

A mass, thermodynamic and lateral boundary inconsistency in the LM equations - the possible origin of a pressure bias ?

Almut Gassmann and Hans-Joachim Herzog

Met. Inst. Univ.Bonn - DWD Potsdam

Contents

- ▶ **Original budget equations**
- ▶ **How to derive the prognostic equations of temperatur and pressure**
- ▶ **Reconsideration of approximations on the way to the LM equations**
- ▶ **Qualitative discussion and possible consequences to reformulate the T – and p' – equation**

**In memory of Günther Doms
and his marvellous LM documentation !**

- Original budget equations for mean flow -

- momentum equations
- total mass equation (continuity equation)
- water constituent equations
- enthalpy equation



elimination of total density



equation of state



$$\bar{\rho} \hat{c}_p \frac{\hat{d} \hat{T}}{d t} = \frac{\hat{d} \bar{p}}{d t} + \bar{Q}_h$$

**continuity equation
hidden behind !**

$$\frac{\hat{d} \bar{p}}{d t} = - \left(\hat{c}_p / \hat{c}_v \right) \bar{p} \nabla \cdot \hat{\mathbf{v}} + \left(\hat{c}_p / \hat{c}_v - 1 \right) \bar{Q}_h + \left(\hat{c}_p / \hat{c}_v \right) \bar{Q}_m$$

- Prognostic equations of temperature and pressure -

Common approximations :

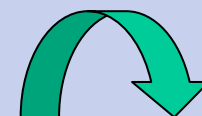
- ▶ Turbulence averaging symbols dropped
- ▶ In heat and moisture source terms molecular fluxes neglected
- ▶ Latent heat of vapourisation and sublimation assumed constant
- ▶ Specific heats of moist air replaced by specific heat of dry air



$$\rho c_{pd} \frac{dT}{dt} = \frac{dp}{dt} + \underline{Q_h}$$

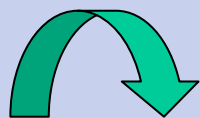
$$\frac{dp}{dt} = -\left(c_{pd}/c_{vd}\right) p \nabla \cdot \mathbf{v} + \left(c_{pd}/c_{vd} - 1\right) \underline{Q_h} + \left(c_{pd}/c_{vd}\right) \underline{Q_m}$$

Q_h , Q_m  heat - and moisture sources



Definition of heat source term

- Günther's (2002) notation -



$$Q_T := \frac{1}{\rho c_{pd}} Q_h$$

$$Q_T = -\frac{1}{\rho c_{pd}} \nabla \cdot (\mathbf{H} + \mathbf{R}) + \left(\frac{L_v}{c_{pd}} S^l + \frac{L_s}{c_{pd}} S^f \right)$$

Turbulent heat flux

Radiation flux

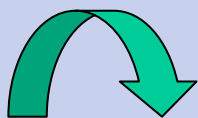
Diabatic heating due to cloud microphysical sources per unit mass

$$-\frac{1}{\rho c_{pd}} \nabla \cdot \mathbf{H} := -\frac{1}{\rho c_{pd}} \nabla \cdot c_{pd} \overline{\rho \mathbf{v}'' T}$$

$$\approx -\frac{1}{\rho c_{pd}} \nabla \cdot c_{pd} \bar{\pi} \bar{\rho} \overline{\mathbf{v}' \theta'} \approx \frac{1}{\rho \sqrt{G}} \frac{\partial}{\partial \zeta} \left(\frac{\pi \rho K_h}{\sqrt{G}} \frac{\partial \theta}{\partial \zeta} \right)$$

$$S^l = I^l / \rho$$

$$S^f = I^f / \rho$$



Definition of moisture source term

- Günther's (2002) notation -

$$Q_M := \frac{1}{\rho c_{pd}} Q_m$$

$$Q_M = \frac{R_d T}{\rho c_{pd}} \left(1 - \frac{R_v}{R_d} \right) \nabla \cdot \mathbf{F}^v - \frac{R_v T}{c_{pd}} (S^l + S^f) + \frac{R_d T}{\rho c_{pd}} \left[\nabla \cdot (\mathbf{F}^l + \mathbf{F}^f) + \nabla \cdot (\mathbf{P}^l + \mathbf{P}^f) \right]$$

