The p*-T* - Dynamics of the Runge-Kutta Numerics

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1st Part:

p*-T* - Dynamics

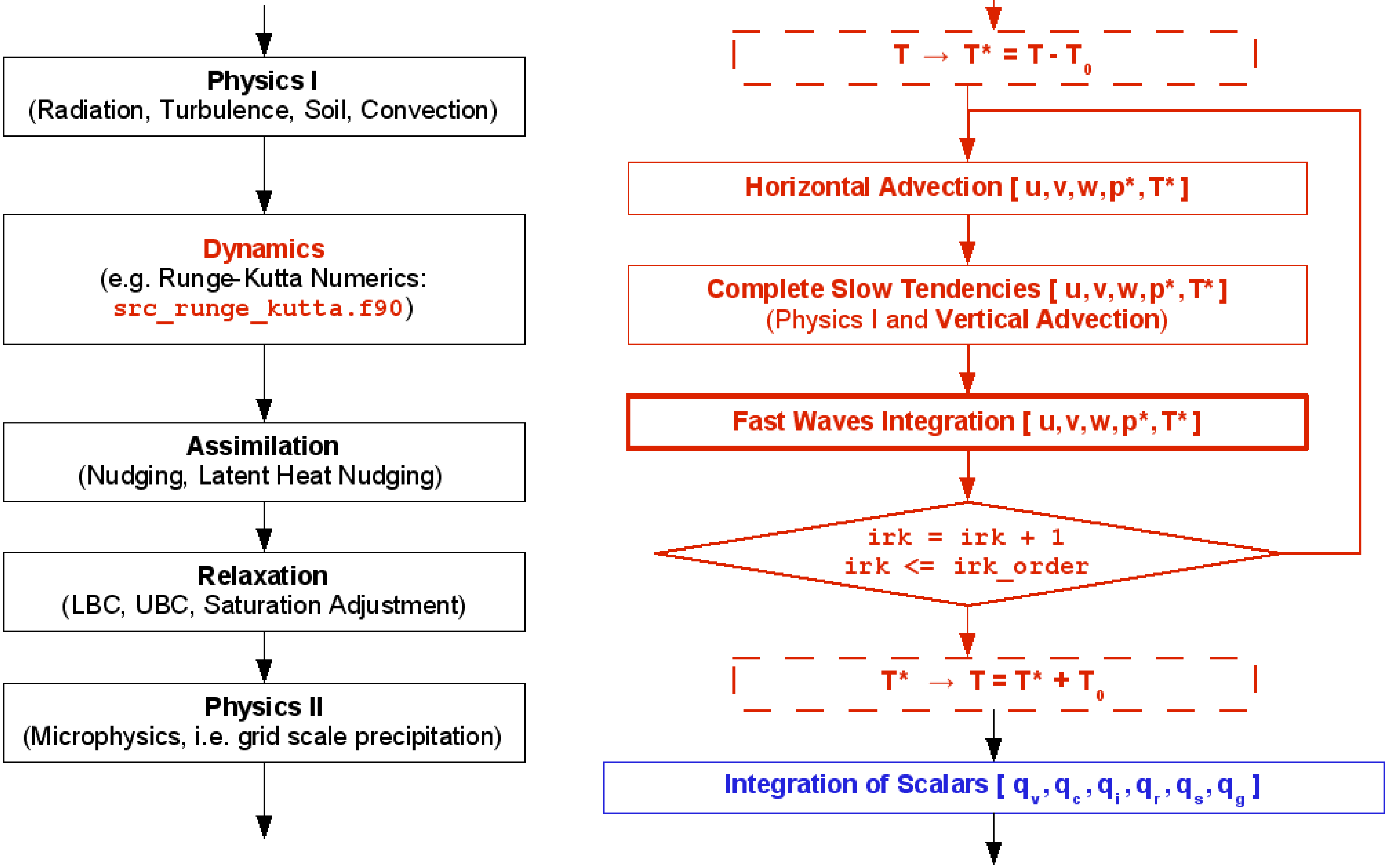
(“Symmetric Thermodynamics”)

a) idealized (Test Case: Atmosphere in Rest)

b) semi-idealized (Inn Valley-Test Case)

c) real (CLM-Test Case)

Flow Chart of an Integration Step



p*-T* - Dynamics (“Symmetric Thermodynamics“)

- Change to a **Prognostic Equation** for the **Temperature Perturbation**

1. Advection of a Decomposed (Reference State + Deviation) Scalar:
   $\psi = \psi_0 + \psi^*$

$$\left(\frac{\partial \psi}{\partial t}\right)^{(n)}_{VADV} = -\overline{\dot{\zeta}^{(n)} \delta_\zeta \psi^{*(n)}}^{\zeta} - \overline{\dot{\zeta}^{(n)} \delta_\zeta \psi_0}^{\zeta}$$

$$= -\overline{\dot{\zeta}^{(n)} \delta_\zeta \psi^{*(n)}}^{\zeta} - \overline{w^{(n)} \frac{\partial \psi_0}{\partial z}}^{\zeta}$$

implicit 2nd order centered-differences

$$+\overline{\left(\frac{J_\lambda}{a\cos\varphi}\overline{u^{(n)}}^{\lambda,\zeta} + \frac{J_\varphi}{a}\overline{v^{(n)}}^{\varphi,\zeta}\right)\frac{\partial \psi_0}{\partial z}}^{\zeta}$$

$$\left(\frac{\partial \psi}{\partial t}\right)^{(n)}_{HADV} = -\frac{1}{a\cos\varphi}\left(\overline{u^{(n)} \delta_\lambda \psi^{*(n)}}^{\lambda} + \cos\varphi\, \overline{v^{(n)} \delta_\varphi \psi^{*(n)}}^{\varphi}\right)$$

explicit 5th order upwind

$$-\overline{\left(\frac{J_\lambda}{a\cos\varphi}\overline{u^{(n)}}^{\lambda,\zeta} + \frac{J_\varphi}{a}\overline{v^{(n)}}^{\varphi,\zeta}\right)\frac{\partial \psi_0}{\partial z}}^{\zeta}$$

2. New Formulation of the Fast Waves Solver

- Vertical Velocity

$$
\begin{aligned}
w^{(\nu+1)} = w^{(\nu)} + \Bigg[& \frac{1}{\overline{\sqrt{G}}^{\zeta}} \frac{1}{\overline{\rho^{(n)}}^{\zeta}} \left\{ \beta^{+} \delta_{\zeta} p^{*(\nu+1)} + \beta^{-} \delta_{\zeta} p^{*(\nu)} \right\} \\
& - g \frac{\overline{\rho_0}^{\zeta}}{\overline{\rho^{(n)}}^{\zeta}} \left\{ \overline{\left(\frac{T_0 \beta^{+}}{T^{(n)} p_0} \right) p^{*(\nu+1)}}^{\zeta} + \overline{\left(\frac{T_0 \beta^{-}}{T^{(n)} p_0} \right) p^{*(\nu)}}^{\zeta} \right\} \\
& + g \frac{\overline{\rho_0}^{\zeta}}{\overline{\rho^{(n)}}^{\zeta}} \left\{ \overline{\left(\frac{\beta^{+}}{T^{(n)}} \right) T^{*(\nu+1)}}^{\zeta} + \overline{\left(\frac{\beta^{-}}{T^{(n)}} \right) T^{*(\nu)}}^{\zeta} \right\} + f_w^{(n)} \Bigg] \Delta\tau
\end{aligned}
$$

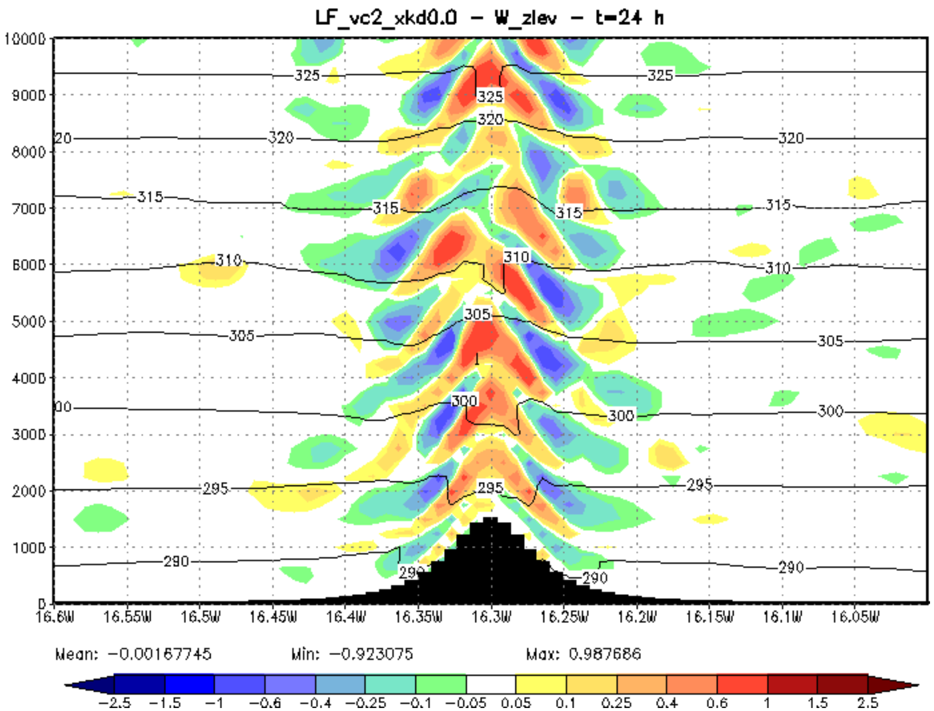
- Pressure Perturbation

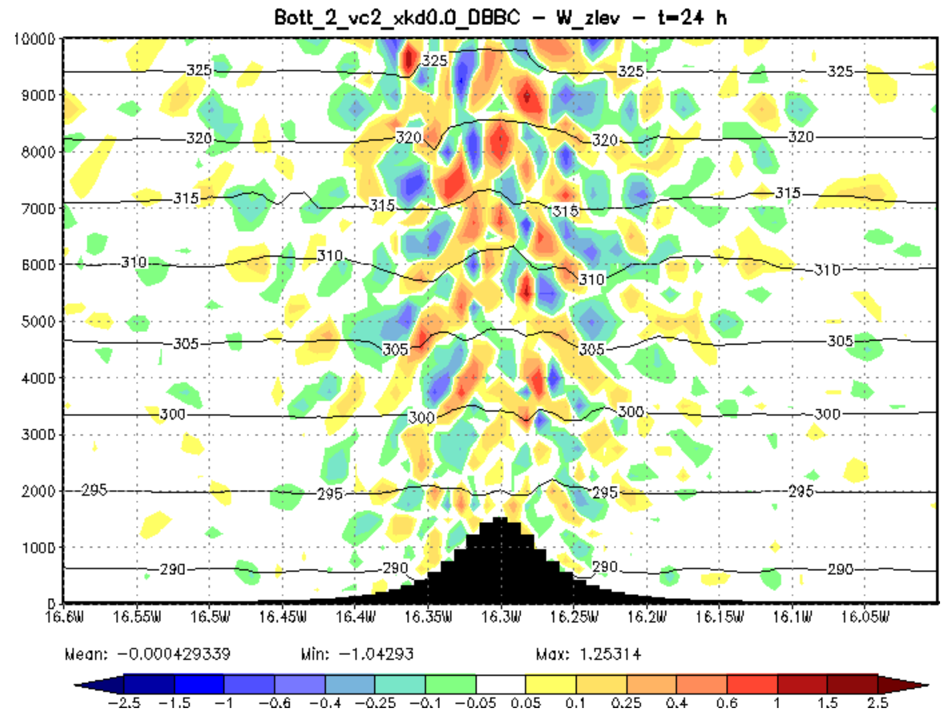
$$
\begin{aligned}
p^{*(\nu+1)} = p^{*(\nu)} + \Bigg[& g \rho_0 \left\{ \beta^{+} \overline{w^{(\nu+1)}}^{\zeta} + \beta^{-} \overline{w^{(\nu)}}^{\zeta} \right\} \\
& + \frac{1}{\sqrt{G}} \frac{p^{(n)} c_{pd}}{c_{vd}} \left\{ \beta^{+} \delta_{\zeta} w^{(\nu+1)} + \beta^{-} \delta_{\zeta} w^{(\nu)} \right\} - \frac{p^{(n)} c_{pd}}{c_{vd}} D_h^{(\nu+1)} + f_{p^*}^{(n)} \Bigg] \Delta\tau
\end{aligned}
$$

