



## Mass Conserving Courant Number Independent Eulerian Advection of the Moisture Quantities for the LMK

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- Grid structure: horizontal: Arakawa-C staggered  
vertical: Lorenz staggered
- Time integration: time-splitting (Klemp-Wilhelmson-splitting) of slow and fast modes  
2-timelevels: **Runge-Kutta: 2<sup>nd</sup> order, 3<sup>rd</sup> order, 3<sup>rd</sup> order TVD**  
(7km LM: 3-timelevels: Leapfrog (+ centered differences))
- **Advection:** for  $u, v, w, p^*, T$ :  
horizontal adv.: upwind 3<sup>rd</sup> or **5<sup>th</sup> order**, centered 4<sup>th</sup> or 6<sup>th</sup> order  
vertical adv.: **implicit centered 2<sup>nd</sup> order**,  
explicit upwind 3<sup>rd</sup> or 5<sup>th</sup> order (J.Förstner),  
implicit upwind 3<sup>rd</sup> order (M. Baldauf)  
for  $q_v, q_c, q_i, q_r, q_s, q_g, TKE$ :  
**Courant-number-independent (CNI) advection:**  
Motivation: no constraint for  $w$  (deep convection!)  
Euler forward (J.Förstner):
  - van Leer-type scheme
  - **Bott-scheme (2<sup>nd</sup>, 4<sup>th</sup> order) in Conservation Form**Semi-Lagrange (M. Baldauf):
  - tri-linear:  $q_r, q_s, q_g$  tri-cubic:  $q_v, q_c, q_i, TKE$  (clipping)
  - tri-cubic:  $q_v, q_c, q_i, q_r, q_s, q_g, TKE$  (multiplicative filling)
- Smoothing: divergence damping

4<sup>th</sup> order horizontal diffusion (to control instabilities near damping layers)

$$\frac{\partial q_x}{\partial t} + \vec{v} \cdot \nabla q_x = \dots$$

Advective Form

$$\frac{\partial q_x}{\partial t} + \nabla \cdot (\vec{v} q_x) - q_x \nabla \cdot \vec{v} = \dots$$

Flux Form – 3D Wind Divergence

$$\frac{\partial \rho q_x}{\partial t} + \nabla \cdot (\rho \vec{v} q_x) = \dots$$

**Conservation Form**

$$\rho_x^{(n)} = \rho^{(n)} q_x^{(n)}$$

$$\rho_x^{(*)} = \rho_x^{(n)} + \Delta t A_x(\rho_x^{(n)})$$

$$\rho_x^{(**)} = \rho_x^{(*)} + \Delta t A_y(\rho_x^{(*)})$$

$$\rho_x^{(n+1)} = \rho_x^{(**)} + \Delta t A_z(\rho_x^{(**)})$$

“diagnostic“



**“prognostic“**



$$\rho^{(n+1)} = \frac{p_0 + p^{*(n+1)}}{R_d T^{(n+1)}} - \left( \left( \frac{R_v}{R_d} - 1 \right) \rho_v^{(n+1)} - \rho_c^{(n+1)} - \rho_s^{(n+1)} \right) \quad \text{or}$$

$$\begin{aligned} \rho^{(*)} &= \rho^{(n)} + \Delta t A_x(\rho^{(n)}) \\ \rho^{(**)} &= \rho^{(*)} + \Delta t A_y(\rho^{(*)}) \\ \rho^{(n+1)} &= \rho^{(**)} + \Delta t A_z(\rho^{(**)}) \end{aligned}$$

$$q_x^{(n+1)} = \frac{\rho_x^{(n+1)}}{\rho^{(n+1)}}$$

$$q_x^{t+\Delta t} = (I + A_z)(I + A_y)(I + A_x) q_x^t$$

3D Strang-Splitting

$$q_x^{t+2\Delta t} = (I + A_x)(I + A_y)(I + A_z) q_x^{t+\Delta t}$$



### Courant Number Independent Formulation:

Standard Courant Number  $Cr_{j+\frac{1}{2}} = u_{j+\frac{1}{2}} \frac{\Delta t}{\Delta x}$

Integer Courant Number  $K_{j+\frac{1}{2}} = \text{INT}(Cr_{j+\frac{1}{2}})$

Fractional Courant Number  $Cr'_{j+\frac{1}{2}} = \text{MOD}(Cr_{j+\frac{1}{2}}, K_{j+\frac{1}{2}})$

Integer Flux  $\frac{\Delta t}{\Delta x} F_{j+\frac{1}{2}} = \begin{cases} \sum_{k=1}^{K_{j+\frac{1}{2}}} \rho_{x_{j-k+1}}, & K_{j+\frac{1}{2}} \geq 1 \\ 0, & K_{j+\frac{1}{2}} = 0 \\ \sum_{k=-1}^{K_{j+\frac{1}{2}}} \rho_{x_{j-k}}, & K_{j+\frac{1}{2}} \leq -1 \end{cases}$

Fractional Flux  $f'_{j+\frac{1}{2}} = f'(Cr'_{j+\frac{1}{2}}, j - K_{j+\frac{1}{2}})$

### Positive Definite Bott - Scheme:

$$i_{j+\frac{1}{2}}^+ = \sum_{k=0}^{l=2/4} \frac{\alpha_{j,k}}{(k+1)2^{k+1}} \left[ 1 - \left( 1 - 2Cr_{j+\frac{1}{2}}^+ \right)^{k+1} \right]$$

$$i_{j+\frac{1}{2}}^- = \sum_{k=0}^{l=2/4} \frac{\alpha_{j+1,k}}{(k+1)2^{k+1}} \left[ 1 - \left( 1 + 2Cr_{j+\frac{1}{2}}^- \right)^{k+1} \right] (-1)^k$$

$$i_j = \sum_{k=0}^{l=2/4} \frac{\alpha_{j,k}}{(k+1)2^{k+1}} \left[ (-1)^k + 1 \right]$$

$$i_{j+\frac{1}{2}}^+ = \max\left(0, i_{j+\frac{1}{2}}^+\right)$$

$$i_{j+\frac{1}{2}}^- = \max\left(0, i_{j+\frac{1}{2}}^-\right)$$

$$i_j = \max\left(i_j, i_{j+\frac{1}{2}}^+ + i_{j-\frac{1}{2}}^- + \epsilon\right)$$

$$f_{j+\frac{1}{2}} = \frac{\Delta x}{\Delta t} \left[ \frac{i_{j+\frac{1}{2}}^+}{i_j} \rho_{x_j} - \frac{i_{j+\frac{1}{2}}^-}{i_{j+1}} \rho_{x_{j+1}} \right]$$

# Deutscher Wetterdienst

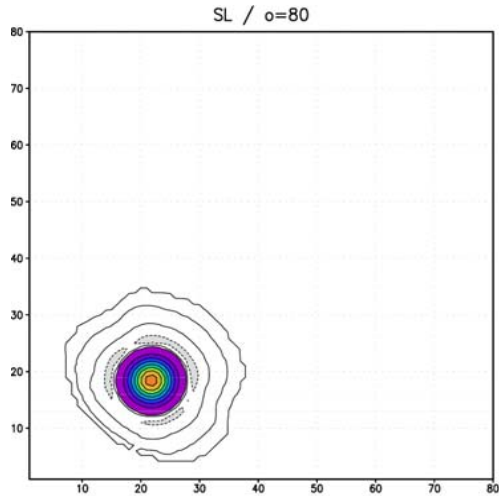
## Aktionsprogramm 2003

### Solid Body Rotation

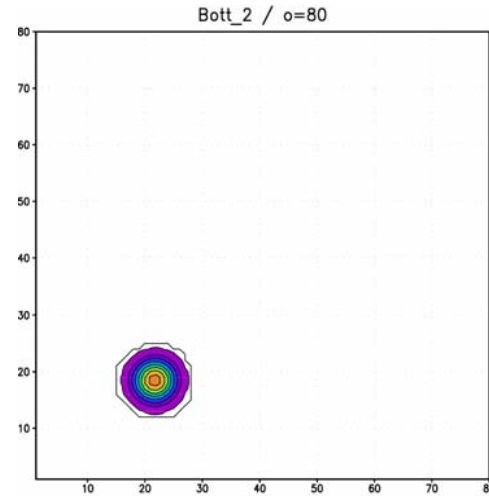
of a tracer cone with an initial maximum of 1