Turbulent flux for water constituents

$$\mathbf{F}^x = \overline{\rho \mathbf{v}'' q^x} \approx \overline{\rho w'' q^x} \mathbf{k} \approx \frac{\rho K_h}{\sqrt{G}} \frac{\partial q^x}{\partial \zeta} \mathbf{k}$$

$x = v, l, f$

Cloud heat sources

Precipitation (gravitational diffusion) fluxes

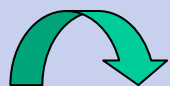
$$\mathbf{P}^{l,f} = -q^{l,f} \left| \mathbf{v}_T^{l,f} \right| \mathbf{k}$$

Reconsidering the derivation of prognostic temperature- and pressure equations in the LM

$$\frac{dT}{dt} = \frac{1}{\rho c_{pd}} \frac{dp}{dt} + Q_T$$

$$\frac{dp}{dt} = -\left(c_{pd}/c_{vd}\right) p \nabla \cdot \mathbf{v} + \left(c_{pd}/c_{vd} - 1\right) \rho c_{pd} Q_T + \left(c_{pd}/c_{vd}\right) \rho c_{pd} Q_M$$

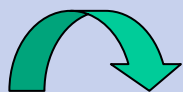
1. In pressure equation heat- and moisture source terms neglected
2. dp/dt in T - equation eliminated after neglecting these terms



$$\frac{dT}{dt} = -\frac{p}{\rho c_{vd}} \nabla \cdot \mathbf{v} + Q_T$$

$$\frac{dp}{dt} = -\left(c_{pd}/c_{vd}\right) p \nabla \cdot \mathbf{v}$$

3. Formal addition of moist convection tendency, computational mixing, lateral and upper boundary relaxation terms



Final prognostic LM - equations for T and p'

$$\frac{dT}{dt} = -\frac{p}{\rho c_{vd}} \nabla \cdot \mathbf{v} + Q_T + M_T^{MC} + M_T^{CM} - \mu_b (T - T_b) - \mu_R(z) (T - T_b)$$

$$\frac{dp}{dt} = -\left(c_{pd} / c_{vd}\right) p \nabla \cdot \mathbf{v} + M_{p'}^{CM} - \mu_b (p' - p'_b) - \mu_R(z) (p' - p'_b)$$

The way to come to this result is wrong and leads to insufficient equations !

We meet with a wrong combination of p' - and T - equation, each used at a different level of approximation, which is physically inconsistent.

This operation is equivalent to the application of a wrong continuity equation producing a mass deficiency

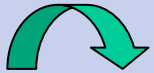
Heating - and moisture source terms are not properly incorporated in the T - equation

The direct influence of heating and moisture terms in the pressure tendency equation is ignored

The lateral and vertical boundary relaxation technique in the T- equation is incomplete.

Consistent derivation of T- and p'- equation

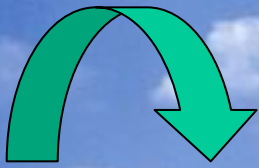
1. Heating- and moisture source terms are retained in the pressure equation
2. Addition of moist convection tendency, computational mixing, lateral and upper boundary relaxation terms in both equations



$$\frac{dT}{dt} = \frac{1}{\rho c_{pd}} \frac{dp}{dt} + \underline{Q_T + M_T^{MC}} + M_T^{CM} - \underline{\mu_b(T' - T'_b) - \mu_R(z)(T' - T'_b)}$$

$$\begin{aligned} \frac{dp}{dt} = & -\left(c_{pd}/c_{vd}\right) p \nabla \cdot \mathbf{v} + \underline{\left(c_{pd}/c_{vd} - 1\right) \rho c_{pd} \left(Q_T + M_T^{MC}\right)} + \\ & + \underline{\left(c_{pd}/c_{vd}\right) \rho c_{pd} Q_M} + M_{p'}^{CM} - \underline{\mu_b(p' - p'_b) - \mu_R(z)(p' - p'_b)} \end{aligned}$$