- Temperature Perturbation

$$
\begin{aligned}
T^{*(\nu+1)} = T^{*(\nu)} + \Bigg[& -\frac{dT_0}{dz} \left\{ \beta^{+} \overline{w^{(\nu+1)}}^{\zeta} + \beta^{-} \overline{w^{(\nu)}}^{\zeta} \right\} \\
& + \frac{1}{\sqrt{G}} \frac{p^{(n)}}{\rho^{(n)} c_{vd}} \left\{ \beta^{+} \delta_{\zeta} w^{(\nu+1)} + \beta^{-} \delta_{\zeta} w^{(\nu)} \right\} - \frac{p^{(n)}}{\rho^{(n)} c_{vd}} D_h^{(\nu+1)} + f_{T^*}^{(n)} \Bigg] \Delta\tau
\end{aligned}
$$

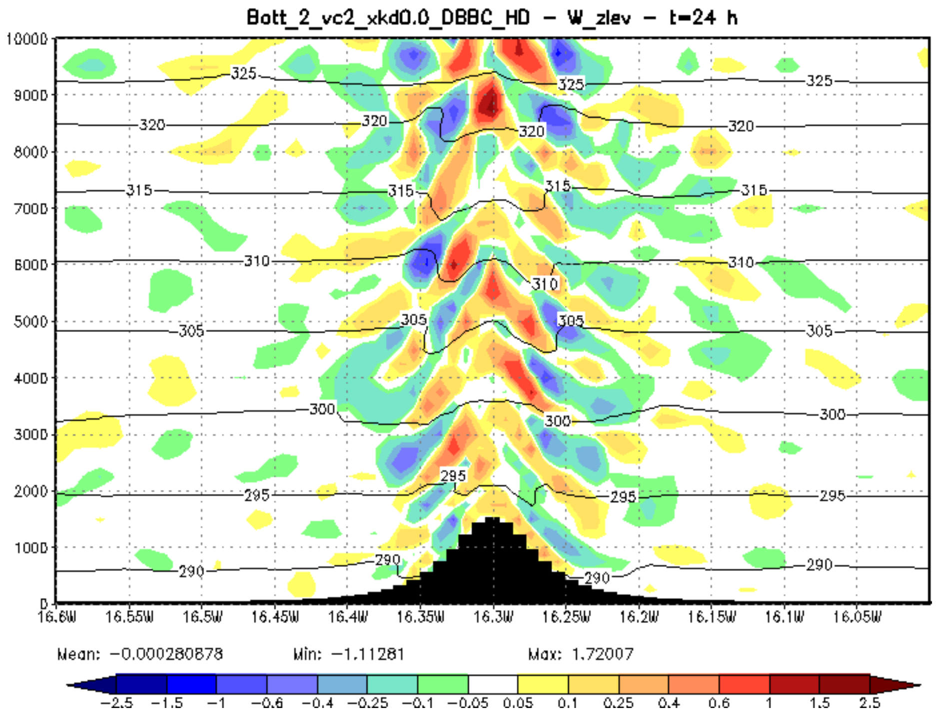
Leapfrog	Runge-Kutta old p*-T-Dynamics



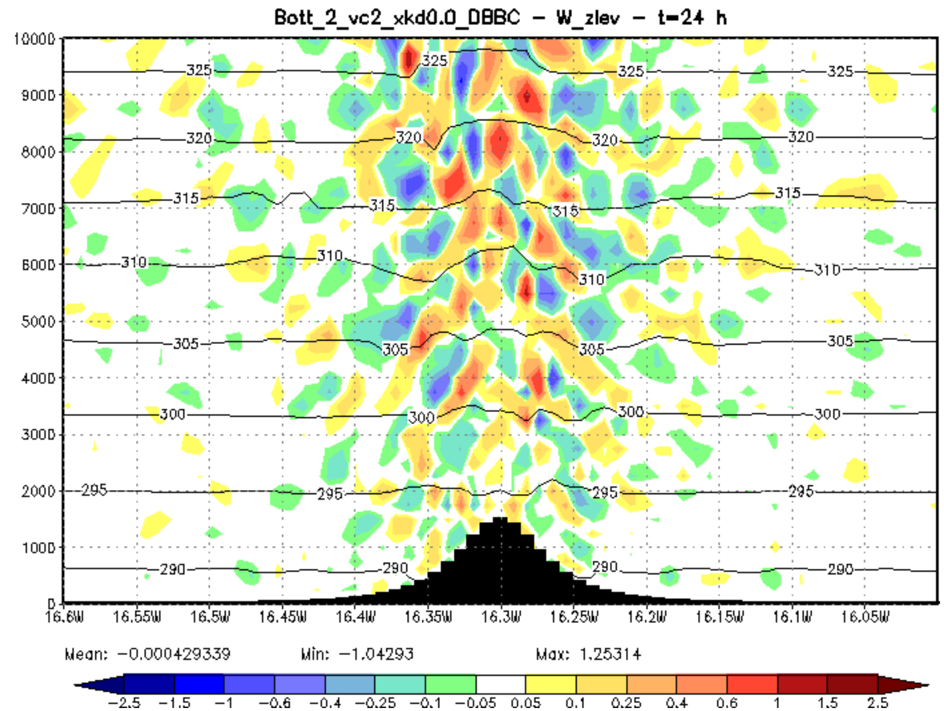


contours: vertical velocity isolines: potential temperature

Runge-Kutta + HD
old p*-T-Dynamics

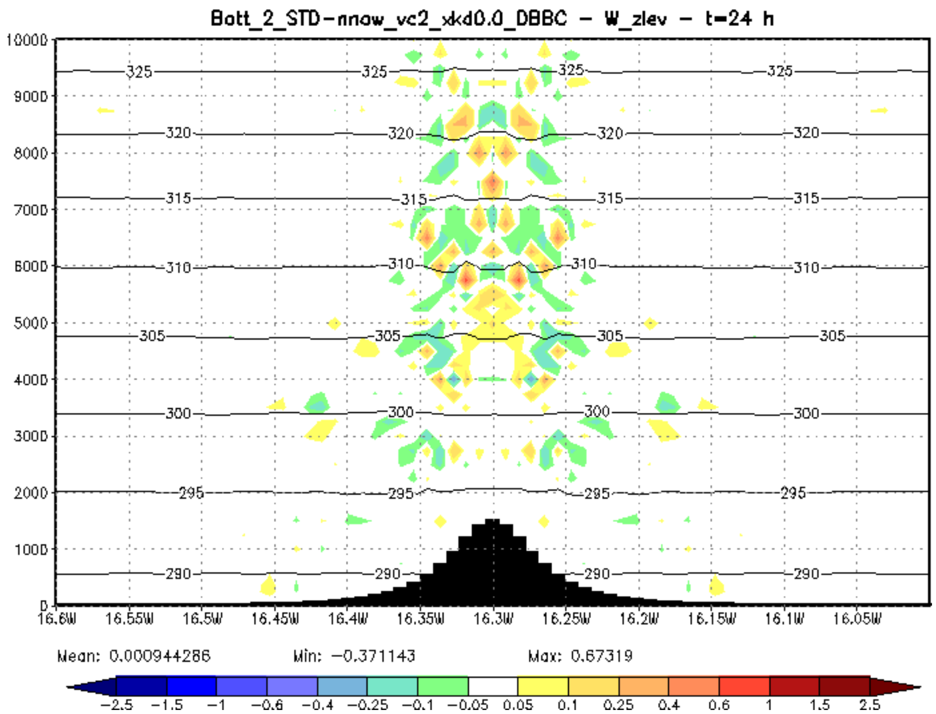


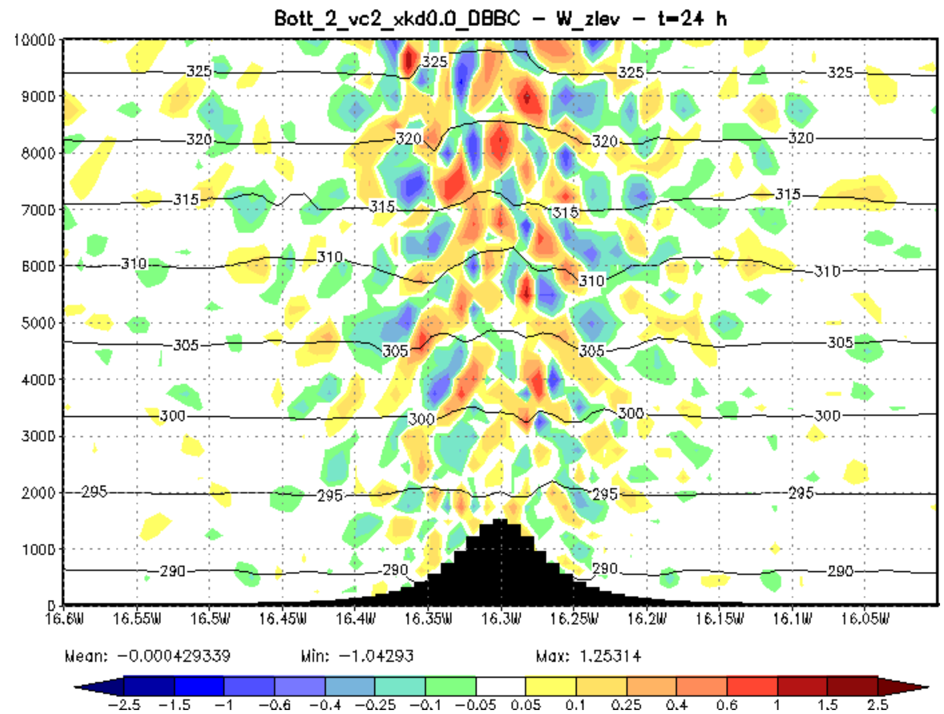
Runge-Kutta
old p*-T-Dynamics



contours: vertical velocity   isolines: potential temperature

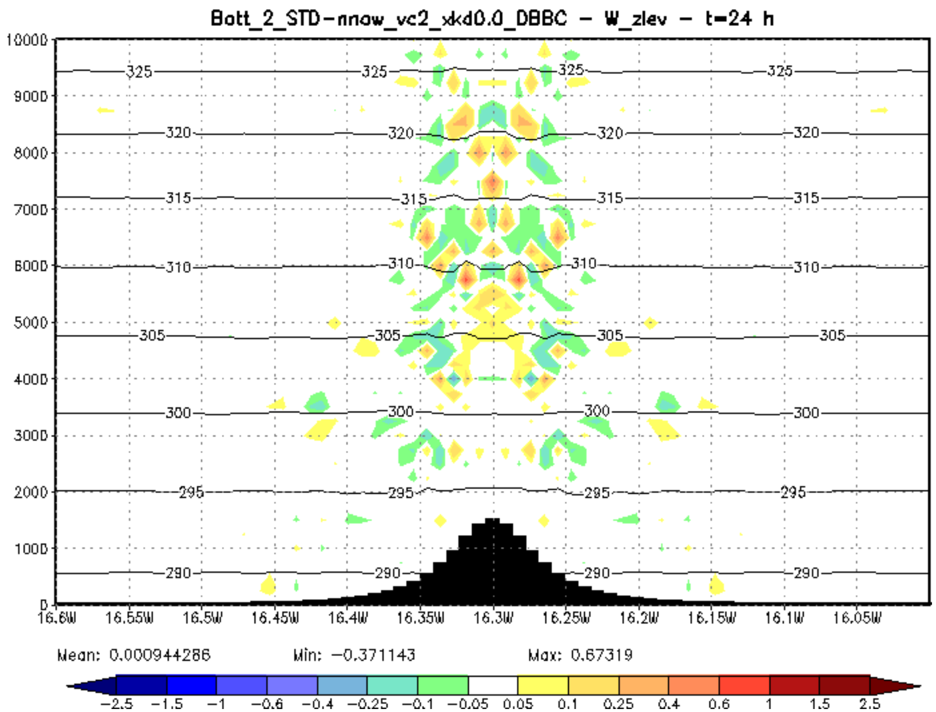
Runge-Kutta new p*-T*-Dynamics	Runge-Kutta old p*-T-Dynamics