**MAX( INT(Cr) ) = 2 (nearly 3)**

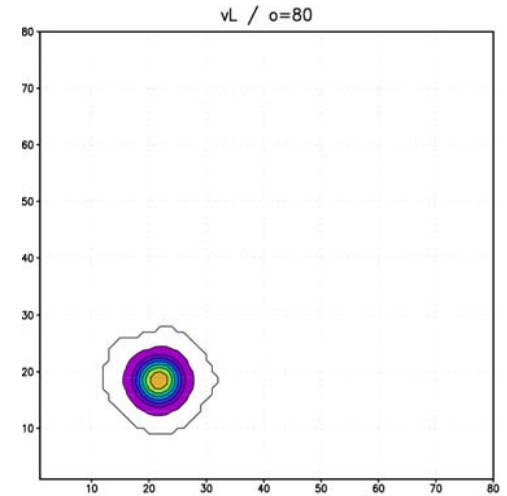
**80** time steps for one turn



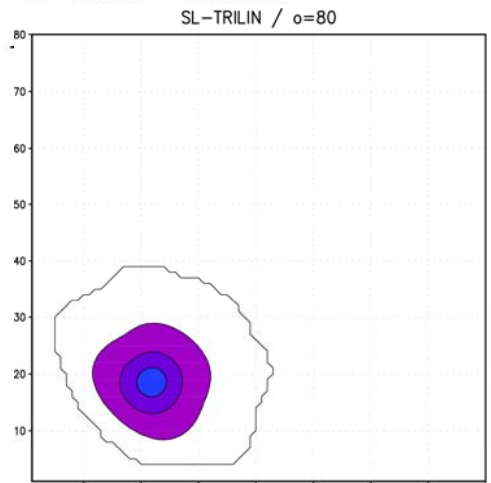
Min: -0.0145539 Max: 0.755817



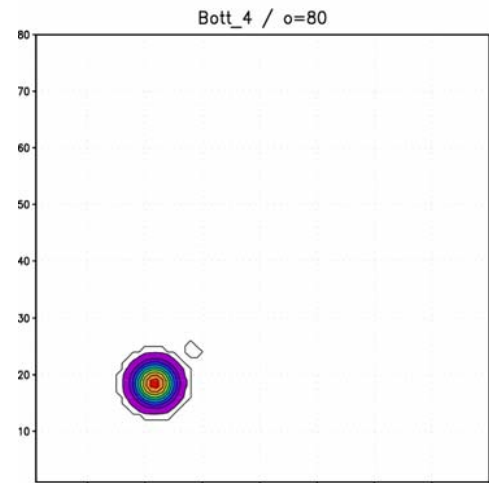
Min: -1.86039e-17 Max: 0.753967



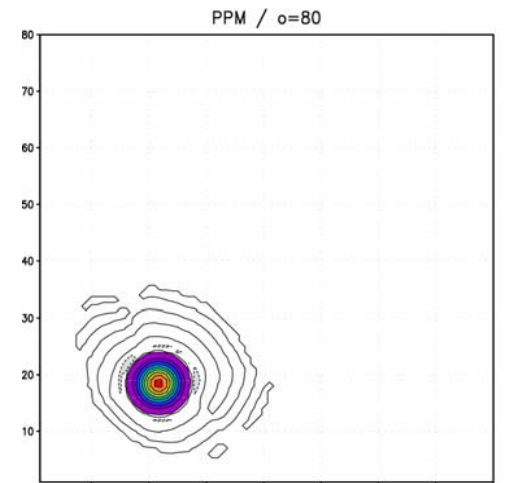
Min: -1.84585e-16 Max: 0.651138



Min: 0 Max: 0.246414



Min: -2.95617e-18 Max: 0.842178



Min: -0.0155748 Max: 0.8326



# Deutscher Wetterdienst

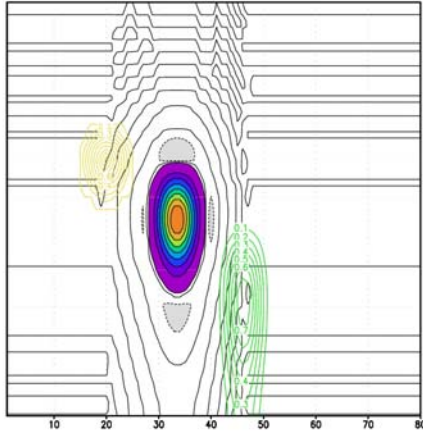
## Aktionsprogramm 2003

**xz - Advection** ( $u = 150 \text{ m/s}$ ,  $w = -10 \text{ m/s}$ )

of a tracer cone with an initial maximum of 1  
vertically stretched grid:

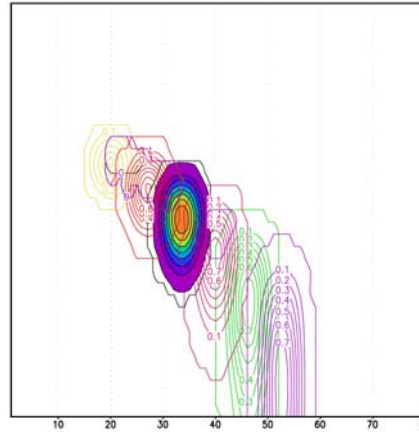
**MAX( INT(Cr) ) = 1** (near the bottom boundary)

SL /  $u=150\text{m/s}$ ,  $w=-10\text{m/s}$



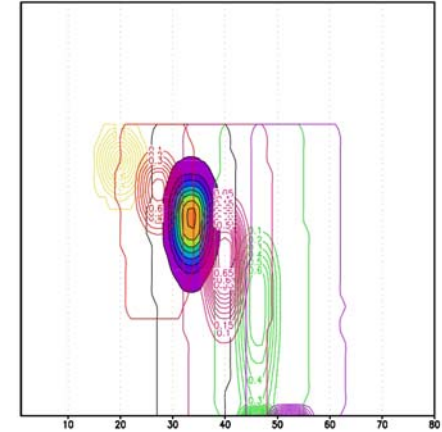
(4) Min: -0.0279751 Max: 0.796995  
(8) Min: -0.0305676 Max: 0.784036

Bott\_2\_NEW\_QXACF-V2\_ke1 /  $u=150\text{m/s}$ ,  $w=-10\text{m/s}$



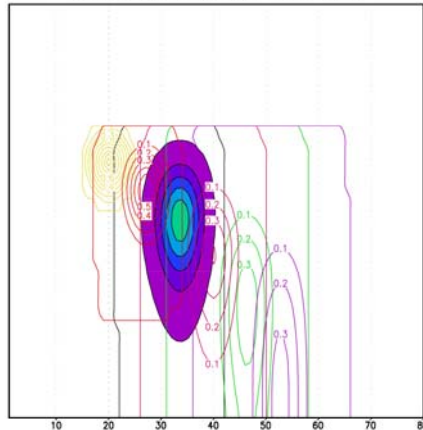
(2) Min: 0 Max: 0.833206  
(4) Min: 0 Max: 0.799759  
(6) Min: 0 Max: 0.795898  
(8) Min: 0 Max: 0.791702  
(10) Min: 0 Max: 0.782394

vL /  $u=150\text{m/s}$ ,  $w=-10\text{m/s}$



(2) Min: -1.39283e-17 Max: 0.769821  
(4) Min: -1.01963e-16 Max: 0.724457  
(6) Min: -6.27683e-21 Max: 0.69783  
(8) Min: 0 Max: 1.18555  
(10) Min: -3.12189e-17 Max: 10.6399

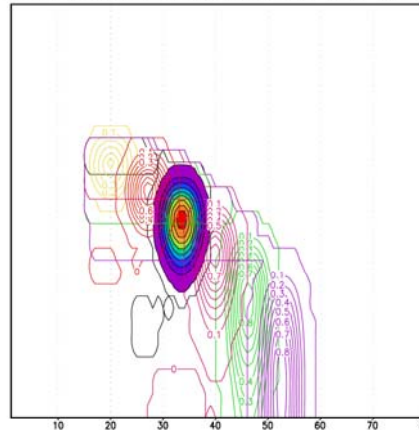
SL-TRILIN /  $u=150\text{m/s}$ ,  $w=-10\text{m/s}$



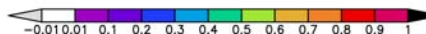
(2) Min: 0 Max: 0.597214  
(4) Min: 0 Max: 0.476059  
(6) Min: 0 Max: 0.407547  
(8) Min: 0 Max: 0.362343  
(10) Min: 0 Max: 0.338043



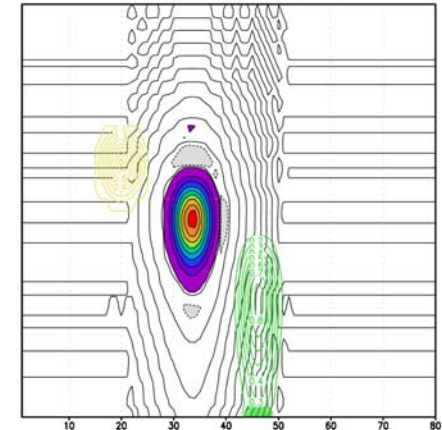
Bott\_4\_NEW\_QXACF-V2\_ke1 /  $u=150\text{m/s}$ ,  $w=-10\text{m/s}$



(2) Min: 0 Max: 0.878433  
(4) Min: 0 Max: 0.85611  
(6) Min: -5.18074e-16 Max: 0.862457  
(8) Min: 0 Max: 0.866653  
(10) Min: 0 Max: 0.86084



PPM /  $u=150\text{m/s}$ ,  $w=-10\text{m/s}$



(4) Min: -0.0321955 Max: 0.850739  
(8) Min: -0.0848058 Max: 1.19989



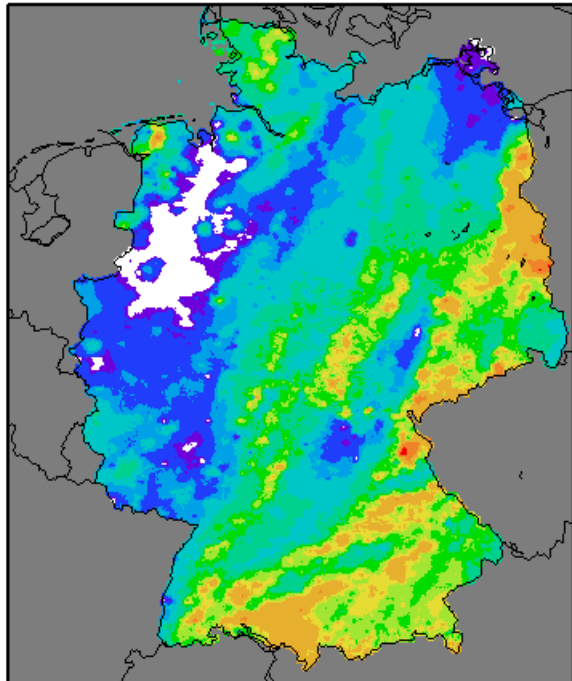
southwesterly flow

Observation

**Bott (2<sup>nd</sup> order)**  
**“Flux Form - DIV”**  
 $\nabla \cdot (\mathbf{v}q_x) - q_x \nabla \cdot \mathbf{v}$

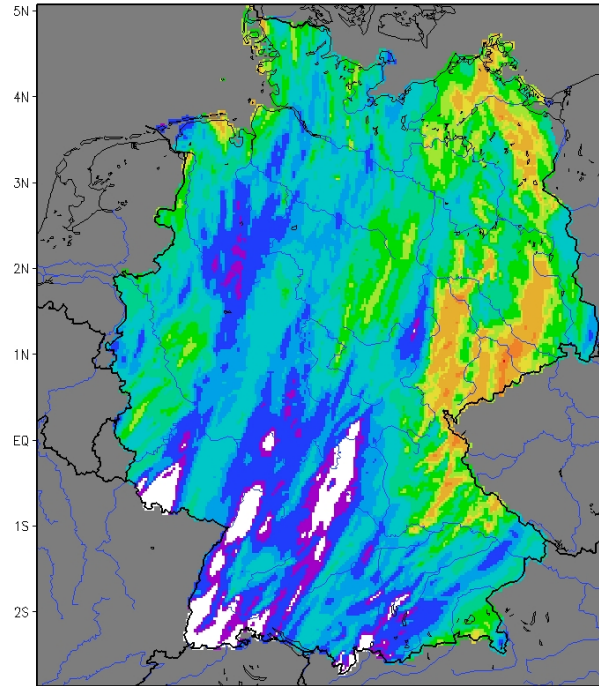
**semi-Lagrange**  
 $q_v, q_c, q_i$ : tri-cubic  
 $q_r, q_s, q_g$ : tri-linear

24-h-Niederschlag 08.07.2004 05:30 UTC + 24h (Beo.)