3. Only now eliminating individual pressure tendency dp/dt in the first law of thermodynamics !



$$\frac{dT}{dt} = -\frac{p}{\rho c_{vd}} \nabla \cdot \mathbf{v} +$$

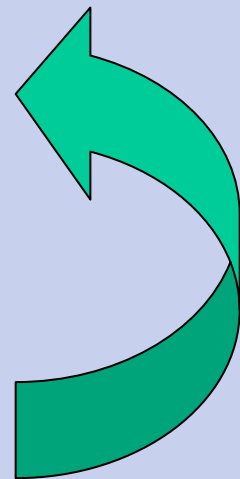
$$+ \left(\underline{c_{pd} / c_{vd}} \right) \left(\underline{Q_T} + \underline{Q_M} + M_T^{MC} \right) + \left(M_T^{CM} + \frac{1}{\underline{\rho c_{pd}}} M_{p'}^{CM} \right) -$$

$$- \mu_b \left[(T' - T'_b) + \frac{1}{\underline{\rho c_{pd}}} (p' - p'_b) \right] - \mu_R(z) \left[(T' - T'_b) + \frac{1}{\underline{\rho c_{pd}}} (p' - p'_b) \right]$$

$$\frac{dp}{dt} = -\left(\underline{c_{pd} / c_{vd}} \right) p \nabla \cdot \mathbf{v} + \left(\underline{c_{pd} / c_{vd} - 1} \right) \rho c_{pd} \left(\underline{Q_T} + M_T^{MC} \right) +$$

$$+ \left(\underline{c_{pd} / c_{vd}} \right) \rho c_{pd} \underline{Q_M} + M_{p'}^{CM} - \mu_b (p' - p'_b) - \mu_R(z) (p' - p'_b)$$

new !



$$\frac{dT}{dt} = -\frac{p}{\rho c_{vd}} \nabla \cdot \mathbf{v} + Q_T + M_T^{MC} + M_T^{CM} - \mu_b (T - T_b) - \mu_R(z) (T - T_b)$$

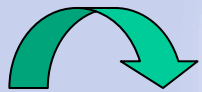
$$\frac{dp}{dt} = -\left(\underline{c_{pd} / c_{vd}} \right) p \nabla \cdot \mathbf{v} + M_{p'}^{CM} - \mu_b (p' - p'_b) - \mu_R(z) (p' - p'_b)$$

old !

- Lateral Davies - boundary relaxation technique - Günther's notation (2002)

For temperature a different lateral boundary treatment compared to all other variables :

$$\frac{T^{n+1} - T^*}{\Delta t} = -\mu_b (T^{n+1} - T_b^{n+1}) - \frac{\mu_b}{c_{pd}\rho^n} (p'^{n+1} - p_b'^{n+1})$$



$$T^{n+1} = T^* - \alpha (T^* - T_b^{n+1}) - \alpha (1 - \alpha) \left(\frac{p'^* - p_b'^{n+1}}{c_{pd}\rho^n} \right)$$

$$\alpha = \frac{\Delta t \mu_b}{1 + \Delta t \mu_b} \Rightarrow 1 - \tanh\left(\frac{d}{2\Delta s}\right) ; 0 < \alpha \leq 1$$

Maximal influence of pressure boundary data on temperature within boundary zone at $\alpha = 0.5$. - Rayleigh damping similar.

- Conclusions -



- The prognostic pressure - and temperature equations have been reconsidered and reformulated.
- It is shown that the well-known heat- and moisture source/sink terms are necessary to be taken into account in the pressure equation in order to allow in small-scale simulations the direct influence of thermal and moisture effects on the pressure field - and so on the wind field too.
- Ignoring these terms or having some inconsistent approximation of them in the pressure equation is equivalent to a hidden mass budget error in the model.
- Furthermore, even the heating terms in the final prognostic temperature equation appear to be not correct, if for the elimination of the individual pressure tendency a pressure equation is applied where the diabatic terms are already ignored or incomplete.
- It is important to note that it is necessary not to apply selective approximations in one equation, which after this approximation is used to combine with another equation having less and/or different (inconsistent) approximations.
- On this ground we are going to pursue a new project in favour of physically improving the given LM equations beyond of any improvement of parameterisation.