Is divergence damping an appropriate tool to filter out acoustic modes in a compressible nonhydrostatic model?

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- **Philosophy of divergence damping**
- **Analysis of continuous fast-mode equations**

  - isotropic approach (Skamarock and Klemp 1992)

  → ARPS-model

  - non-isotropic approach

  → MM5 , LM , Advanced Research WRF-NCAR model

- **Numerical analysis**
- **Conclusions**

• Philosophy of divergence damping

Transformation of common perturbation variables ►

$$(u \;\; w \;\; T\,)^t := \sqrt{\varrho_s \bar{\varrho}}\, \left(u' \;\; w' \;\; c_p T'\right)^t \;\;,\;\; p := \sqrt{\frac{\varrho_s}{\bar{\varrho}}}\; p'$$

▼

Complete linear fast-mode system :

$$\begin{aligned}
\frac{\partial u}{\partial t} + \frac{\partial p}{\partial x} &= 0 \\
\frac{\partial w}{\partial t} + \left(\frac{\partial}{\partial z} - \frac{1}{2H}\right) p + \left(\frac{g}{R\bar{T}} p - \frac{g}{c_p \bar{T}} T\right) &= 0 \\
\frac{\partial p}{\partial t} - g w + c_s^2\, D &= 0 \\
\frac{\partial T}{\partial t} - g\left(1 - \frac{N^2}{N_0^2}\right) w + c_s^2\, D &= 0
\end{aligned}$$

Vertical Scale-Height

$$\frac{1}{H} = -\frac{1}{\bar{\varrho}}\frac{d\bar{\varrho}}{dz} = \frac{N^2}{g} + \frac{g}{c_s^2}$$

Divergence

$$D = \frac{\partial u}{\partial x} + \left(\frac{\partial}{\partial z} + \frac{1}{2H}\right) w$$

Acoustic Filtering

$$c_s^2 \to \infty$$

$$\frac{1}{c_s^2}\left(\frac{\partial p}{\partial t} - gw\right) + D = 0$$

$$D \to 0$$

Introduction of a divergence damping term :

$$\frac{\partial u}{\partial t} + \frac{\partial p}{\partial x} - \underline{\gamma_h}\frac{\partial D}{\partial x} = 0$$

$$\frac{\partial w}{\partial t} + \left(\frac{\partial}{\partial z} - \frac{1}{2H}\right)p + \left(\frac{g}{R\bar{T}}p - \frac{g}{c_p\bar{T}}T\right) - \underline{\gamma_v}\left(\frac{\partial}{\partial z} + \frac{1}{2H}\right)D = 0$$

isotropic case

$$\gamma_h = \gamma_v \neq 0$$

non-isotropic case

$$\gamma_h \neq 0, \gamma_v = 0$$

- **Analysis of continuous fast-mode equations**

wave solution: $\psi \sim \exp\left(i\left(kx + mz - \omega t\right)\right)$

Reference case : $\gamma_h = \gamma_v = 0.$ without divergence damping

frequency equation

$$\frac{1}{c_s^2}\omega^4 - \kappa^2\omega^2 + k^2N^2 = 0$$

gravity frequency

$$\omega_g^{+,-} \approx \pm \frac{kN}{\kappa}$$

sound frequency

$$\omega_s^{+,-} \approx \pm c_s\kappa$$

$\kappa^2 = k^2 + m^2 + \frac{1}{4H^2}$

- **Isotropic case (Skamarock and Klemp 1992) :** $\gamma_h = \gamma_v \neq 0$

frequency equation

$$\frac{1}{c_s^2}\omega^4 + i\gamma\left(k^2 - \left(im + \frac{1}{2H}\right)^2\right)\frac{\omega^3}{c_s^2} - \kappa^2\omega^2 + k^2N^2 = 0\,.$$

gravity frequency

$$\omega_g^{+,-} \;\approx\; \pm\frac{kN}{\kappa}\,.$$

sound frequency

$$\omega_s^{+,-} \;\approx\; \pm\kappa c_s\left(1 - \frac{\gamma^2\kappa^2}{4c_s^2}\right)^{1/2} - i\frac{1}{2}\gamma\kappa^2$$

retained !

- **wave propagation retarded**
- **amplitude selectively damped**

Application recommendable !