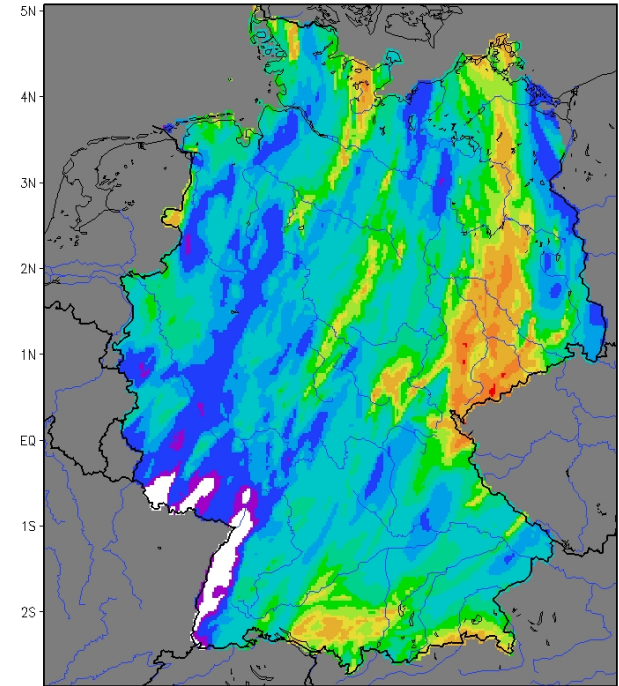
Mean: 12.3503 Min: 0 Max: 118.3 Var: 112.661

24 h PRECIPITATION - B2\_08\_TS\_2.2\_b



Mean: 10.4671 Min: 0.00195312 Max: 81.6357

24 h PRECIPITATION - SL\_08\_TS\_2.2



Mean: 11.7978 Min: 0.0195312 Max: 109.348

07/08/2004



southwesterly flow

Observation

**Bott (2<sup>nd</sup> order)**  
**“Conservation Form”**

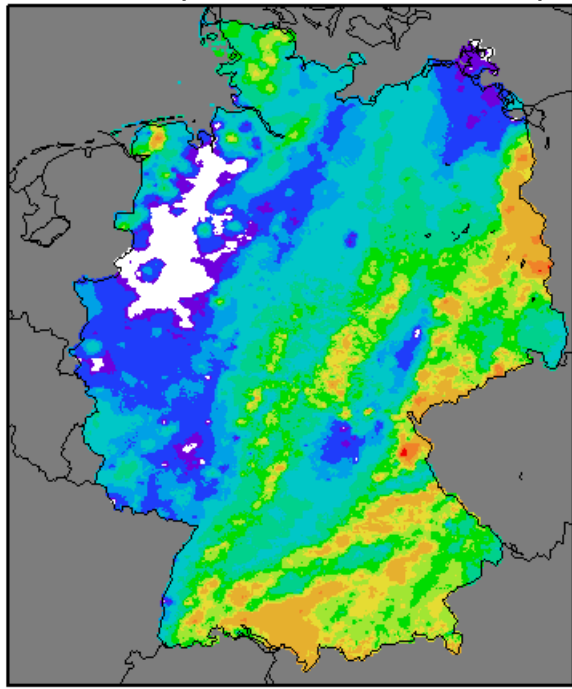
$$\nabla \cdot (\rho \mathbf{v} q_x) = \nabla \cdot (\mathbf{v} \rho_x)$$

moist turbulence with

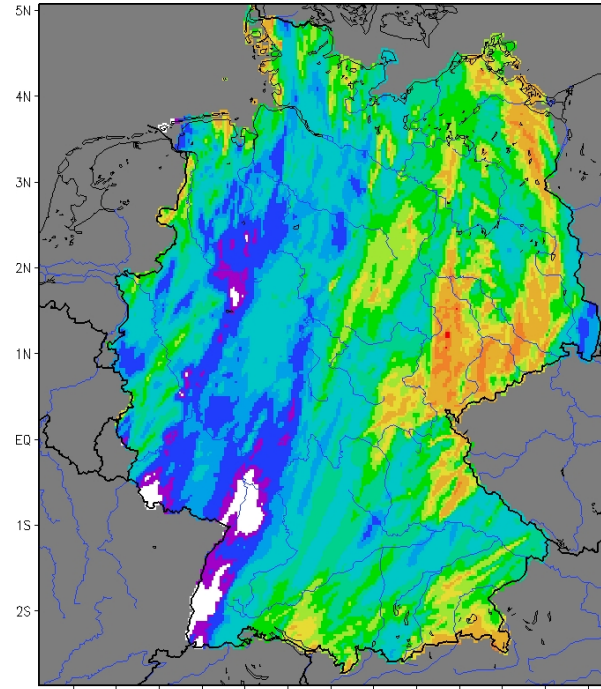
(subscale) cloud diagnostic

(only) scale clouds

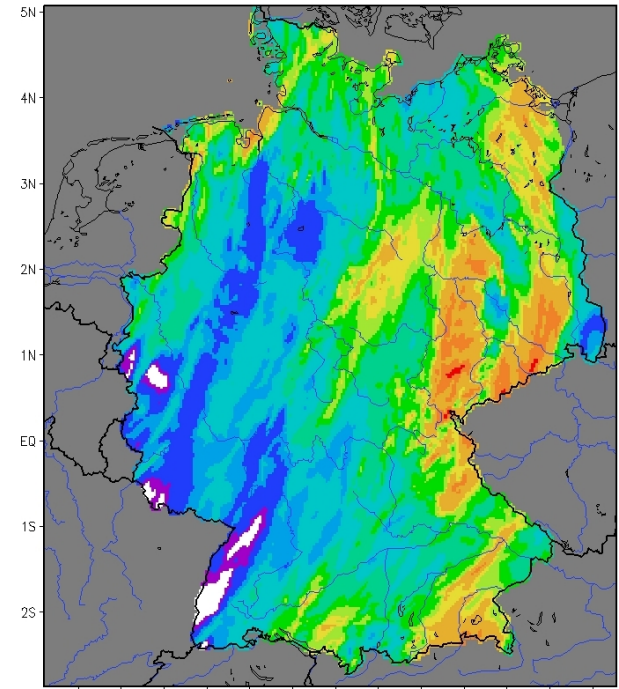
24-h-Niederschlag 08.07.2004 05:30 UTC + 24h (Beo.)



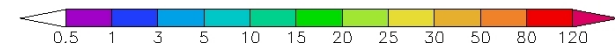
Mean: 12.3503 Min: 0 Max: 118.3 Var: 112.661



Mean: 13.3845 Min: 0.00878906 Max: 112.042



Mean: 14.8777 Min: 0.0332031 Max: 118.532



07/08/2004

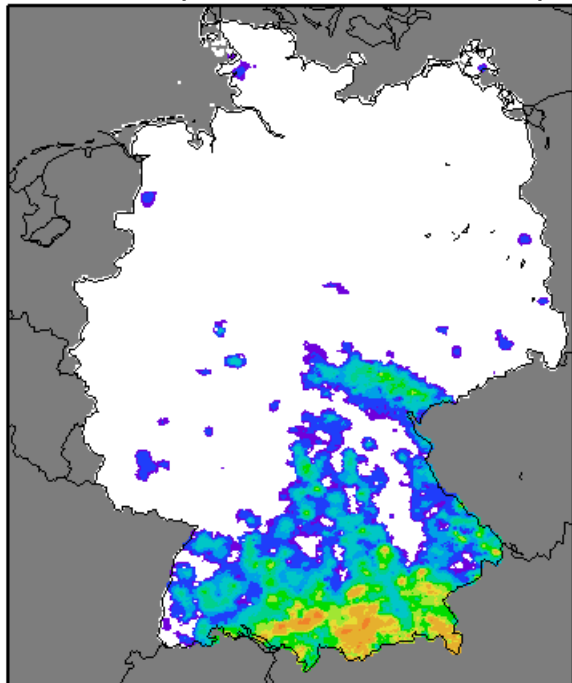
(north)westerly flow

Observation

**Bott (2<sup>nd</sup> order)**  
**“Flux Form - DIV”**  
 $\nabla \cdot (\mathbf{v}q_x) - q_x \nabla \cdot \mathbf{v}$

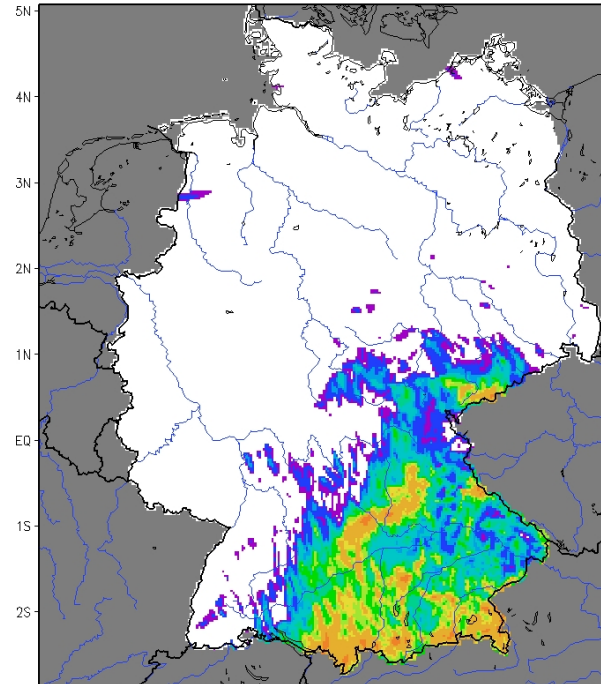
**semi-Lagrange**  
 $q_v, q_c, q_i$ : tri-cubic  
 $q_r, q_s, q_g$ : tri-linear

24-h-Niederschlag 24.07.2004 05:30 UTC + 24h (Beo.)