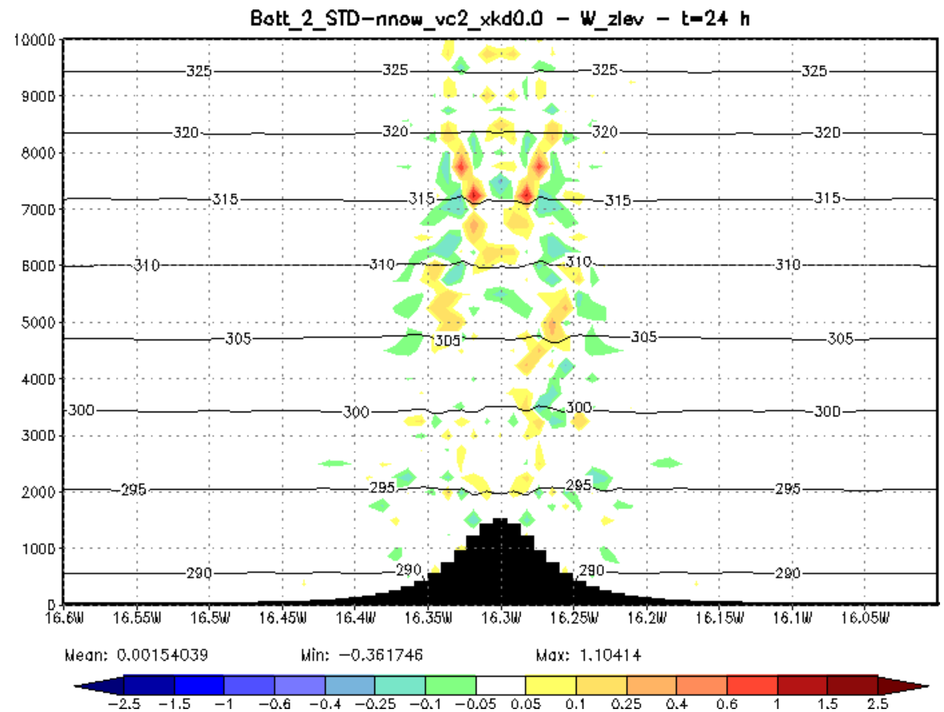


contours: vertical velocity isolines: potential temperature

Runge-Kutta
new p*-T*-Dynamics

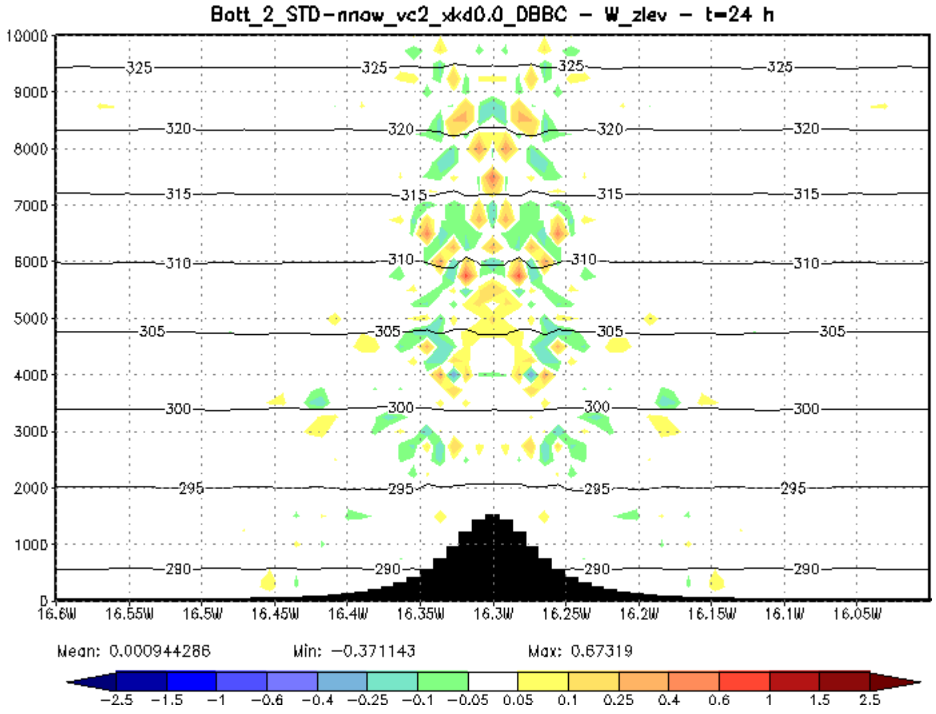


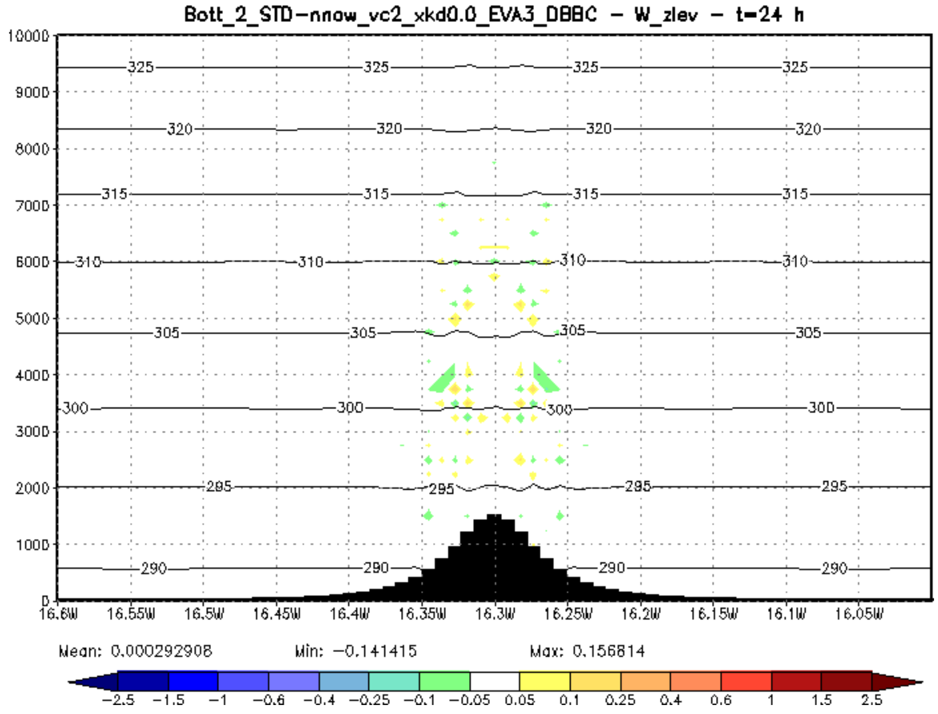
Runge-Kutta
new p*-T*-Dynamics
old bottom BC



contours: vertical velocity isolines: potential temperature

Runge-Kutta new p*-T*-Dynamics	Runge-Kutta new p*-T*-Dynamics explicit upwind 3rd order VA





contours: vertical velocity   isolines: potential temperature

semi-idealized Inn Valley - Test Case
(suggested by Günther Zängl during visit of MeteoCH in March 2006)

- Orography of Inn Valley region at a resolution of 800 m.
- Physics: radiation - and turbulence parameterization.
- Initialization: very dry atmosphere at rest, 15 Oct. 00 UTC.

- old p*-T-Dynamics: |(u,v)| in k=3 T in k=1 qv in k=1 w at x=55
- new p*-T*-Dynamics: |(u,v)| in k=3 T in k=1 qv in k=1 w at x=55
- new p*-T*-Dynamics *): |(u,v)| in k=3 T in k=1 qv in k=1 w at x=55

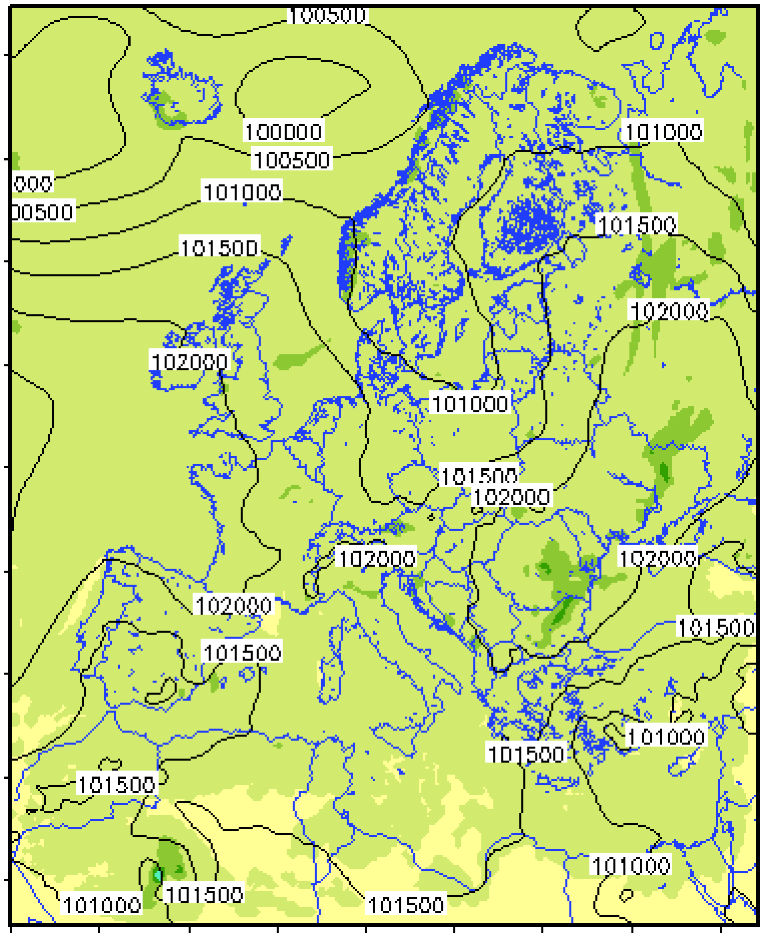
  *) with explicit upwind 3rd order vertical advection

CLM - Test Case
(simulations carried out by U. Schättler)

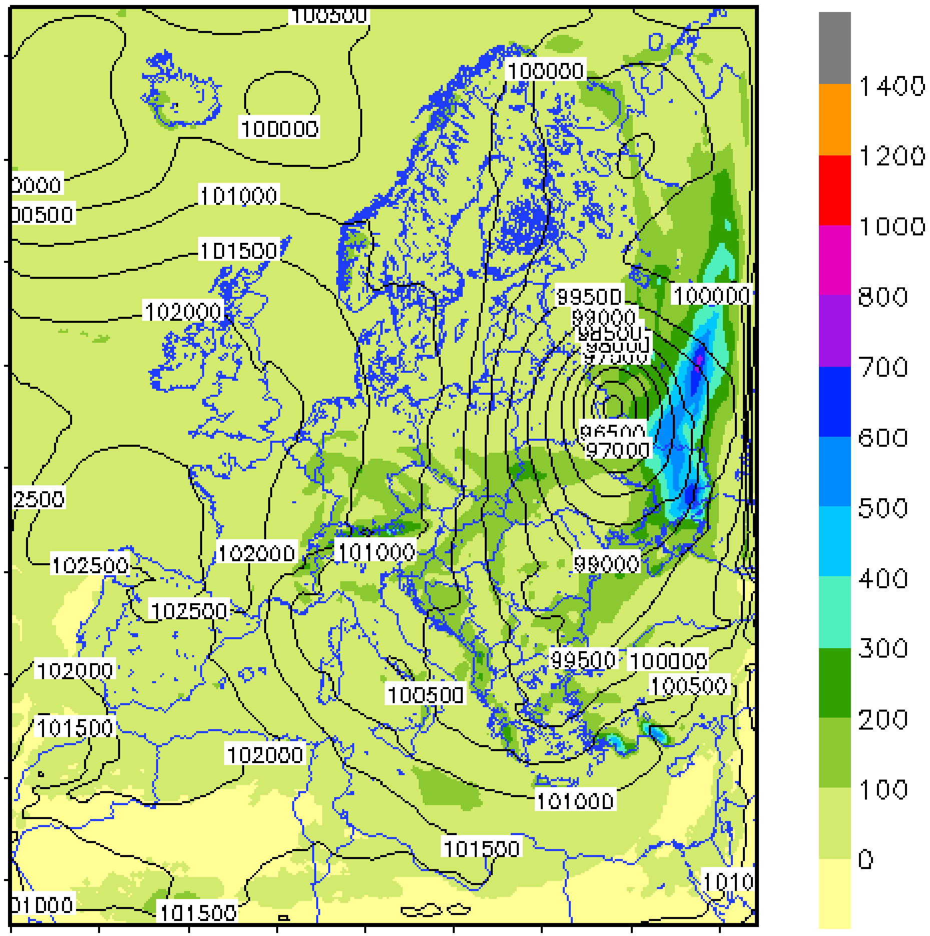
- Problem: unrealistic forecasts of pressure and precipitation fields in long term climate simulations with the LM.
- Problem depends on the chosen time step.

- Start date: 1 July 1979.
- Shown results:
  - accumulated precipitation (TOT_PREC)
  - pressure at mean sea level (PMSL) after a forecast time of 324 h (nearly 2 weeks).