- **Non - isotropic case :** $\gamma_h \neq 0, \gamma_v = 0$

$$\frac{1}{c_s^2}\omega^4 + i\gamma_h k^2 \frac{\omega^3}{c_s^2} - \kappa^2\omega^2 - i\gamma_h k^2 \left(im + \frac{1}{2H}\right)\frac{g}{c_s^2}\omega + k^2 N^2 = 0\,.$$

sound frequency

$$\omega_s^{+,-} \approx \pm\kappa c_s \left(1 - \frac{\gamma_h^2 k^4}{4c_s^2\kappa^2}\right)^{1/2} - i\frac{1}{2}\gamma_h k^2$$

- **Phase speed becomes retarded**
- **Selective amplitude damping, not limited to horizontal waves**
- **Degree of damping depends only on horizontal wavenumber**

• **Non - isotropic case :** $\gamma_h \neq 0, \gamma_v = 0$

$$\frac{1}{c_s^2}\omega^4 + i\gamma_h k^2 \frac{\omega^3}{c_s^2} - \kappa^2\omega^2 - i\gamma_h k^2 \left(im + \frac{1}{2H} \right) \frac{g}{c_s^2}\omega + k^2 N^2 = 0\,.$$

gravity frequency

$$\omega_g^{+,-} \approx \pm \frac{kN}{\kappa}\left(1 + \frac{1}{4}\gamma_h^2 \frac{\kappa^2}{k^2 N^2}\left(\frac{g}{c_s^2}\frac{k^2 m}{\kappa^2}\right)^2\right)^{1/2} + \frac{1}{2}\gamma_h \frac{g}{c_s^2}\frac{k^2 m}{\kappa^2}\,.$$

• **Pair of opposite phase speeds appears erroneously greater or smaller compared to the exact solution**

• **These deviations behave asymmetric due to the second term**

→ **not recommendable !**

- **Numerical analysis** **(cf. Gassmann und Herzog 2006)**

Numerical fast-waves time scheme

$$\gamma_{h,v} = \alpha_{h,v}\Delta\tau c_s^2.$$

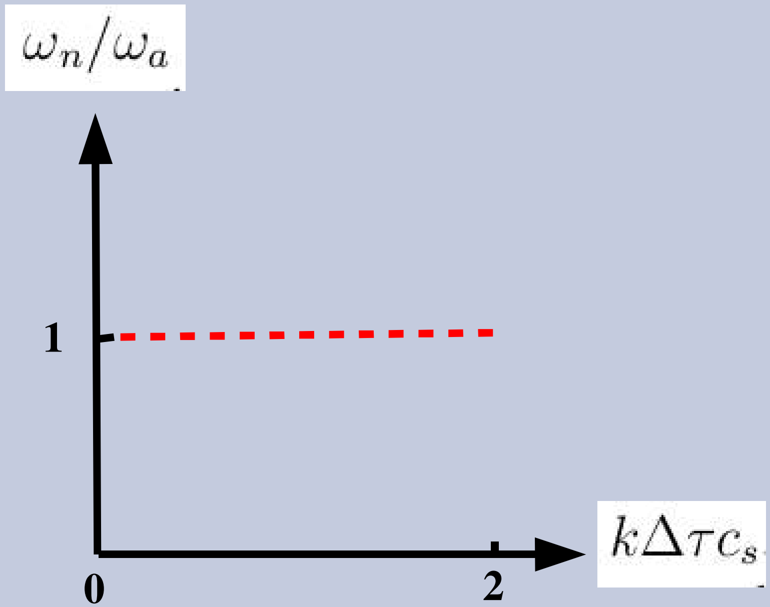
$$\frac{u^{\nu+1}-u^{\nu}}{\Delta\tau} = -\frac{\partial p^{\nu}}{\partial x} + \alpha_h \Delta\tau c_s^2 \frac{\partial D^{\nu}}{\partial x}$$