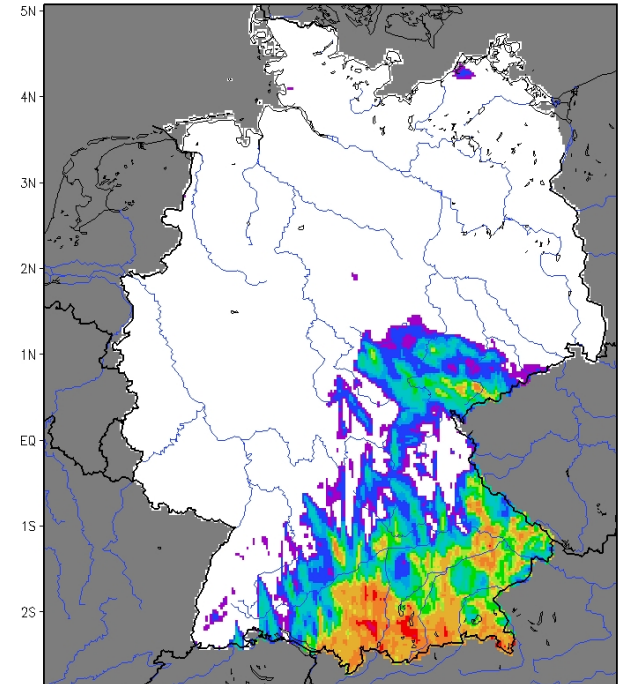
Mean: 2.31457 Min: 0 Max: 70.2 Var: 41.7218

24 h PRECIPITATION - B2\_24\_TS\_2.2\_b



Mean: 3.67557 Min: -0.00195312 Max: 78.3418

24 h PRECIPITATION - SL\_24\_TS\_2.2



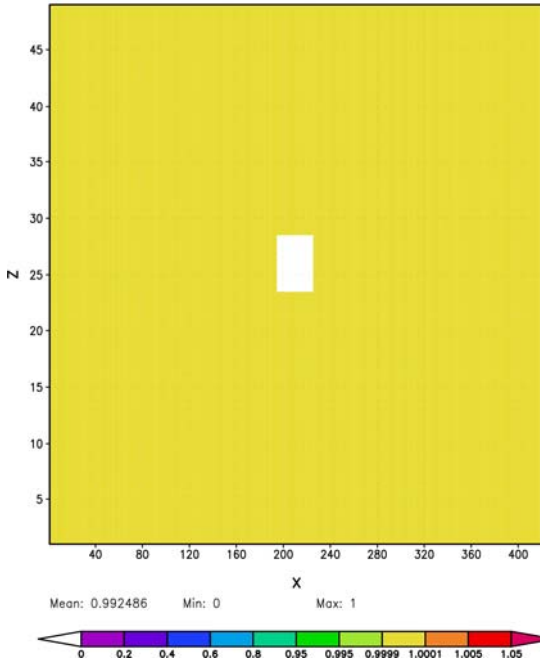
Mean: 4.20635 Min: -0.00195312 Max: 146.521

07/24/2004

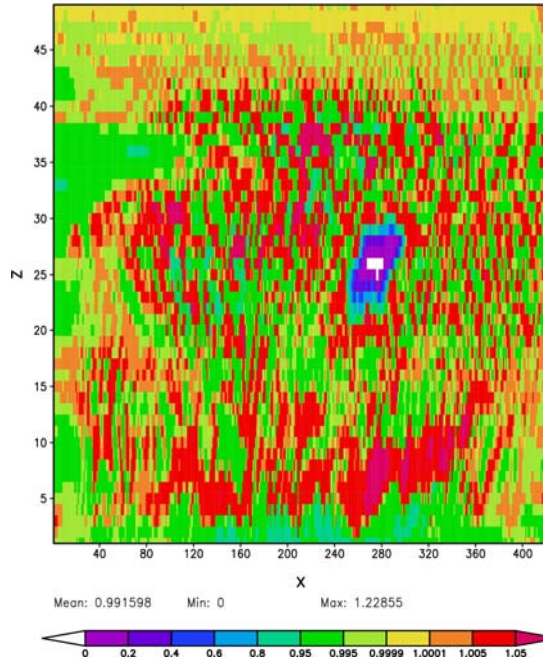


# Transport of Tracer in a Real Case Flow Field

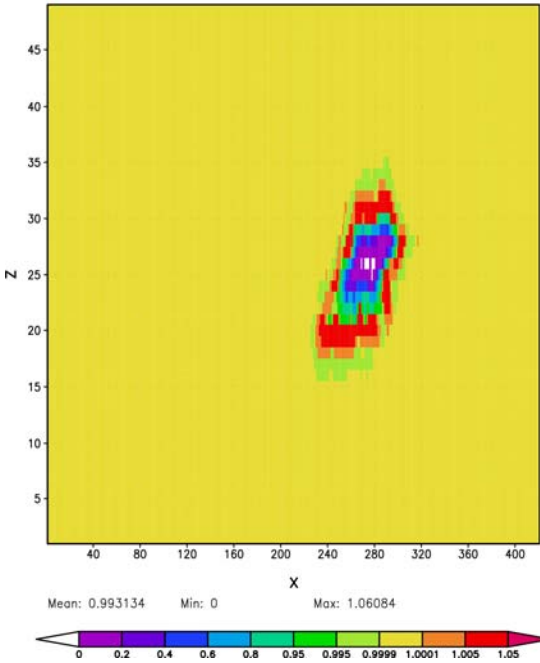
Initialized TRACER Distribution (y = 230)



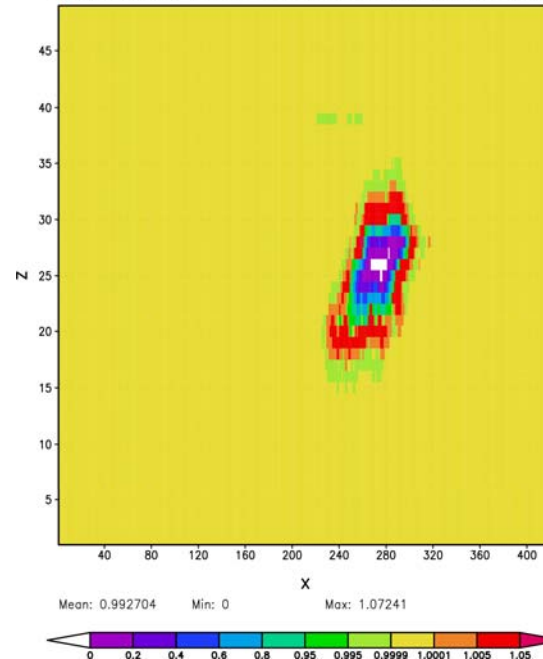
TRACER (y=260 / t=3h) – Bott (2nd) Non-Conserv. Form with Clipping



TRACER (y=260 / t=3h) – semi-Lagrange (tri-cubic) with Clipping



TRACER (y=260 / t=3h) – Bott (2nd) Conserv. Form with Clipping

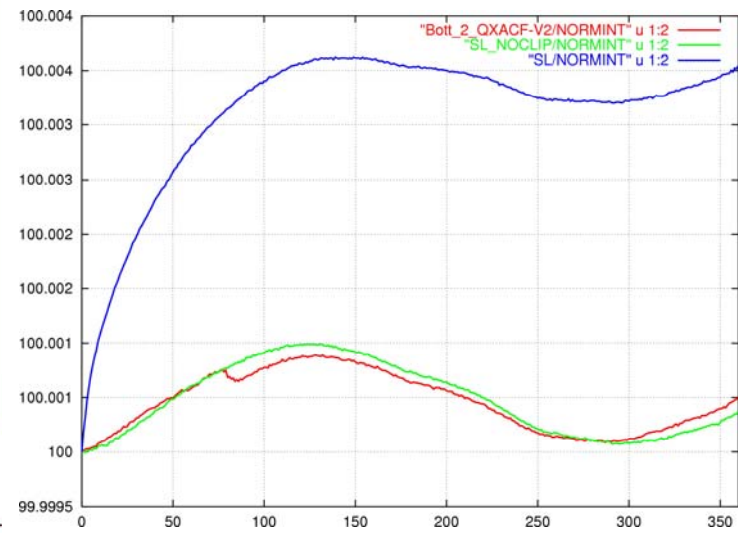


init

**Bott (2<sup>nd</sup>)  
“Flux Form  
- DIV”  
+ Clipping**

**semi-  
Lagrange  
(tri-cubic)  
+ Clipping**

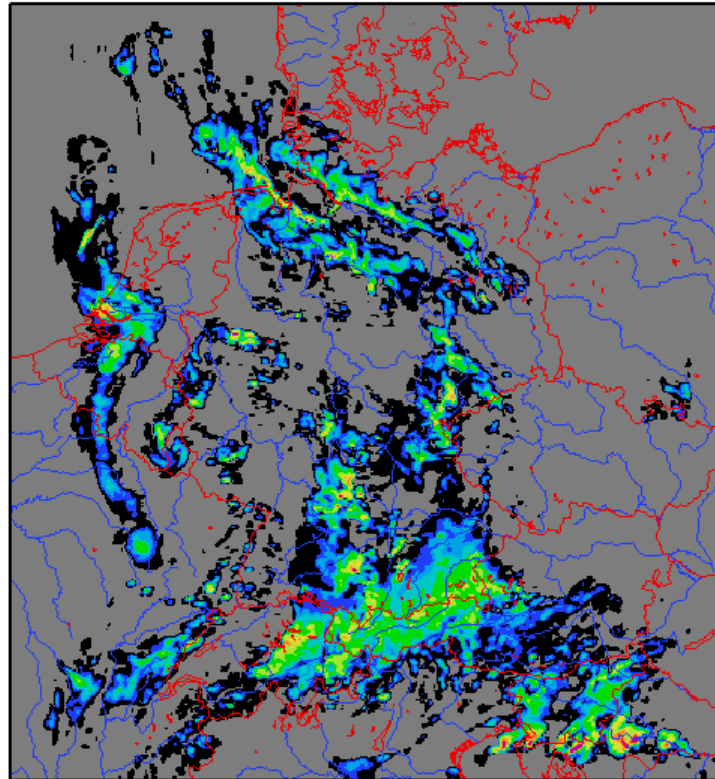
**Bott (2<sup>nd</sup>)  
“Conserv.  
Form”**



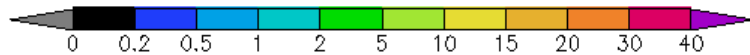
initial: 30 JUN 2005 12 UTC  
 valid: 01 JUL 2005 06 UTC

### Bott (2<sup>nd</sup> order)

CURRENT TOTAL PRECIPITATION RATE [mm/h]