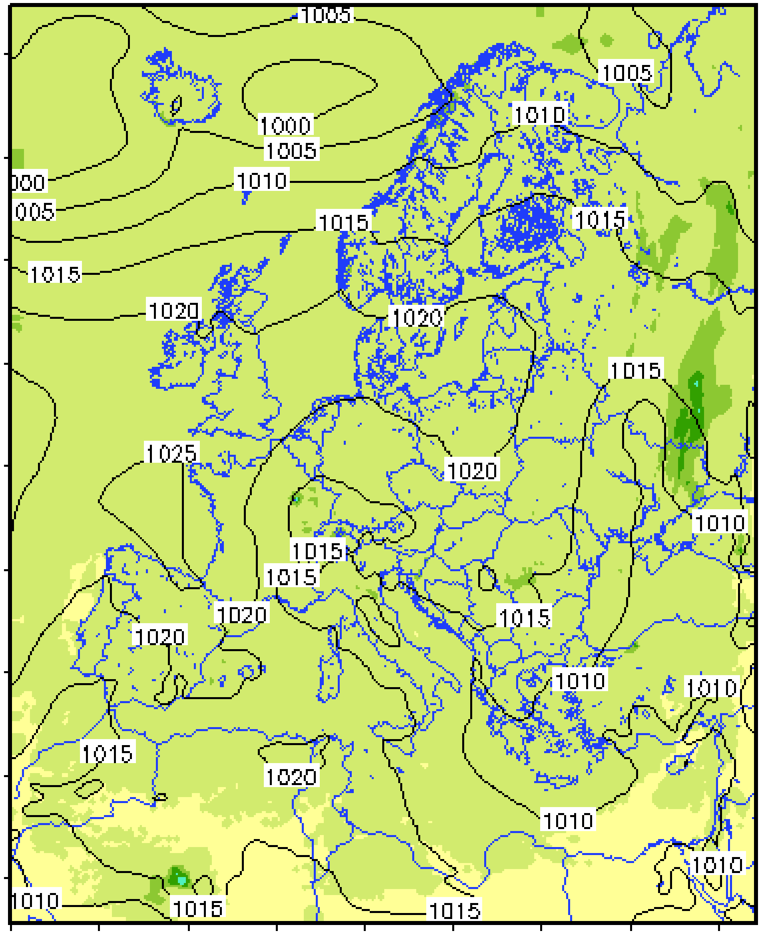
Leapfrog – Δt = 75s



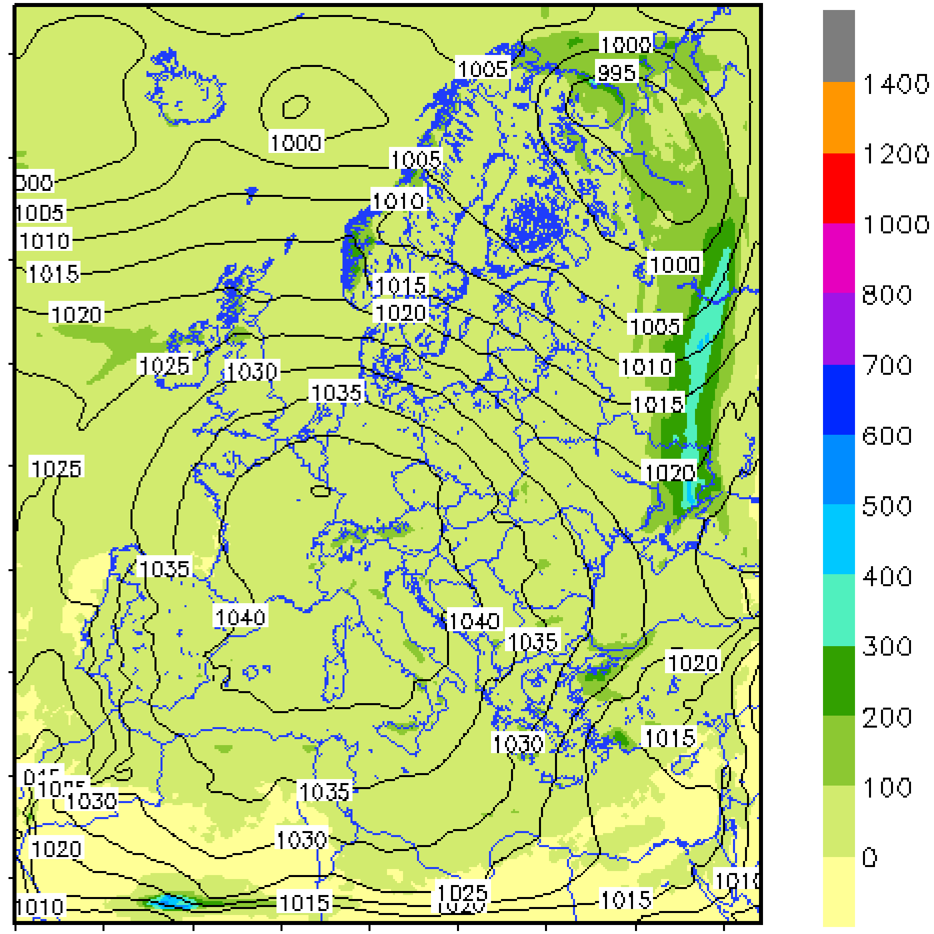
Leapfrog – Δt = 90s



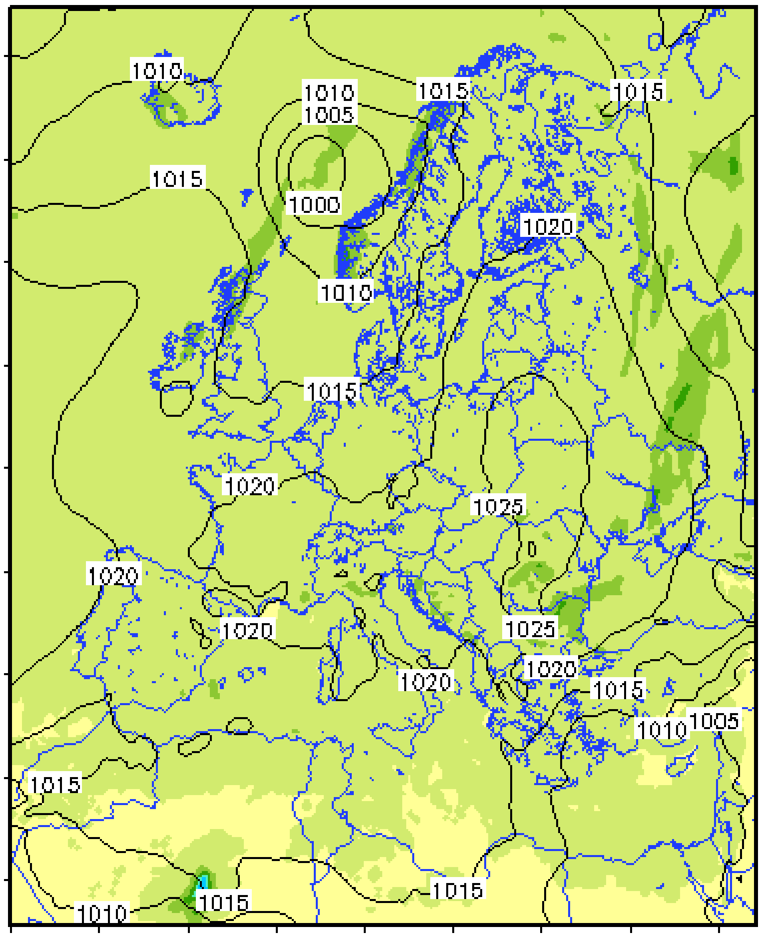
RK (p*-T) – Δt = 150s



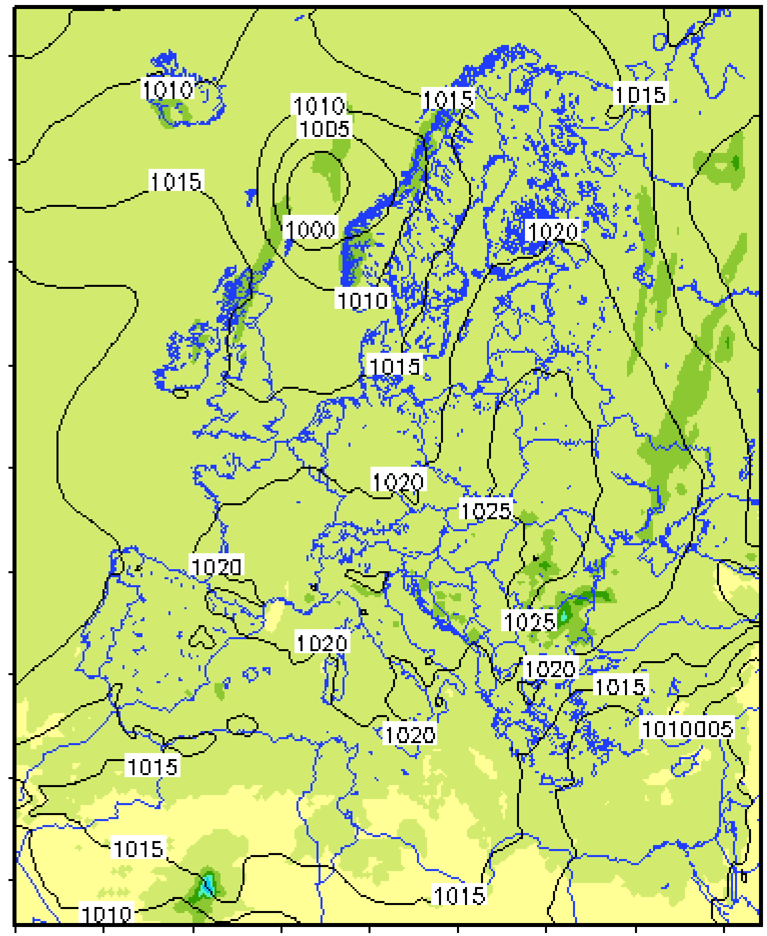
RK (p*-T) – Δt = 180s

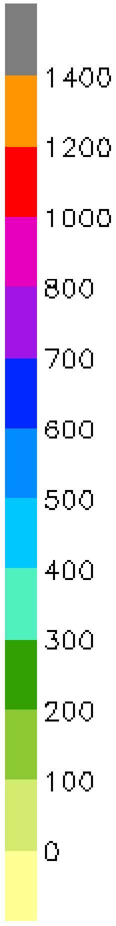


LF (semi-implicit) – Δt = 75s

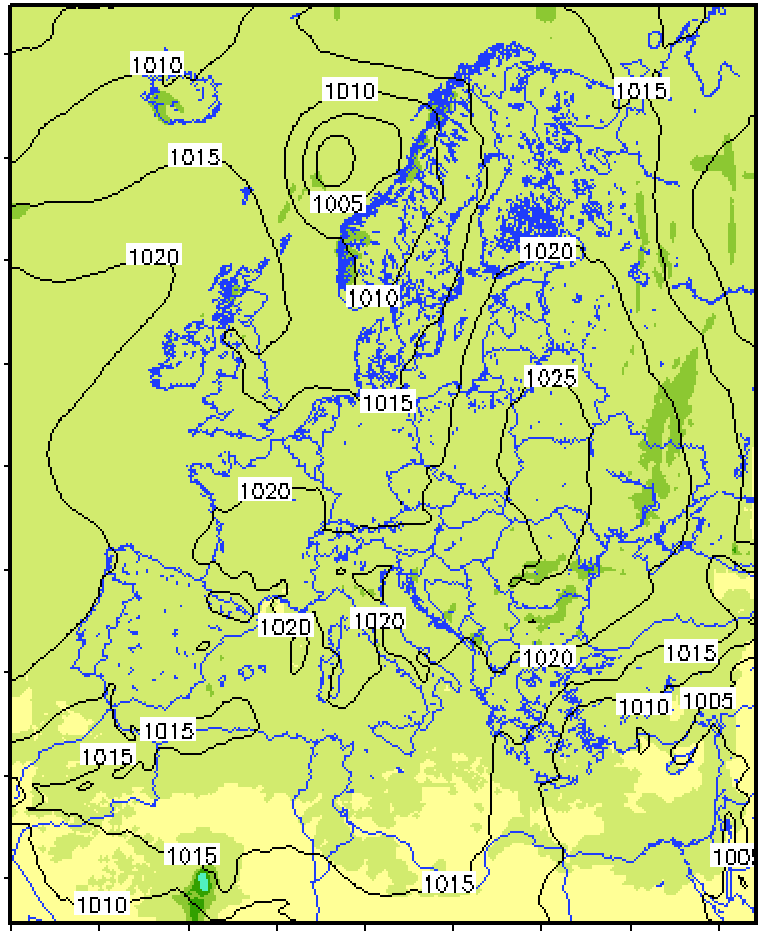


LF (semi-implicit) – Δt = 90s

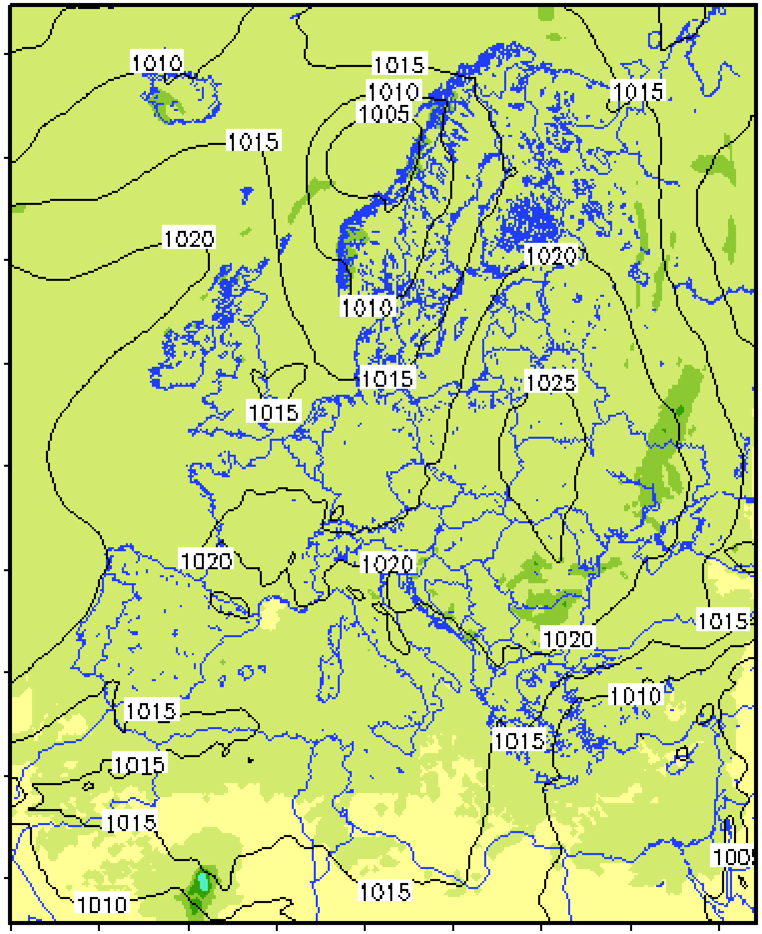


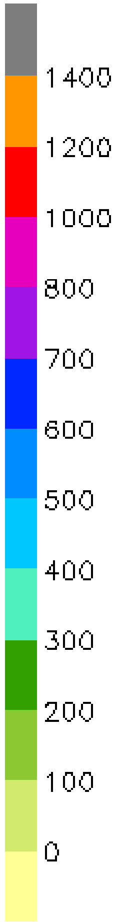