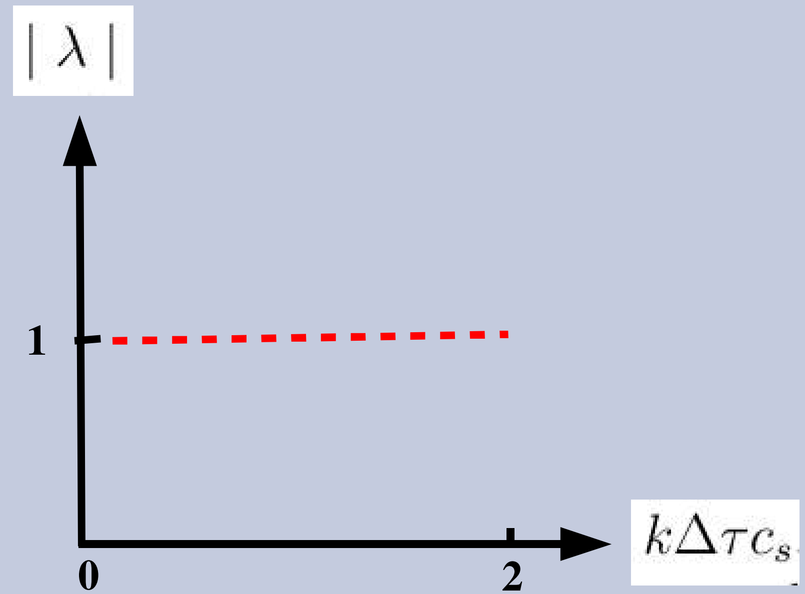
$$\frac{w^{\nu+1}-w^{\nu}}{\Delta\tau} = -\left(\frac{\partial}{\partial z} - \frac{1}{2H}\right)\left(\beta^+ p^{\nu+1} + \beta^- p^{\nu}\right) - \\ -\frac{g}{R\bar{T}}\left(\beta^+ p^{\nu+1} + \beta^- p^{\nu}\right) + \frac{g}{c_p\bar{T}}\left(\beta^+ T^{\nu+1} + \beta^- T^{\nu}\right) + \\ +\alpha_v \Delta\tau c_s^2 \left(\frac{\partial}{\partial z} + \frac{1}{2H}\right)\left(\beta^+ D^{\nu+1} + \beta^- D^{\nu}\right)$$

$$\frac{T^{\nu+1}-T^{\nu}}{\Delta\tau} = -c_s^2\left(\frac{\partial}{\partial z} + \frac{1}{2H}\right)\left(\beta^+ w^{\nu+1} + \beta^- w^{\nu}\right) - c_s^2 \frac{\partial u^{\nu+1}}{\partial x} + \\ +g\left(1 - \frac{N^2}{N_0^2}\right)\left(\beta^+ w^{\nu+1} + \beta^- w^{\nu}\right)$$

$$\frac{p^{\nu+1}-p^{\nu}}{\Delta\tau} = \left(g - c_s^2\left(\frac{\partial}{\partial z} + \frac{1}{2H}\right)\right)\left(\beta^+ w^{\nu+1} + \beta^- w^{\nu}\right) - c_s^2 \frac{\partial u^{\nu+1}}{\partial x}$$