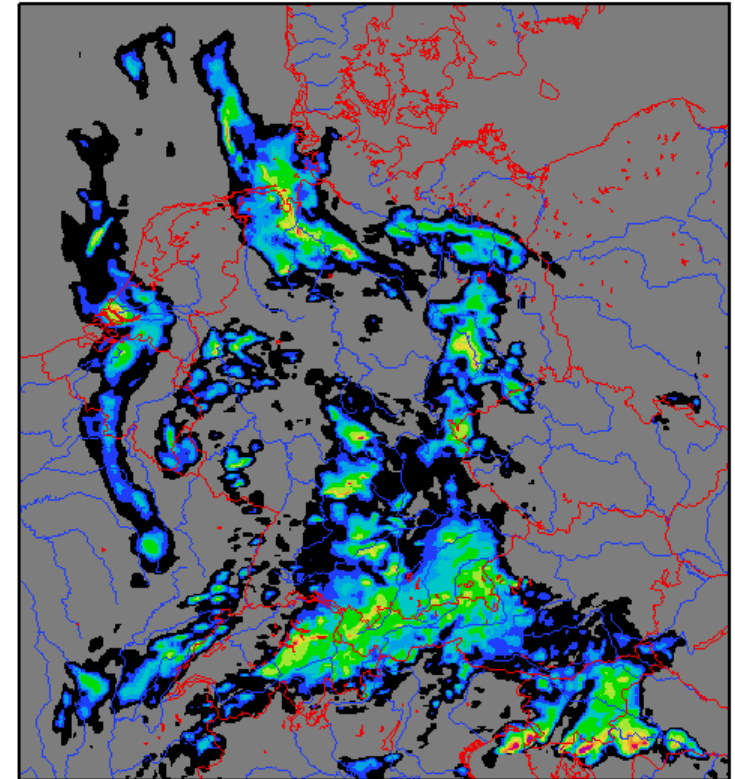
(T)	Mean: 0.324285	Min: 0	Max: 55.0218	Var: 2.41462
(R)	Mean: 0.322602	Min: 0	Max: 55.0218	Var: 2.40799
(S)	Mean: 0.00168304	Min: 0	Max: 7.44045	Var: 0.00548038



initial: 30 JUN 2005 12 UTC  
 valid: 01 JUL 2005 06 UTC

### semi-Lagrange

CURRENT TOTAL PRECIPITATION RATE [mm/h]



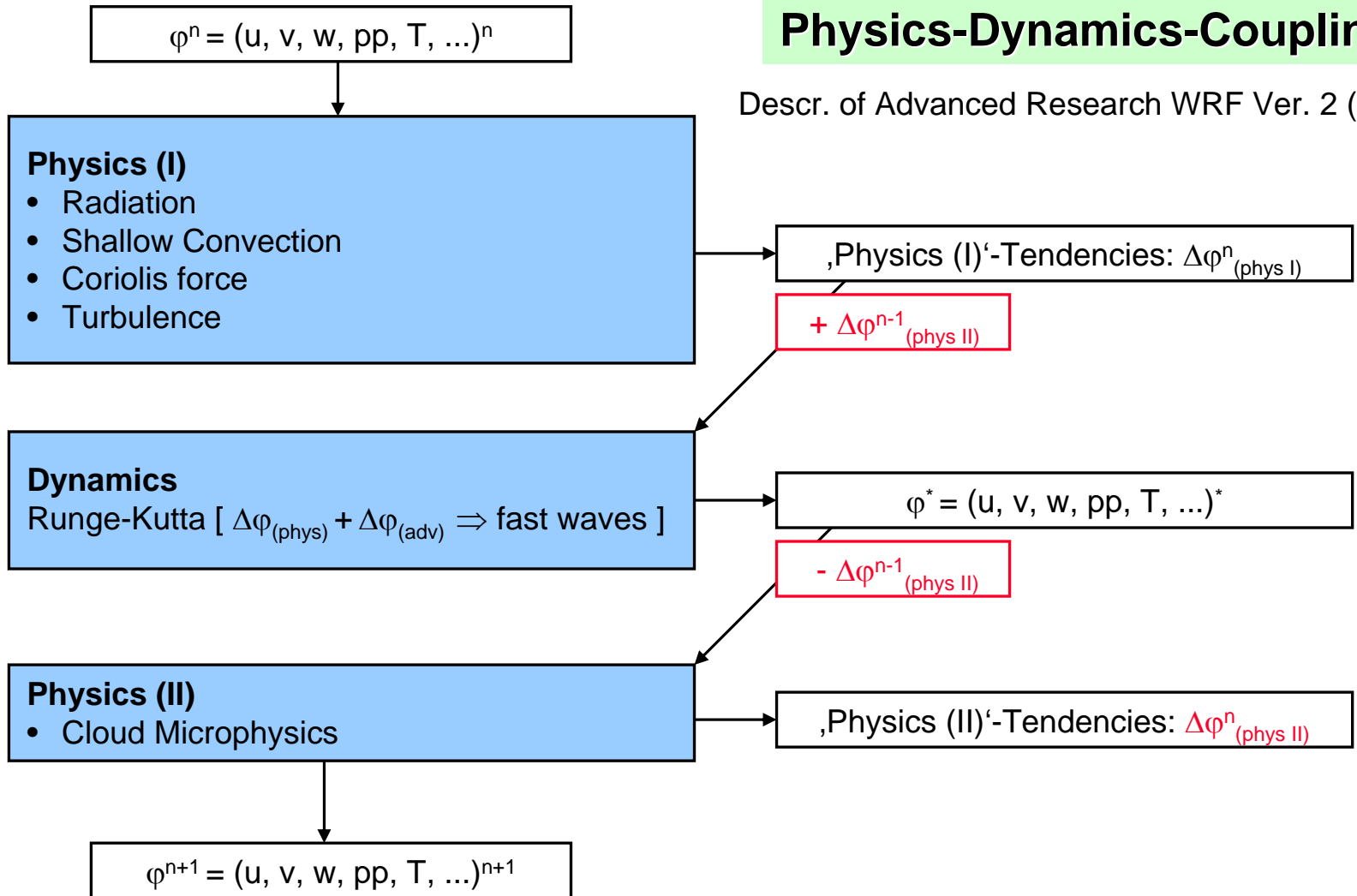
(T)	Mean: 0.328335	Min: 0	Max: 55.1239	Var: 2.24873
(R)	Mean: 0.327088	Min: 0	Max: 55.1239	Var: 2.24436
(S)	Mean: 0.00124755	Min: 0	Max: 8.84292	Var: 0.00368246





### Physics-Dynamics-Coupling

Descr. of Advanced Research WRF Ver. 2 (2005)





### Problems with New Physics-Dynamics-Coupling (NPDC)

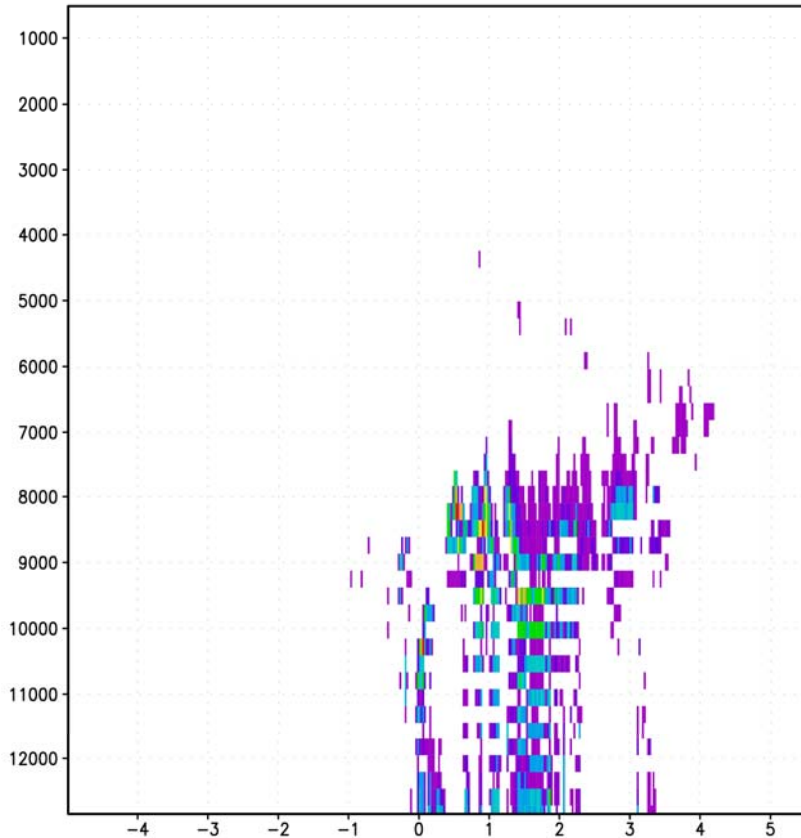
- Negative feedback between NPDC and operational **moist** turbulence parameterization (... not present in dry turbulence parameterization)
- 2- $\Delta z$  - structures in the specific cloud water field ( $q_c$ )
- 2- $\Delta z$  - structures in the TKE field, unrealistic high values, where  $q_c > 0$

### Measurements (K. Stephan, J. Förstner and M. Raschendorfer)

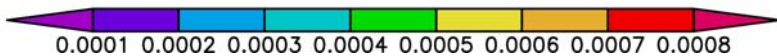
- Turbulence Parameterization
  - Bug - correction in formulation of “explicit correction”
  - No upper value for  $q_c(\text{scale}) + q_c(\text{subscale})$  in SUBROUTINE `cloud_diag` (statistical scheme for cloud diagnostic)
  - No vertical smoothing in calculation of “explicit correction”  $\Rightarrow$  **M. 1** (`wichfakt = 0.0` in PHYCTL namelist)
- NPDC
  - Latent heat conversions in SUBROUTINE `satad` (saturation adjustment) directly after Runge-Kutta dynamics are not taken into account  $\Rightarrow$  **M. 2**

### neither M. 1 nor M. 2

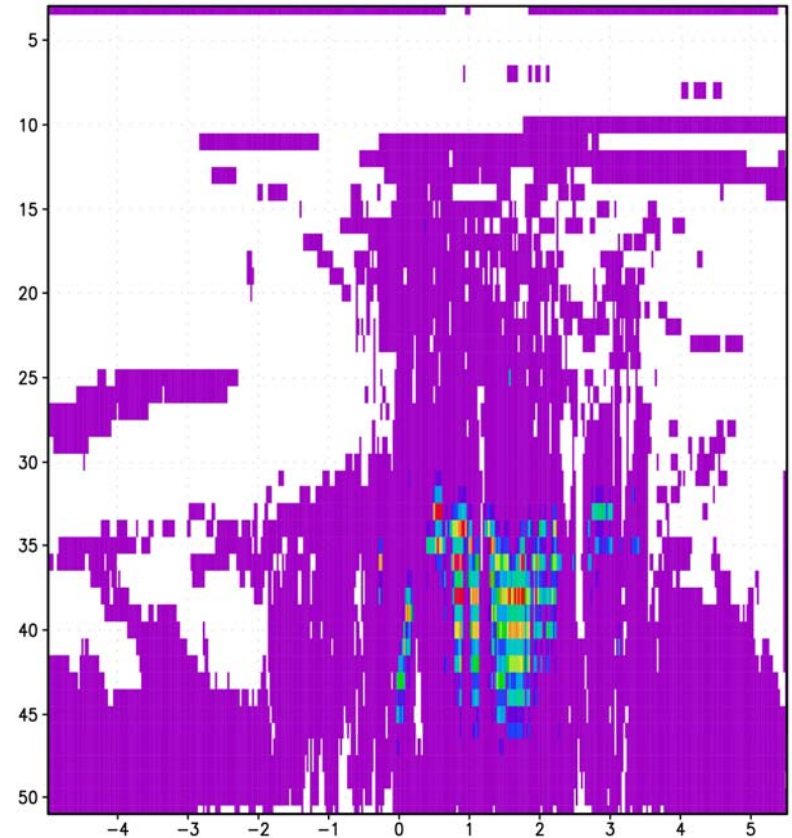
QC ( $> 1.0E-06$ ) - t=24h, lat=-2.8 - 3FT2 (wf=0.15) - NPDC (CP+SATAD)