RK (p*-T*) – Δt = 150s



RK (p*-T*) – Δt = 180s





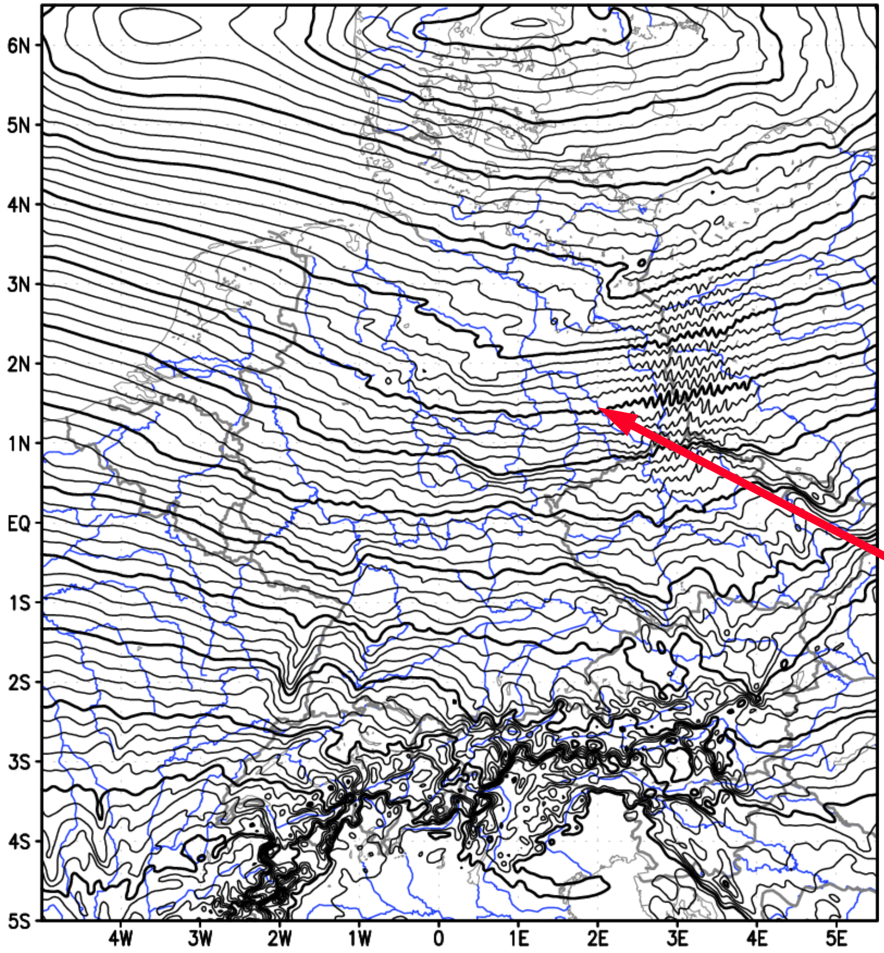
2nd Part:

Pre-Operational Testing Phase of COSMO-LMK

a) Kyrill (18 January 2007)

b) lateral boundary conditions for q_r and q_s

PMSL [hPa] – LMK 2007011806 – nach 12 h



Mean: 993.024 Min: 957.86 Max: 1029.62

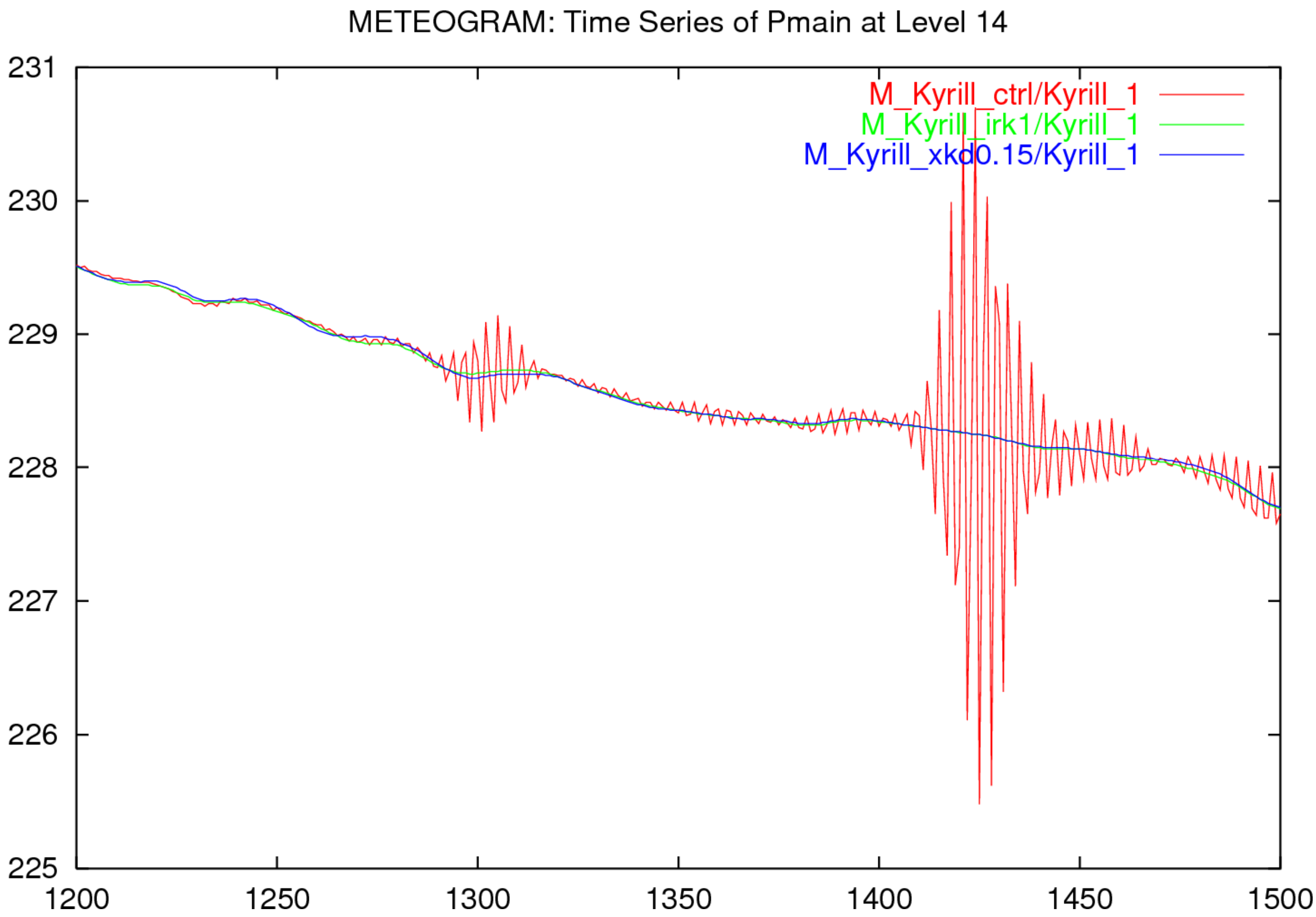
Kyrill (18 January 2007)

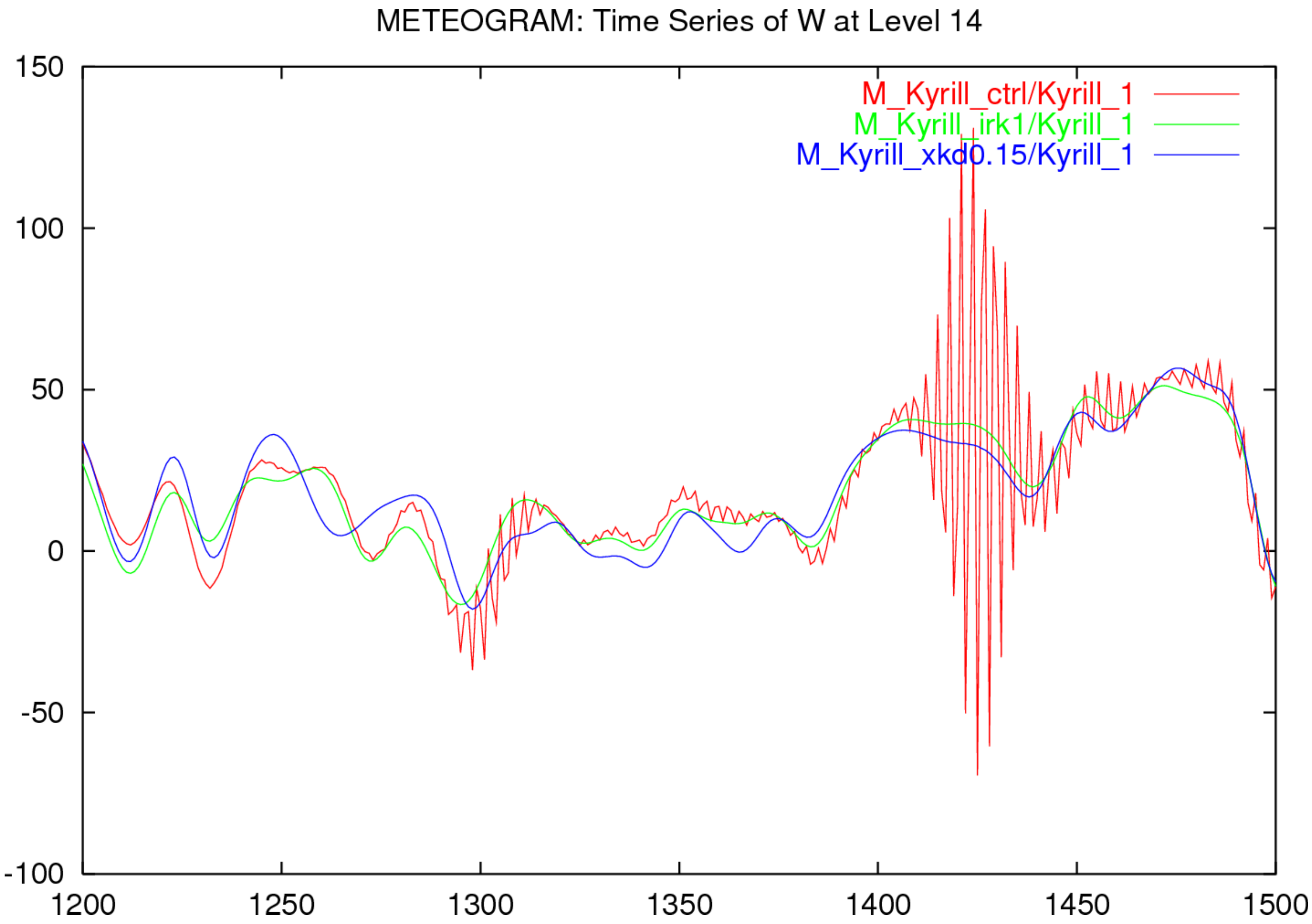
Meteogram-Output „Kyrill_1“ (**`n0gp=1200`**, **`nincqg=1`**)