Fast-mode Stability Analysis - Phase-Amplitude Diagram



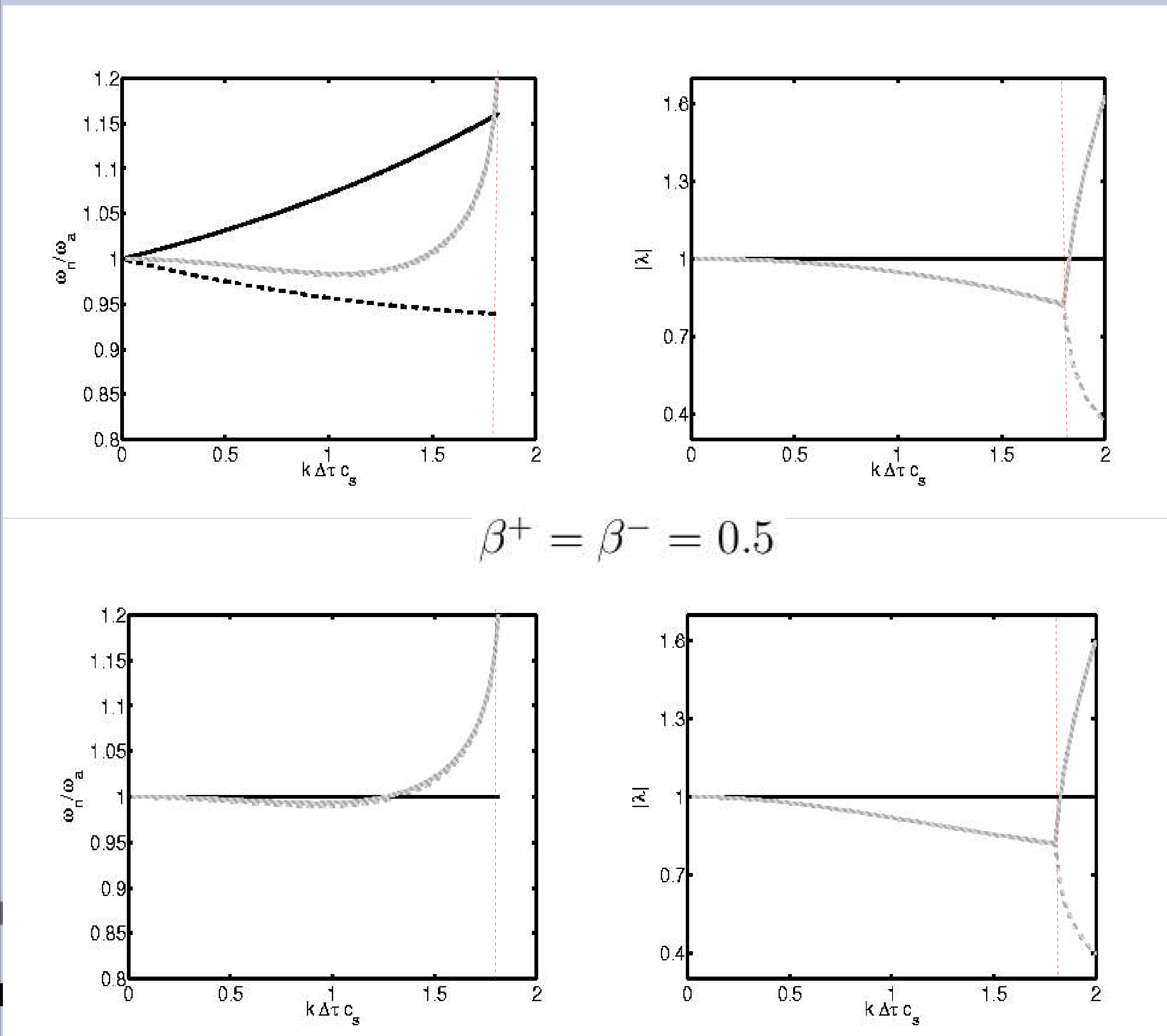


Amplitude amplification rate $\mid\lambda\mid$ **and phase ratio** ω_n/ω_a

against Courant-Number $k\Delta\tau c_s$

$$N^2 = g^2/(c_p\bar{T}) \qquad \bar{T} = 250K \qquad k = m = 2\pi/3000m$$

Divergence damping with a vertically implicit neutral scheme



$$\beta^+ = \beta^- = 0.5$$

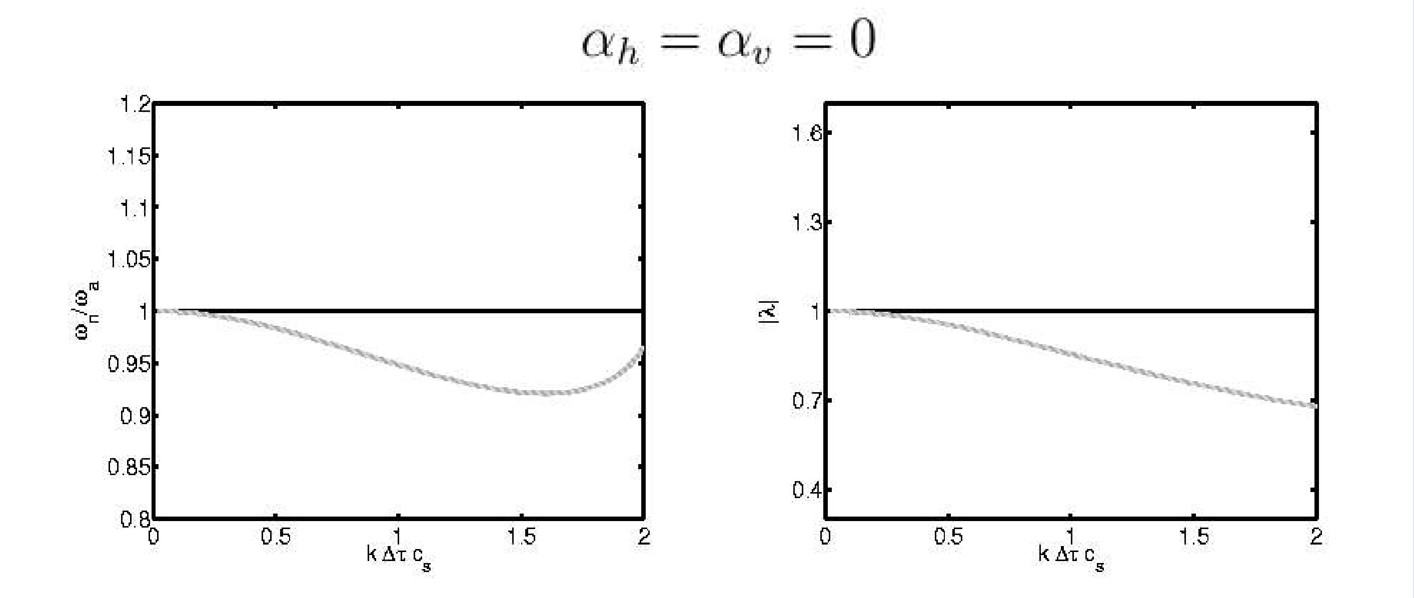
non-isotropic

$$\alpha_h = 0.1 \; ; \; \alpha_v = 0$$

isotropic

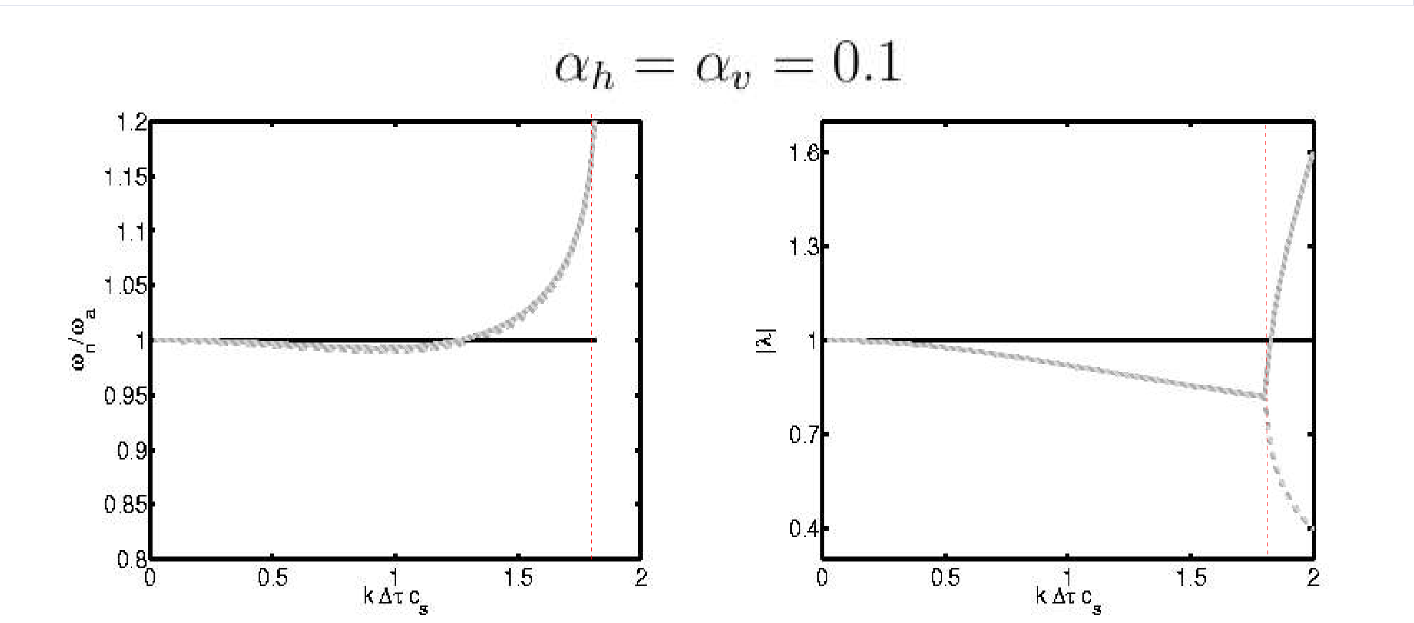
$$\alpha_h = \alpha_v = 0.1$$

Vertically implicit off-centered scheme



off-centered