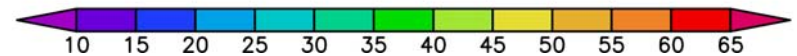
Mean: 9.86073e-06 Min: 0 Max: 0.000810772



TKE ( $> 0.1$ ) - t=24h, lat=-2.8 - 3FT2 (wf=0.15) - NPDC (CP+SATAD)

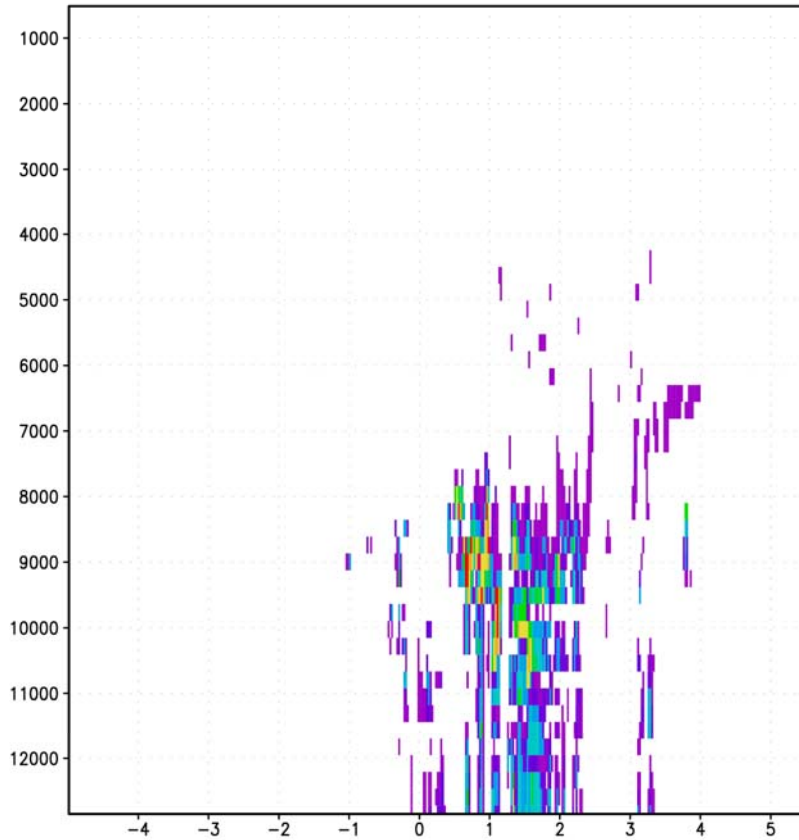


Mean: 1.25314 Min: 5e-05 Max: 71.6334

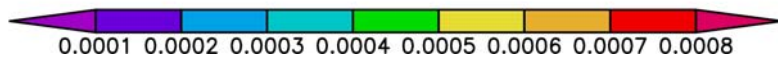


### only M. 1

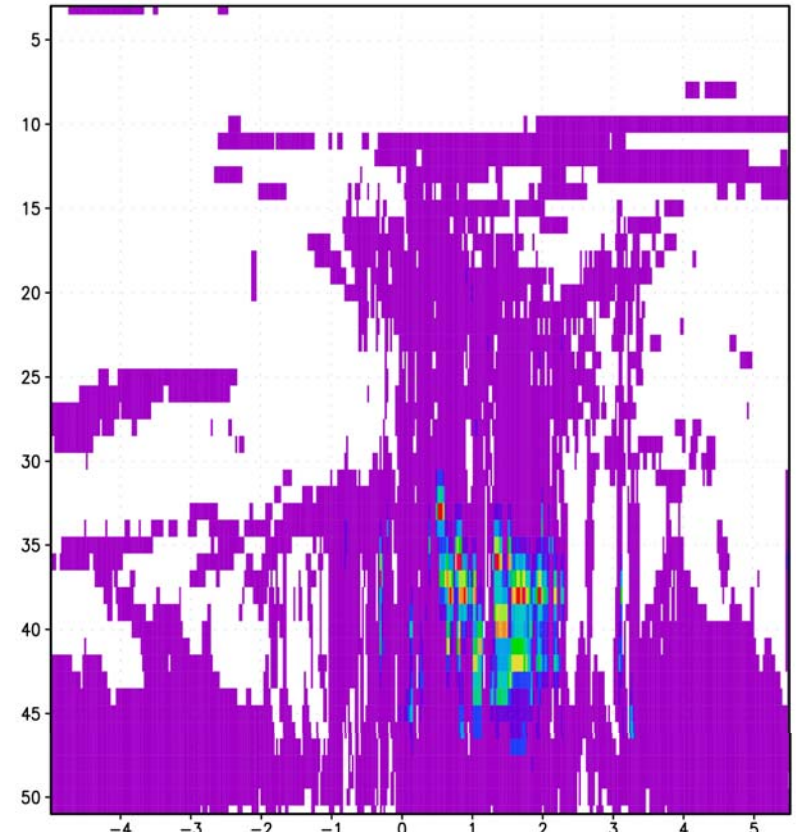
QC ( $> 1.0E-06$ ) - t=24h, lat=-2.8 - 3FT2 (wf=0.0) - NPDC (CP+SATAD)



Mean:  $1.12409e-05$  Min: 0 Max: 0.000873238



TKE ( $> 0.1$ ) - t=24h, lat=-2.8 - 3FT2 (wf=0.0) - NPDC (CP+SATAD)

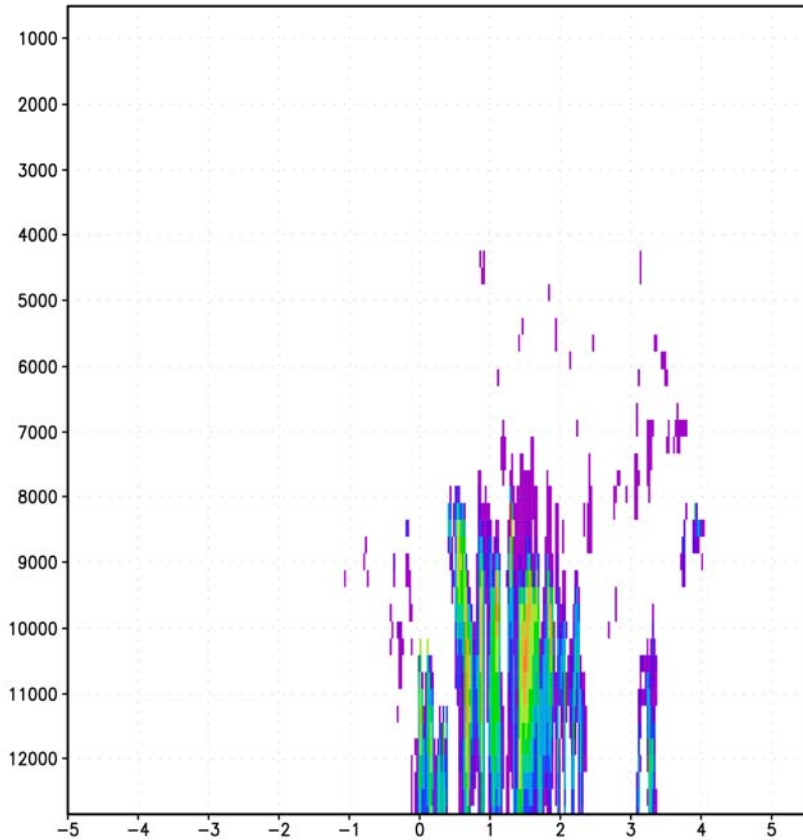


Mean: 0.88697 Min:  $5e-05$  Max: 63.1449

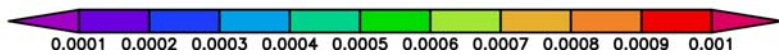


### M. 1 and M. 2

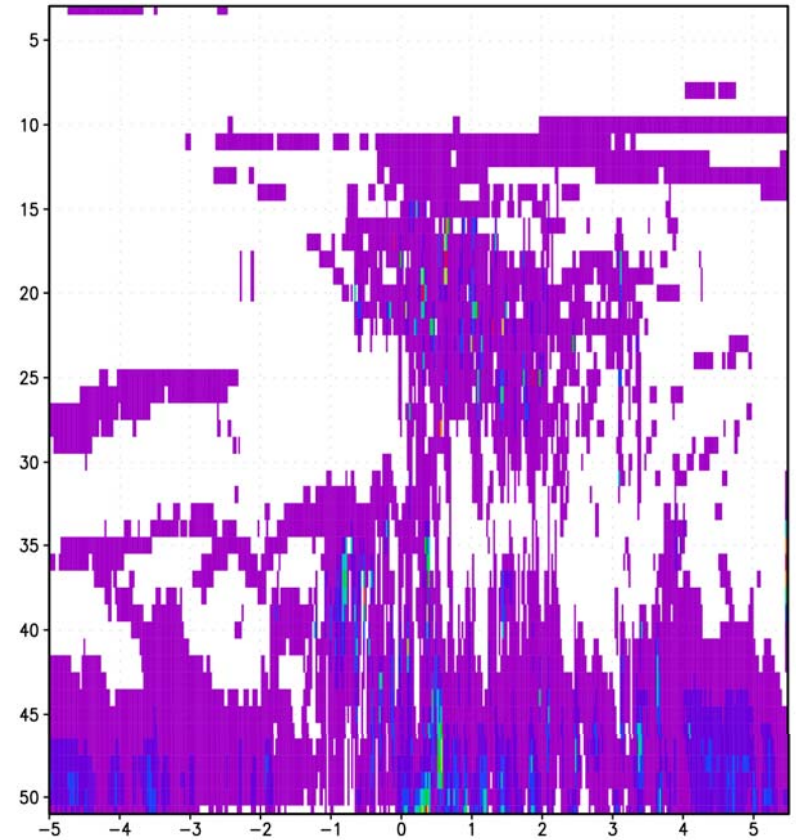
QC ( $> 1.0E-06$ ) - t=24h, lat=-2.8 - 3FT2 (wf=0.0) - NPDC (CLOUD-PHYS)