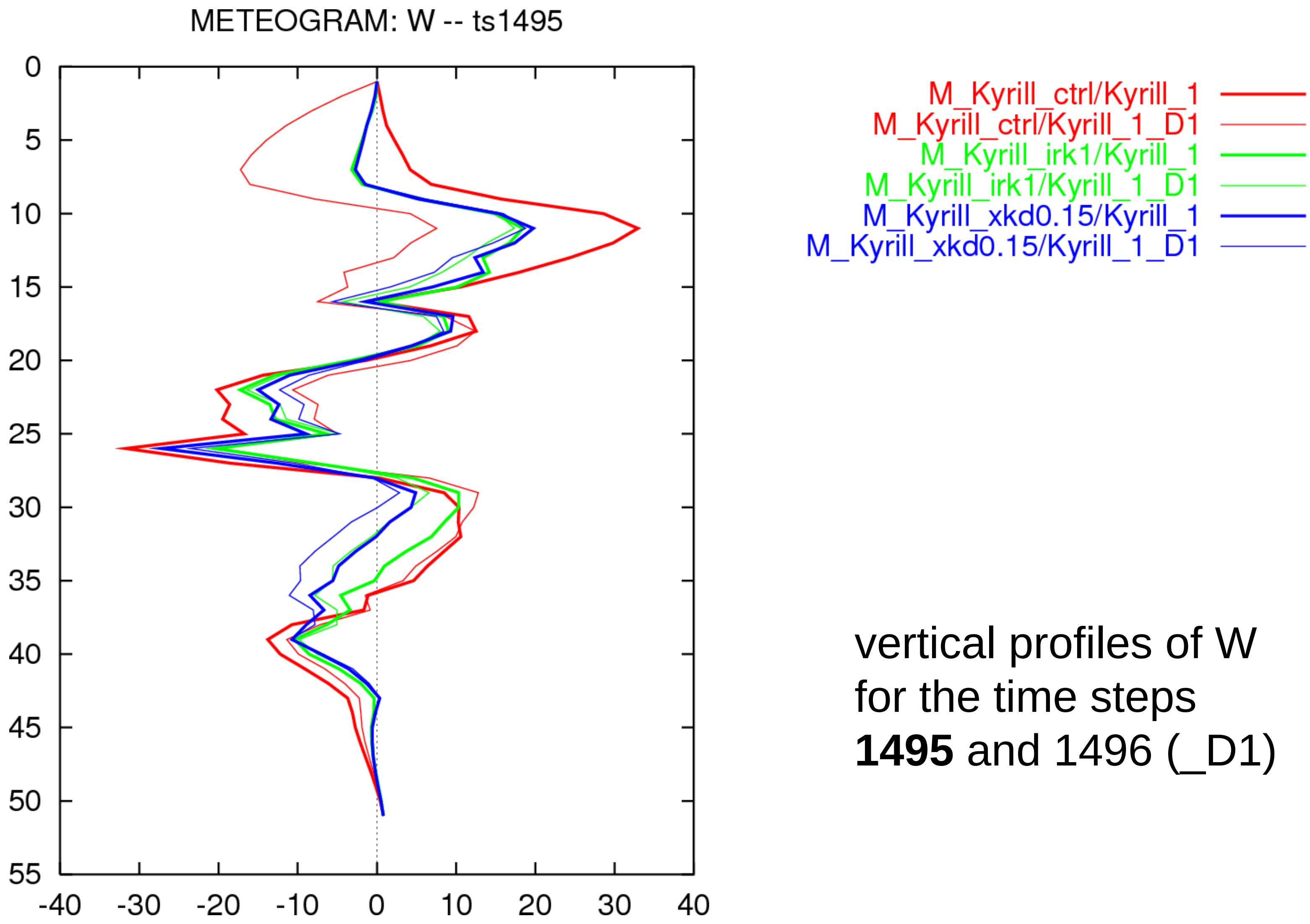
Kyrill (18 January 2007)

Tests carried out:

1. CTRL: new binary based on LM Version 3.21 (**dt=30.0**, **irunge_kutta=2**, **xkd=0.1**)
2. semi-Lagrangian advection (**lsl_adv_qx=.TRUE.**)
3. smaller time step (**dt=25.0** and **dt=20.0**)
4. turbulence of Herzog and Vogel FE14-Potsdam (6FT2)
5. stricter criterion for small time step (Vorfaktor 0,9 → 0,75)
6. HD in whole domain (**lhdiff_mask=.FALSE.**)
7. upwind 3rd order horizontal advection (**iadv_order=3**)
8. without „new phys.-dyn.-coupling“ (**ldiabf_lh=.FALSE.**)
9. standard (WRF-) RK instead of TVD-RK (**irunge_kutta=1**)
10. more divergence damping (**xkd=0.15**)
11. ... ditto (**xkd=0.125**)
12. no divergence damping (**xkd=0.0**)
13. TVD-RK with (same) $\Delta\tau$=2,5 s in every RK step
14. explicit vertical advection (**lva_impl_dyn=.FALSE.**)
15. longer relaxation time in upper damping layer (**nrdtau=10**)
16. explicit formulation with EXP-function in the damping layer
17. corrected CFL-criterion für horizontal advection
18. ... ditto with smaller time step (**dt=25.0**)


METEOGRAM: Time Series of Pmain at Level 14
M_Kyrill_ctrl/Kyrill_1
M_Kyrill_irk1/Kyrill_1
M_Kyrill_xkd0.15/Kyrill_1
231
230
229
228
227
226
225
1200
1250
1300
1350
1400
1450
1500

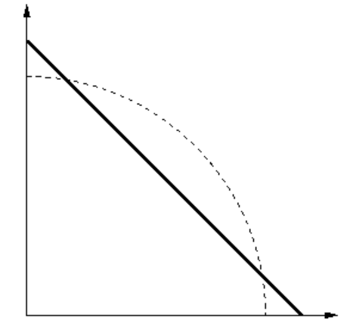

METEOGRAM: Time Series of W at Level 14
M_Kyrill_ctrl/Kyrill_1
M_Kyrill_irk1/Kyrill_1
M_Kyrill_xkd0.15/Kyrill_1
150
100
50
0
-50
-100
1200
1250
1300
1350
1400
1450
1500



vertical profiles of W for the time steps **1495** and 1496 (_D1)

corrected CFL-criterion (Baldauf)**:**

relevant velocity = MAX(ABS(u)+ABS(v))



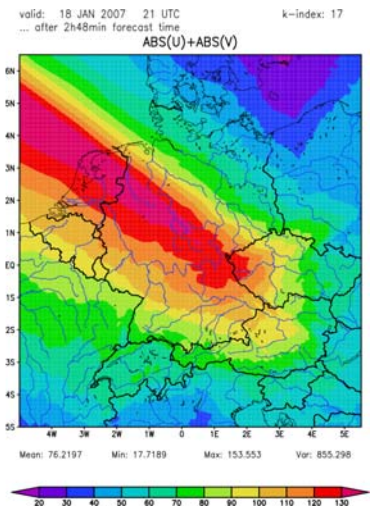
3rd order RK with **5th order** horizontal advection: 0,95 · **1,42** = 1,35
3rd order RK with **3rd order** horizontal advection: 0,95 · **1,61** = 1,53

LMK (MIN(Δx,Δy) = 2762 m):

- Δt = 30 s: stable up to 131 m/s
  Rayleigh-damp.: 124 m/s
- Δt = 25 s: stable up to 157 m/s
  Rayleigh-damp.: 149 m/s

LME (MIN(Δx,Δy) = 6488 m):

- Δt = 72 s: stable up to 128 m/s (145 m/s)
  Rayleigh-damp.: 122 m/s (138 m/s)
- Δt = 60 s: stable up to 154 m/s (174 m/s)
  Rayleigh-damp.: 146 m/s (165 m/s)