$$\beta^+ = 0.7 \quad ; \quad \beta^- = 0.3$$



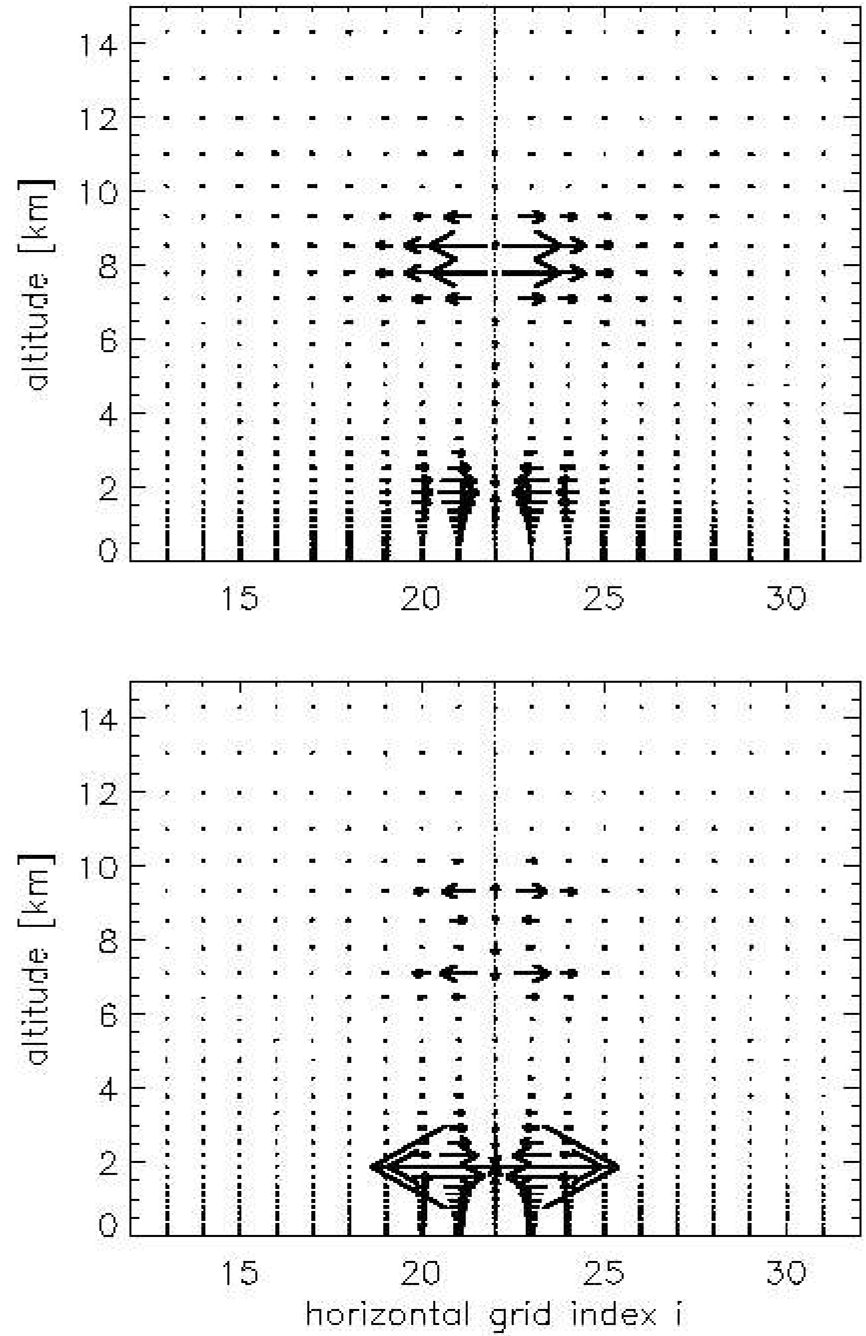
isotropic divergence damping

$$\beta^+ = \beta^- = 0.5$$

without ➡

non-isotropic divergence damping

with ➡

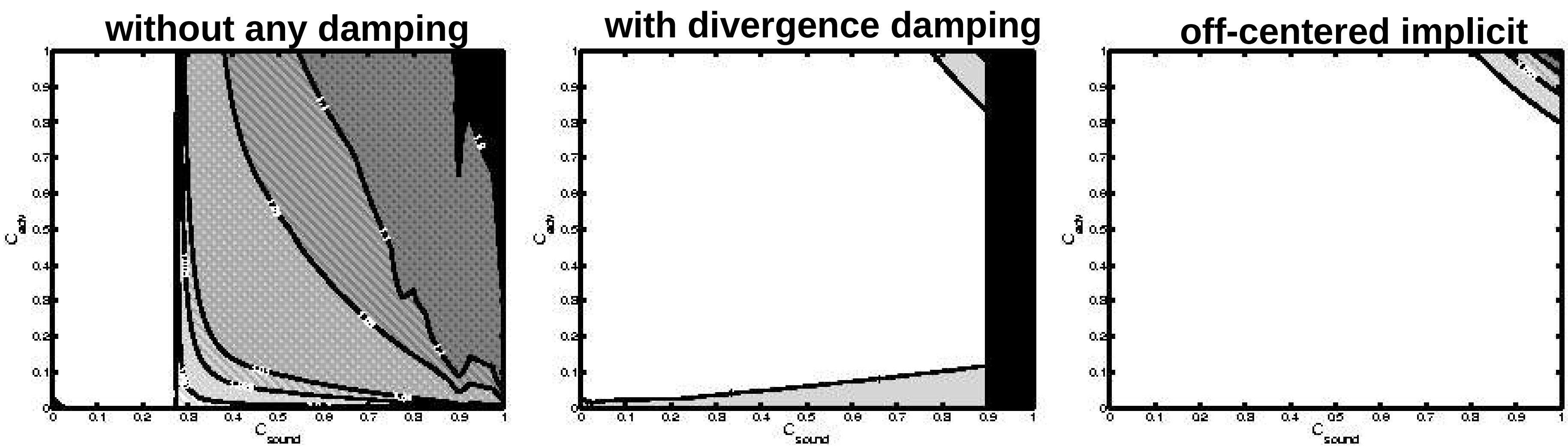


Simulation with vertical redistribution of mass at two selected locations without/with divergence damping (upper/lower) panel. Length of wind arrows in relative units. Arrow length in the lower panel is reduced and must be multiplied by factor five to compare with the upper panel.

(seminar contribution by Kuell, Gassmann, Bott)