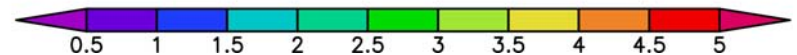
Mean:  $2.07324e-05$  Min: 0 Max: 0.00100881



TKE ( $> 0.1$ ) - t=24h, lat=-2.8 - 3FT2 (wf=0.0) - NPDC (CLOUD-PHYS)



Mean: 0.181602 Min:  $5e-05$  Max: 11.2599



### Dynamic Bottom Boundary...

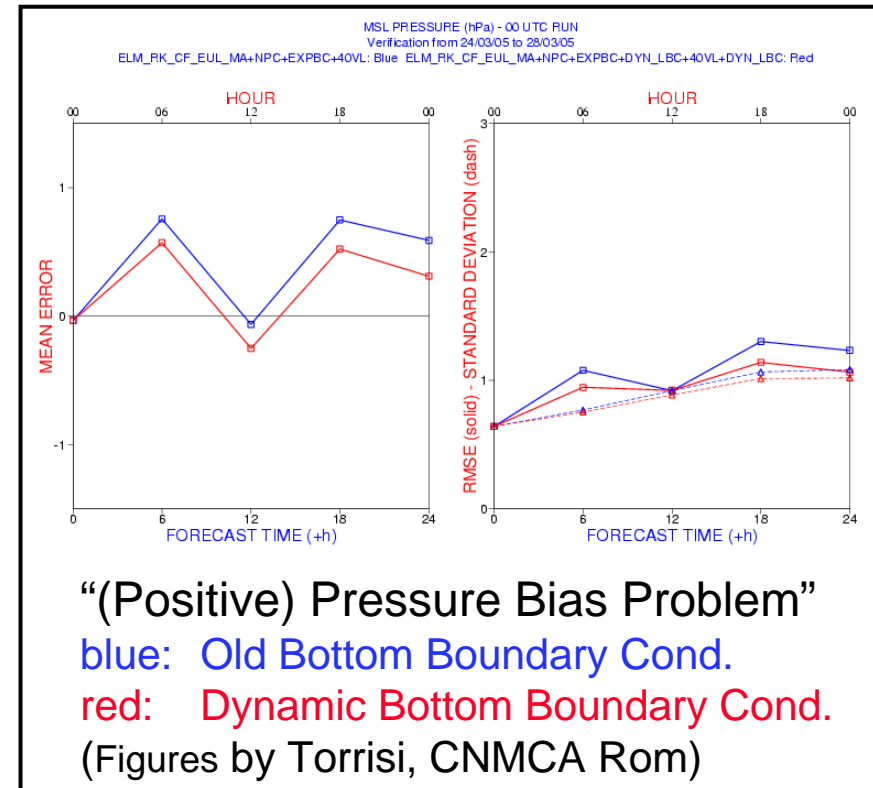
$$\frac{\partial u}{\partial t} = -\frac{1}{\rho a \cos \varphi} \left( \frac{\partial p^*}{\partial \lambda} + \frac{J_\lambda}{\sqrt{G}} \frac{\partial p^*}{\partial \zeta} \right) + f_u,$$

$$\frac{\partial v}{\partial t} = -\frac{1}{\rho a} \left( \frac{\partial p^*}{\partial \varphi} + \frac{J_\varphi}{\sqrt{G}} \frac{\partial p^*}{\partial \zeta} \right) + f_v,$$

$$\frac{\partial w}{\partial t} = \frac{1}{\rho \sqrt{G}} \frac{\partial p^*}{\partial \zeta} + g \frac{\rho_0}{\rho} \left( \frac{T - T_0}{T} - \frac{T_0 p^*}{T p_0} \right) + f_w.$$

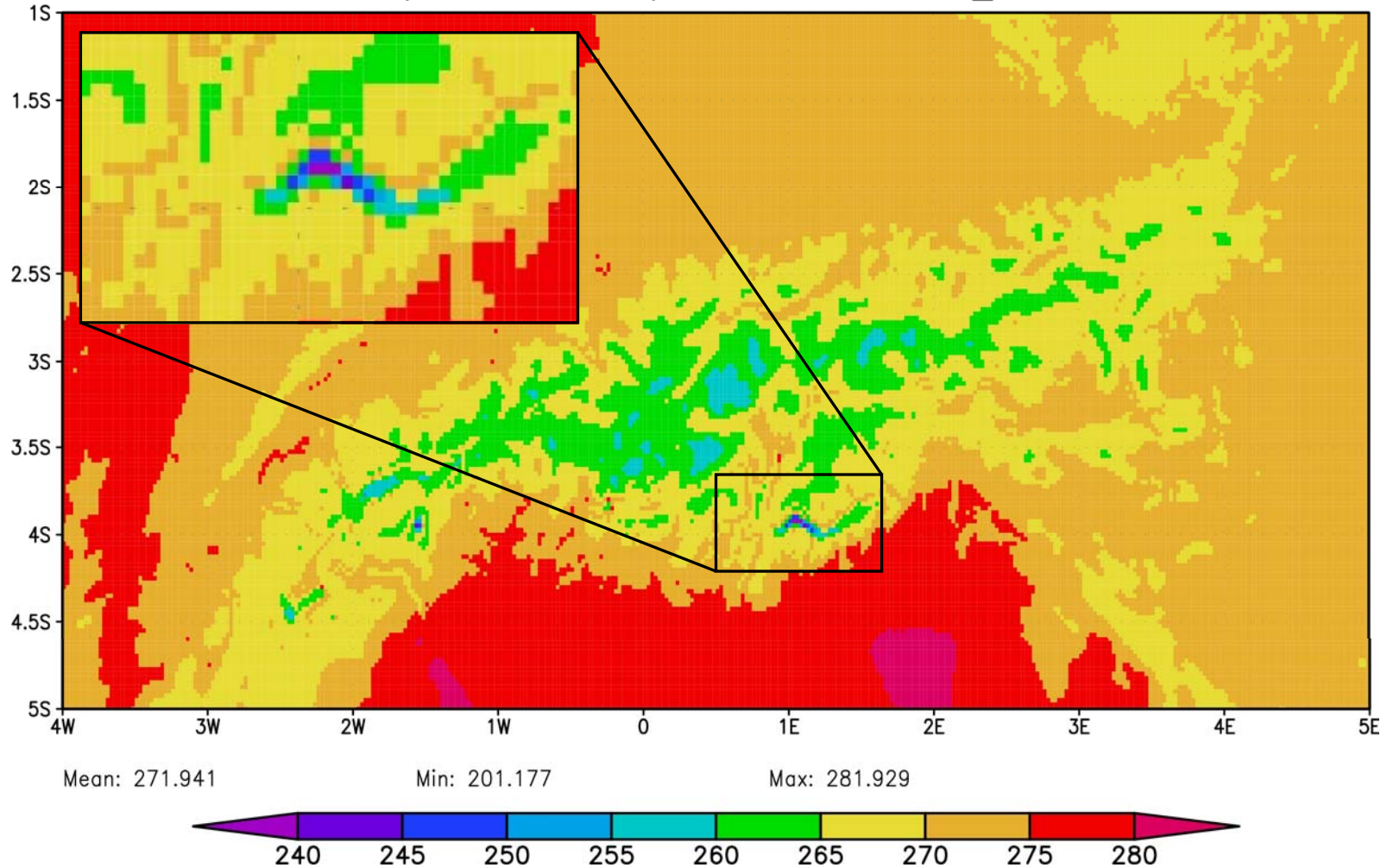
### ... Condition...

$$\frac{\partial \zeta}{\partial t} = \frac{1}{\sqrt{G}} \left( \frac{1}{a \cos \varphi} \frac{\partial z}{\partial \lambda} \frac{\partial u}{\partial t} + \frac{1}{a} \frac{\partial z}{\partial \varphi} \frac{\partial v}{\partial t} - \frac{\partial w}{\partial t} \right) = 0$$



... for metric pressure gradient term in equation for u- and v-component.  
Gaßmann (COSMO Newsletter No. 4)