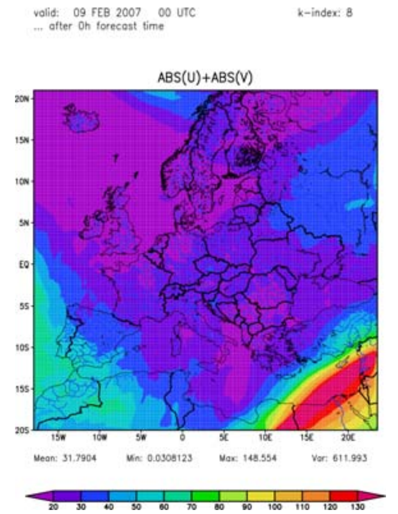


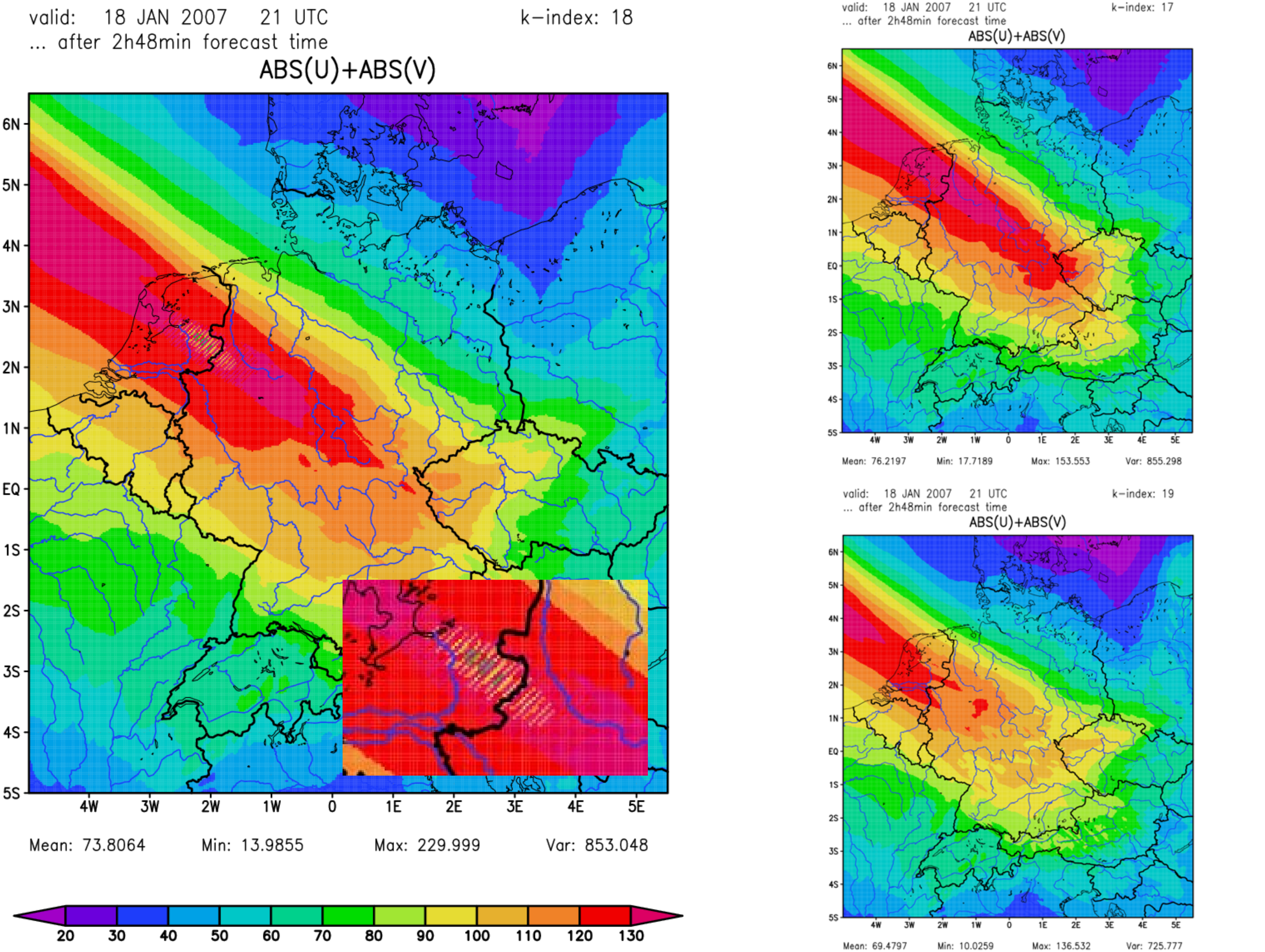
LMK - main run-analysis
d=2007011821

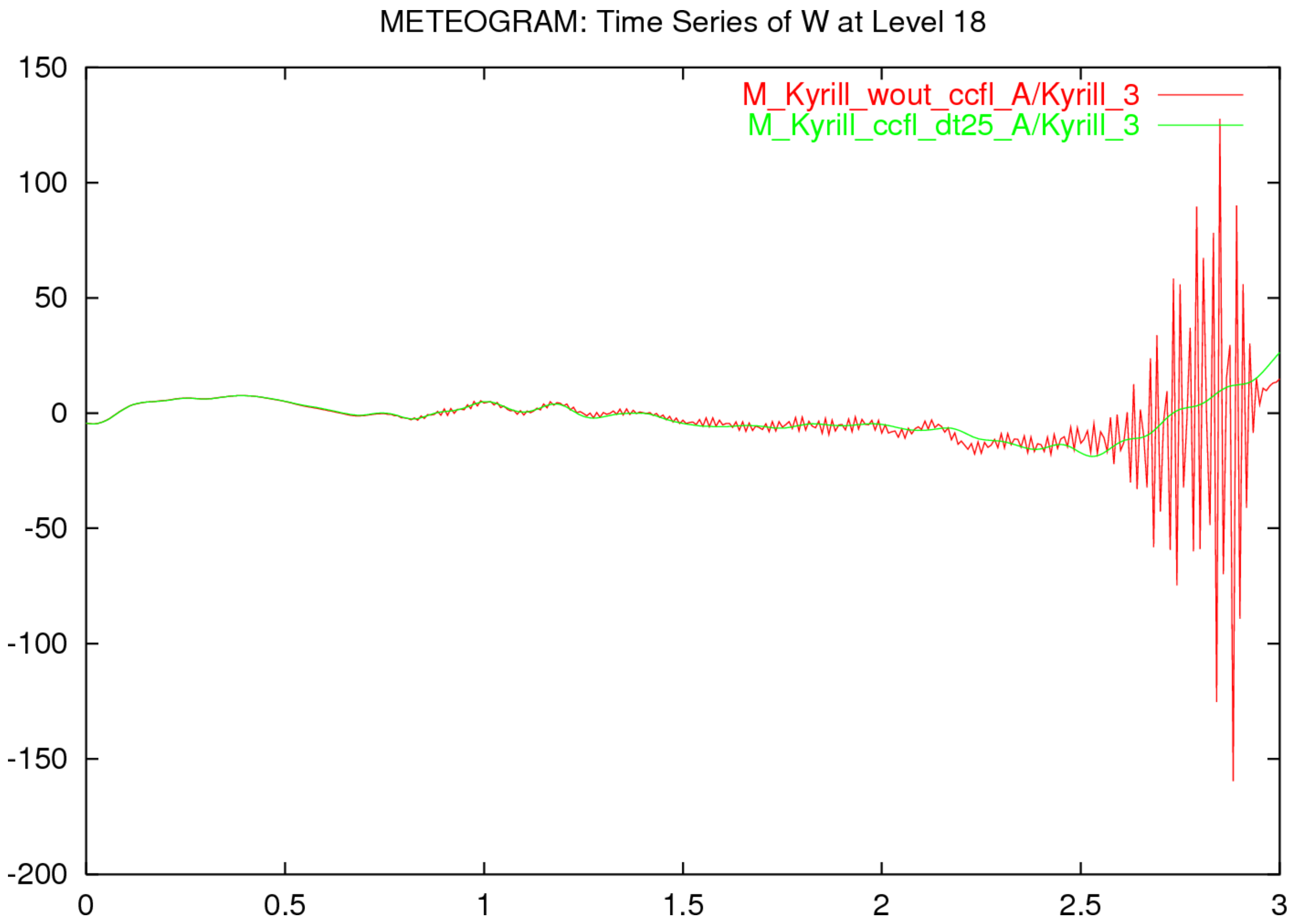
MAX(ABS(u)+ABS(v)) = 147 m/s

LME (parallel routine) - main run
d=2007020900

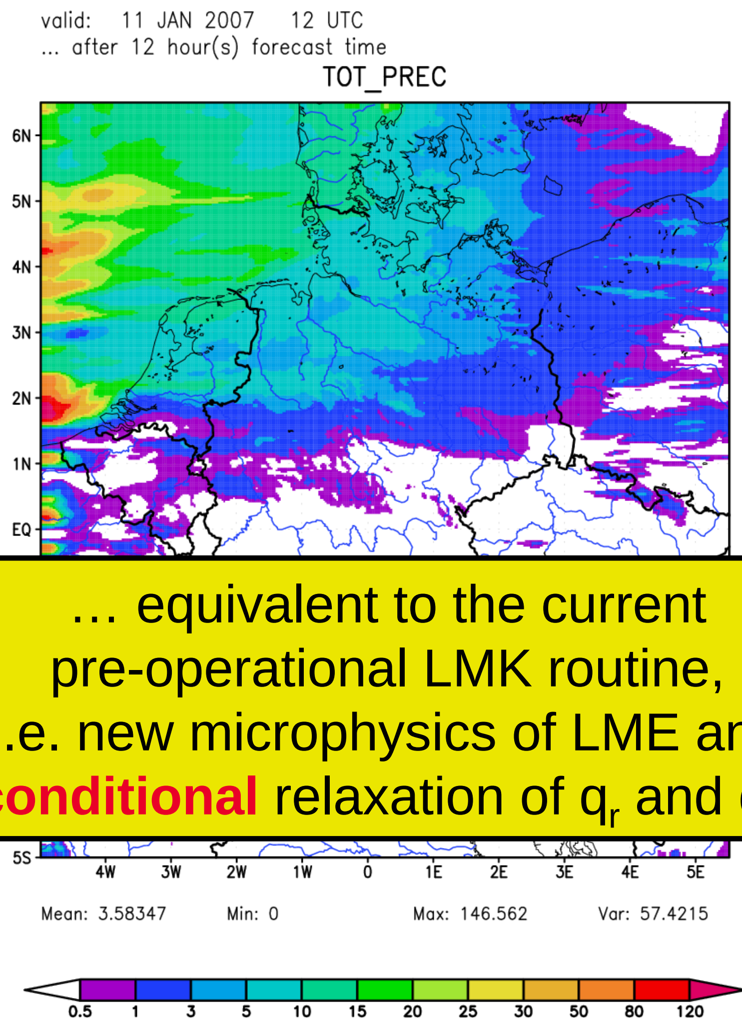
MAX(ABS(u)+ABS(v)) = 152 m/s



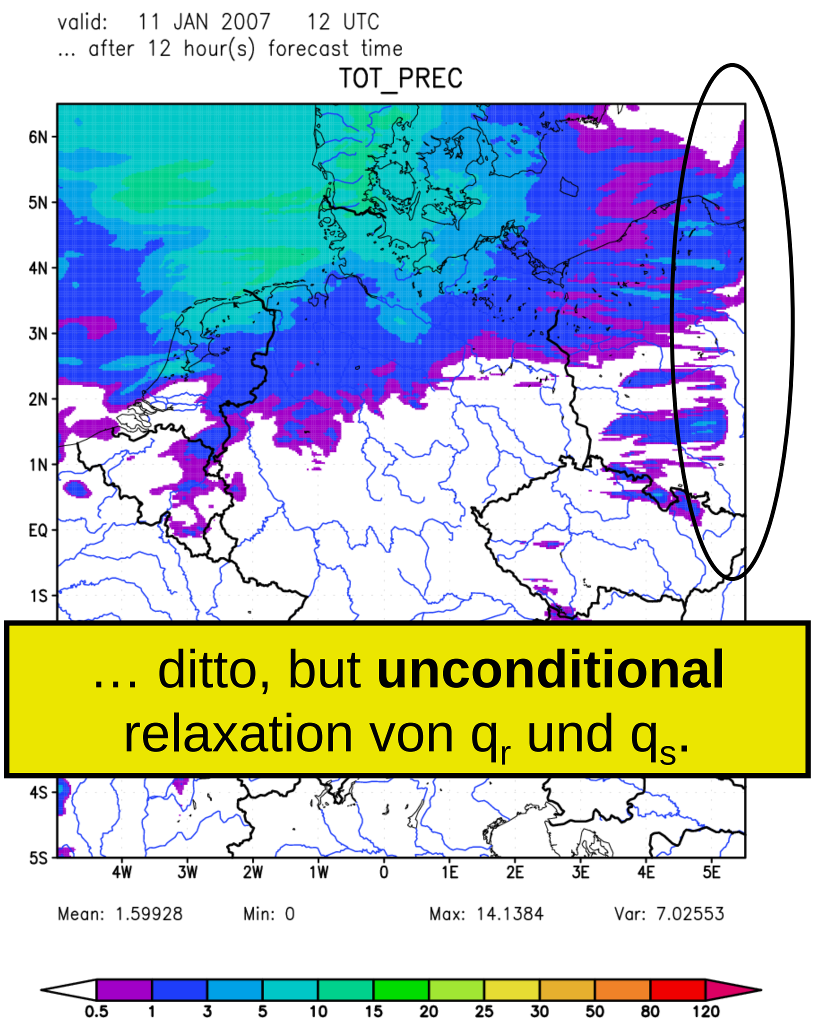

valid: 18 JAN 2007 21 UTC
... after 2h48min forecast time
k-index: 18
ABS(U)+ABS(V)
6N
5N
4N
3N
2N
1N
EQ
1S
2S
3S
4S
5S
4W
3W
2W
1W
0
1E
2E
3E
4E
5E
Mean: 73.8064
Min: 13.9855
Max: 229.999
Var: 853.048
20
30
40
50
60
70
80
90
100
110
120
130
valid: 18 JAN 2007 21 UTC
... after 2h48min forecast time
k-index: 17
ABS(U)+ABS(V)
Mean: 76.2197
Min: 17.7189
Max: 153.553
Var: 855.298
valid: 18 JAN 2007 21 UTC
... after 2h48min forecast time
k-index: 19
ABS(U)+ABS(V)
Mean: 69.4797
Min: 10.0259
Max: 136.532
Var: 725.777


METEOGRAM: Time Series of W at Level 18
M_Kyrill_wout_ccfl_A/Kyrill_3
M_Kyrill_ccfl_dt25_A/Kyrill_3
150
100
50
0
-50
-100
-150
-200
0
0.5
1
1.5
2
2.5
3

lateral boundary conditions for q_r and q_s (11 January 2007 – 00 UTC)

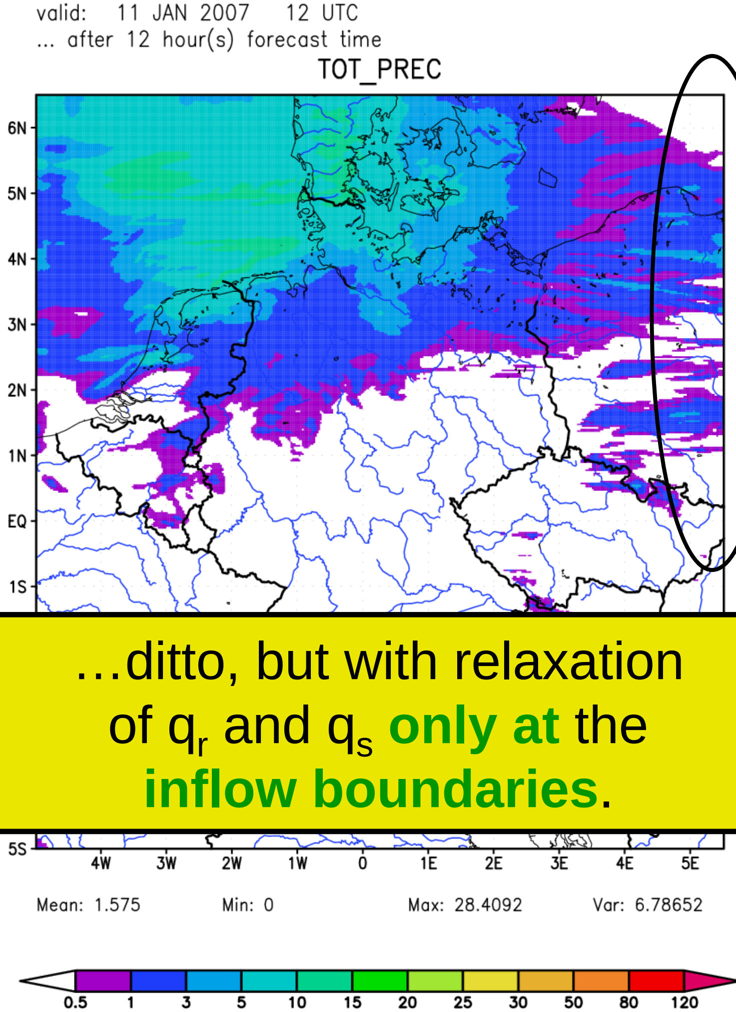


… equivalent to the current pre-operational LMK routine, i.e. new microphysics of LME and **conditional** relaxation of q_r and q_s.