Linear Stability Analysis for Gassmann Splitting Method with RK 2/3 for horizontal advection

Courant-number diagram for $|\lambda_{max}|$



$$C_{sound} = c_s \Delta\tau / \Delta x \ (abcissa) \quad - \quad C_{adv} = U_{max} \Delta t / \Delta x \ (ordinate)$$

$$0 \leq C_{sound,adv} \leq 1$$

Parameter field :

$$\Delta x = 1000m \qquad \Delta z = 300m$$

$$2\Delta z \leq L_z \leq 20\Delta z \qquad 2\Delta x \leq L_x \leq 20\Delta x$$

$$0 \leq \Delta\tau \leq \Delta x / c_s \qquad \Delta t = N_s \Delta\tau (N_s = 4)$$

$$0 \leq U \leq c_s/4$$

Conclusions

- ► Divergence damping is a possible tool to suppress sound waves in compressible nonhydrostatic models – provided the original method from Skamarock and Klemp (1992) is applied (e.g. ARPS-model).
- ► A non-isotropic divergence damping (application only to the horizontal momentum equations) should be avoided, since internal gravity modes are not properly retained (MM5 , LM , Advanced Research WRF-NCAR model).
- ► The application of an off-centered implicit scheme in the fast-waves part is found to be the favorite scheme in connection with our splitting algorithm, which makes a divergence technique avoidable.
- ► We emphasise that our results are based on the necessity to have a complete and consistent fast-waves scheme !