = -K^{\phi} \cdot \partial_z \overline{\phi}$$

$$K^{\phi} = q \cdot \begin{Bmatrix} S^H \\ S^M \end{Bmatrix} \cdot \ell$$

$q = \sqrt{2 \cdot TKE}$

S^H, S^M: **stability function** dependent on $(\partial_z u)^2$ and $\frac{g}{\overline{\theta}_v} \partial_z \overline{\theta}_v$

turbulent diffusion coefficient for scalars (S^H) / momentum (S^M)

ℓ: **turbulent length scale**

$$SP = K^u \cdot (\partial_z u)^2$$ vertical shear production

$$AP = -K^{\theta} \cdot \frac{g}{\overline{\theta}_v} \partial_z \overline{\theta}_v$$ buoyant production

$$Ri := -\frac{AP}{SP} = \frac{g}{\overline{\theta}_v} \frac{S^H \partial_z \overline{\theta}_v}{S^M (\partial_z u)^2}$$

Ri: Richardson-number

What is the idea of turbulence closure?

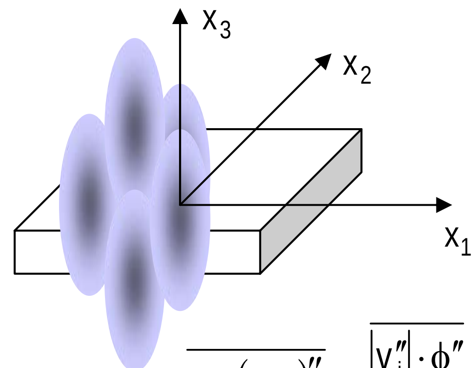
1. Primary turbulent closure assumptions:

Neglect of $D_t\left(\overline{\rho\phi''\psi''}\right)$, in particular all transport terms (equilibrium) except in the SKE equation

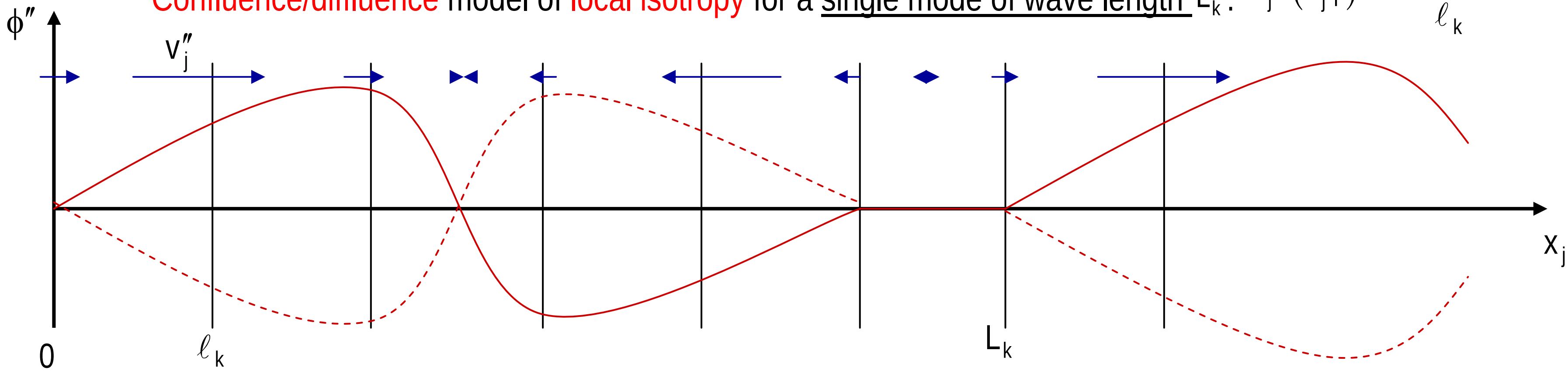
Neglect of sub grid scale phase change production $\left(\overline{\psi'' Q^{\phi}} + \overline{\phi'' Q^{\psi}}\right)$

Neglect of pressure transport $-\bar{\partial}_i \overline{\phi'' p'}$

Bernoulli approximation $p_{tot} := p + \rho \frac{v \cdot v}{2} + \rho g z \approx \text{const}$



Confluence/diffluence model of local isotropy for a single mode of wave length L_k: $\overline{v_j'' \cdot \left(\partial_j \phi\right)''} \approx \frac{\overline{|v_j''| \cdot \phi''}}{\ell_k}$



2. Secondary turbulent closure assumptions:

- **spectral density** of contributing modes follows a power law in terms of wave length in **each direction**:

inertial sub range spectrum

→ - whole spectrum in a given direction is determined by a single **peak wave length**

- the **peak wave length** is the same for samples in all directions: isotropic length scale

→ - pressure correlation and dissipation can be closed using a single turbulent master length scale for each location

Turbulence is that class of sub grid scale structures being in agreement with turbulence closure assumptions!

What is the remaining difficulty with circulations?

- they are related with at least one additional spectral peak

- or they cause different peak wavelengths in vertical direction compared to the horizontal directions:

anisotropic peak wave length

- **larger** peak wave length in **vertical** direction in case of **labile** stratification

at least a two scale problem