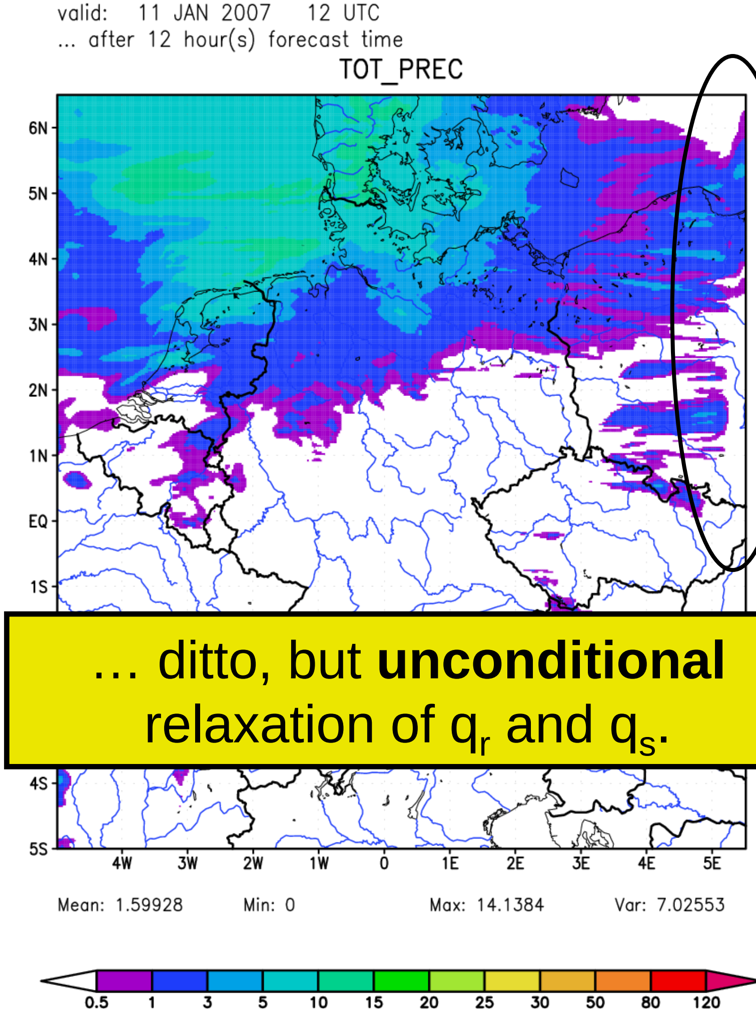


… ditto, but **unconditional** relaxation von q_r und q_s.

lateral boundary conditions for q_r and q_s (11 January 2007 – 00 UTC)

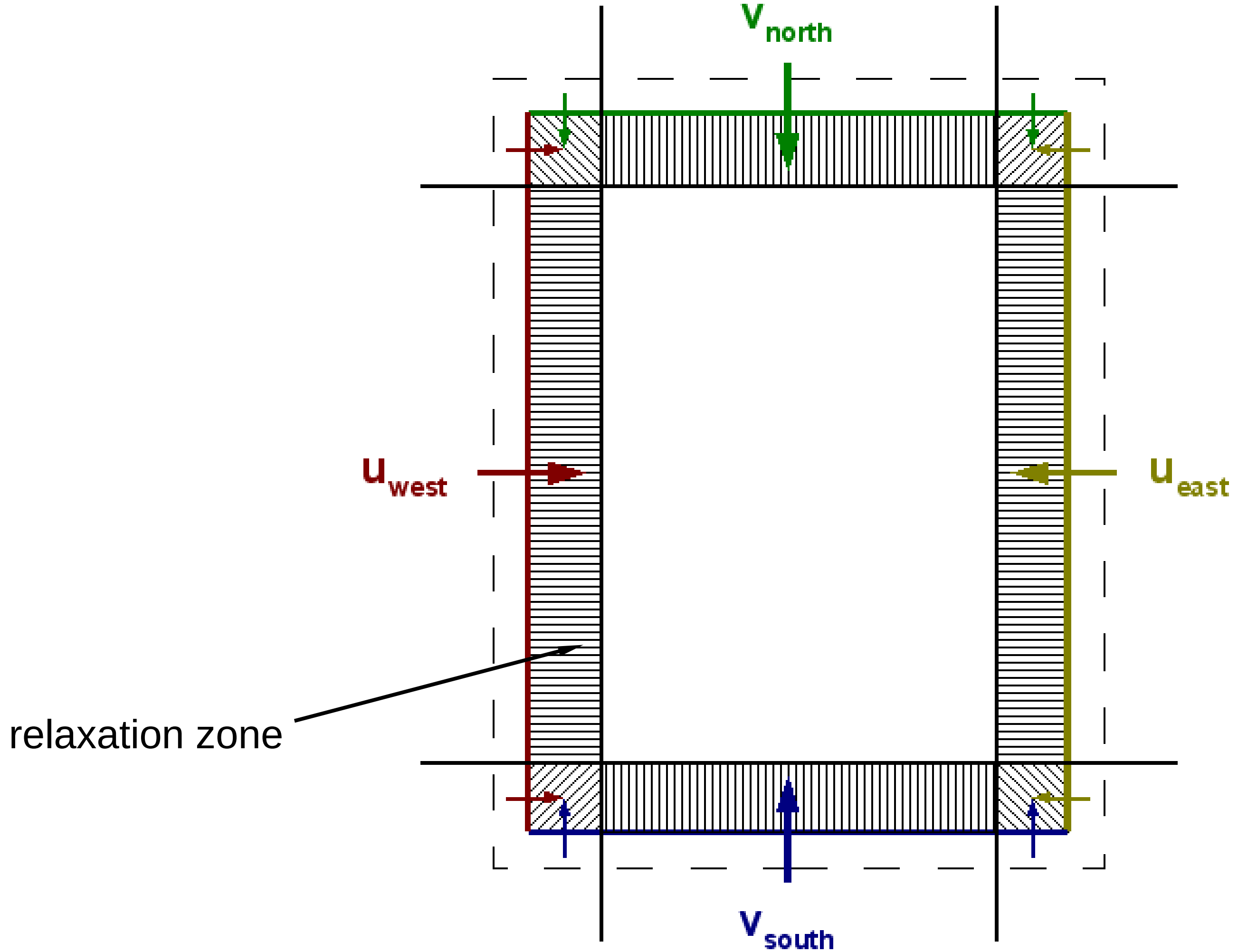


…ditto, but with relaxation of q_r and q_s **only at** the **inflow boundaries**.



… ditto, but **unconditional** relaxation of q_r and q_s.

lateral inflow boundary conditions for q_r and q_s



Outlook:

Runge-Kutta Numerics for COSMO-LMK...

Runge-Kutta Numerics also for COSMO-LME

a) Eulerian advection of q_x with Bott (1989 $\rightarrow$ 1993) scheme:

- monotone scheme

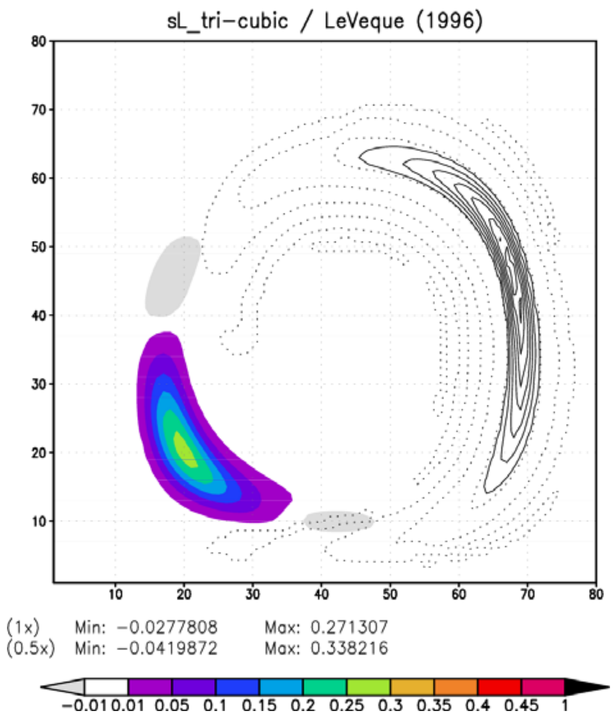
- reduced splitting error

... and investigate problems which arose in combination with steep orography (d=2006102812)

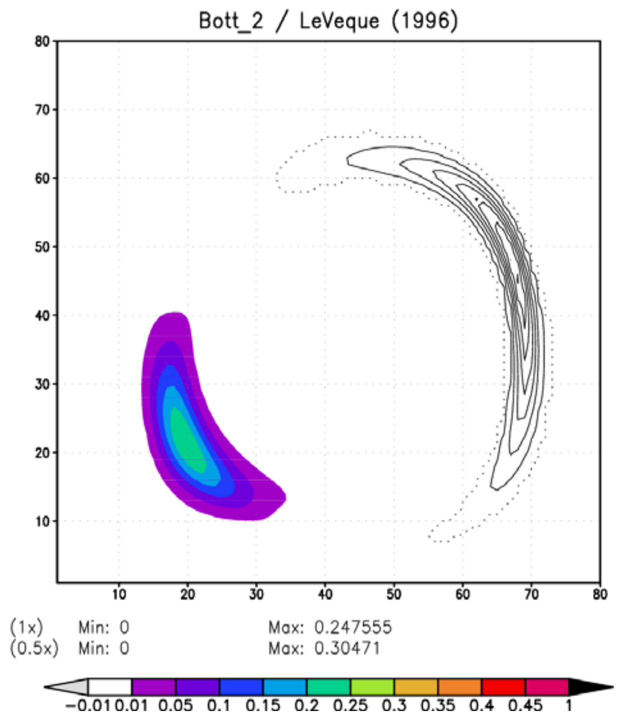
b) Digital filter initialization (DFI) for Runge-Kutta scheme

idealized advection of tracer in a deformational flow field

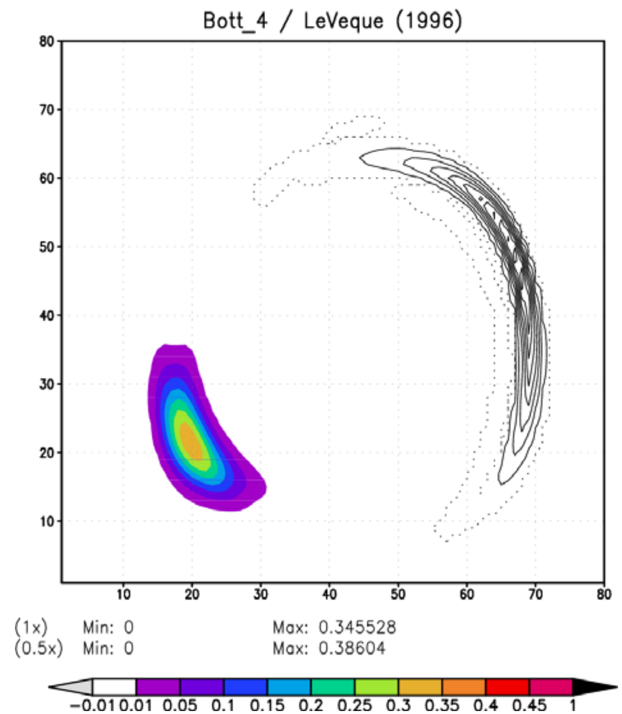
semi-Lagrange (tri-cubic)



2nd order Bott (1989)



4th order Bott (1989)



inititial condition respectively **exact solution**:
cone with maximum of 1.0 at (20,20)