T (d=2005020317) - XS0250.0-LPF6\_LPF4

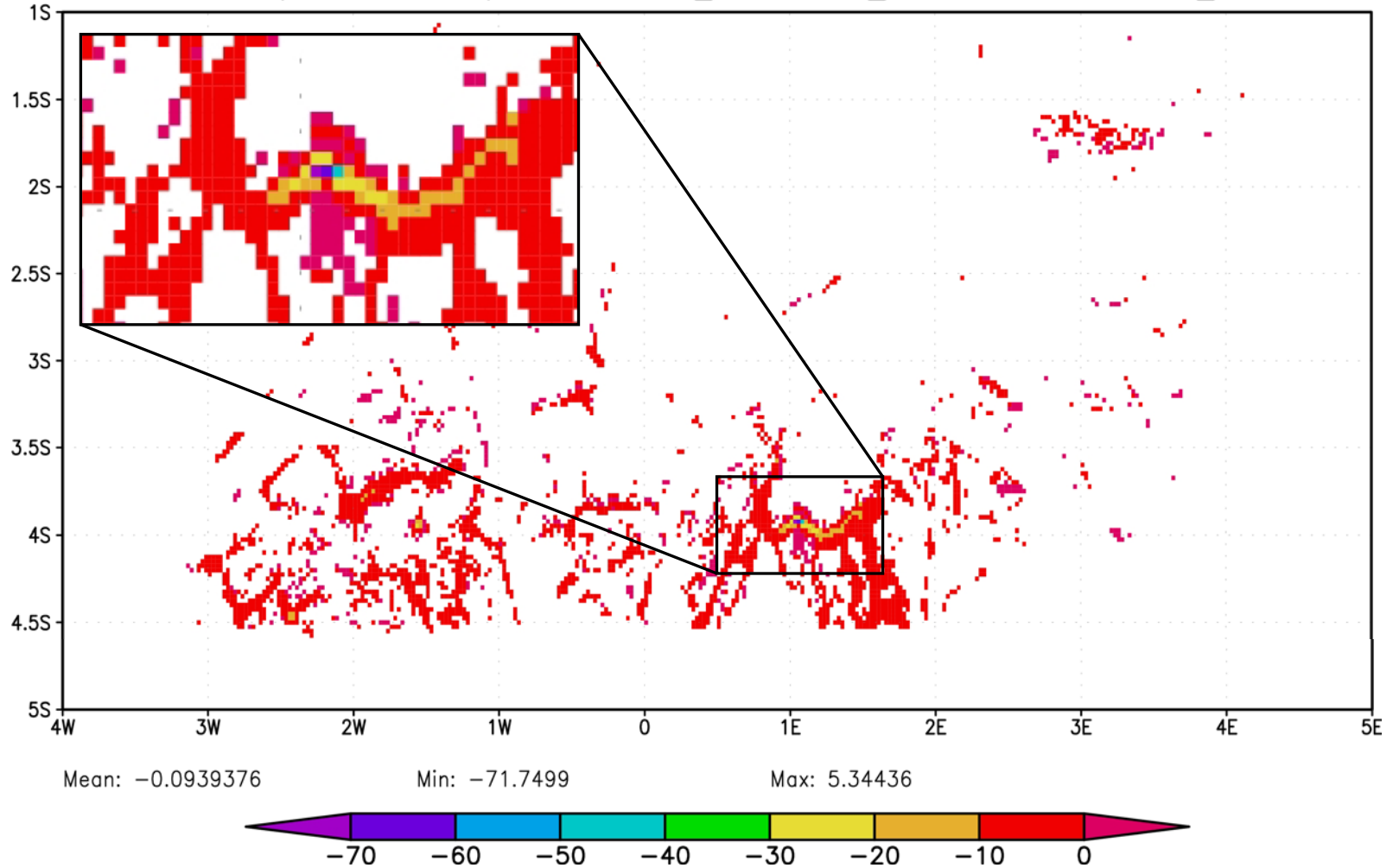


# Deutscher Wetterdienst

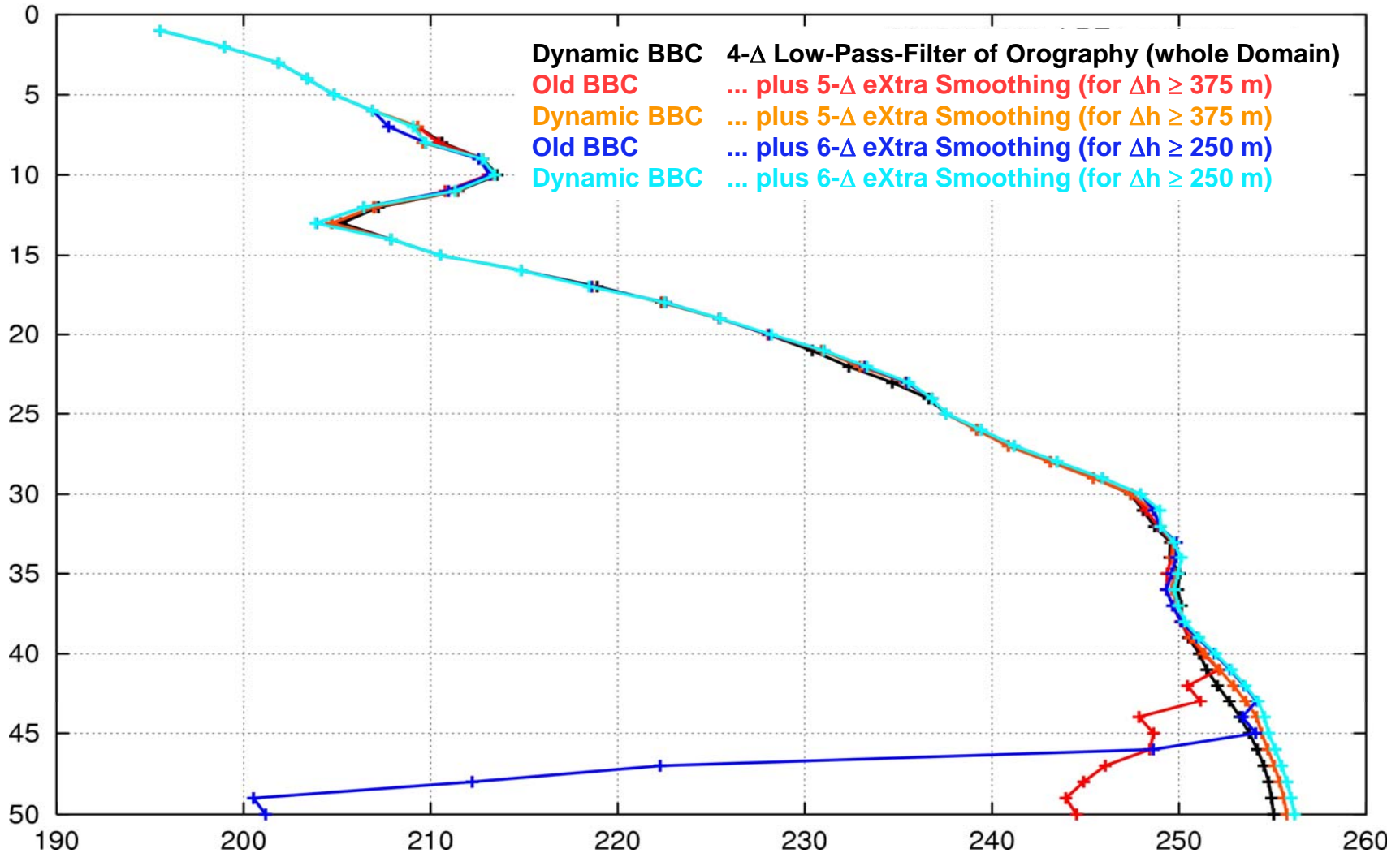
## Aktionsprogramm 2003



Delta T (d=2005020317) > 1 K - XS0\_250.0-LPF6\_LPF4 - STANDARD-DYN\_BBC



OUTPUT DATA: min of T (leveltyp = model) after 17 h



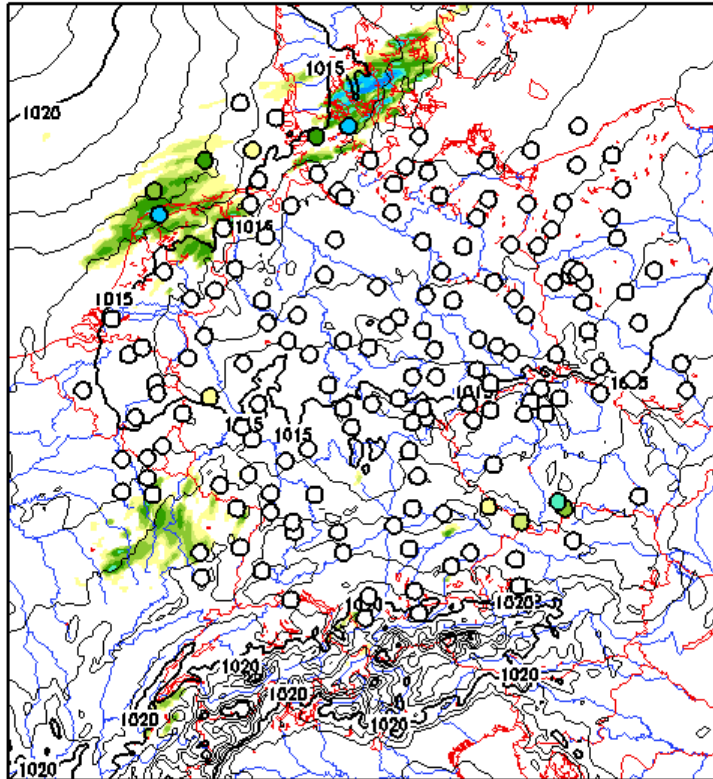
# Deutscher Wetterdienst

## Aktionsprogramm 2003



initial: 25 JUN 2005 00 UTC  
valid: 25 JUN 2005 06 UTC

(1) 1h PRECIPITATION (>0.1mm, MODEL and SYNOP) (2) PMSL



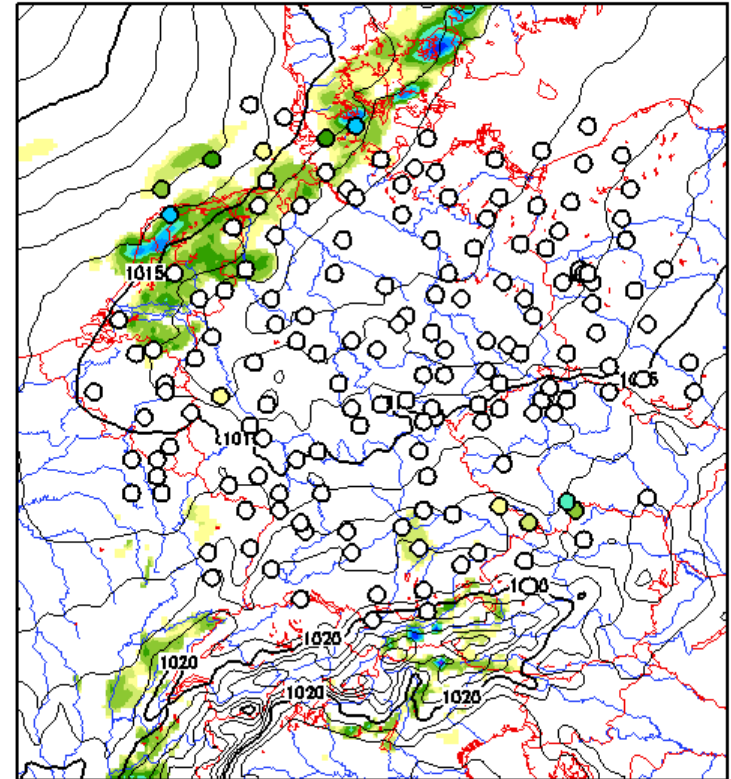
(1) MODEL: Mean: 0.0903425      Min: 0      Max: 15.958  
(1) SYNOP: Mean: 0.23865      Min: 0      Max: 9.9



**LMK**

initial: 25 JUN 2005 00 UTC  
valid: 25 JUN 2005 06 UTC

(1) 1h PRECIPITATION (>0.1mm, MODEL and SYNOP) (2) PMSL



(1) MODEL: Mean: 0.172617      Min: 0      Max: 20.9375  
(1) SYNOP: Mean: 0.23865      Min: 0      Max: 9.9



**LME**

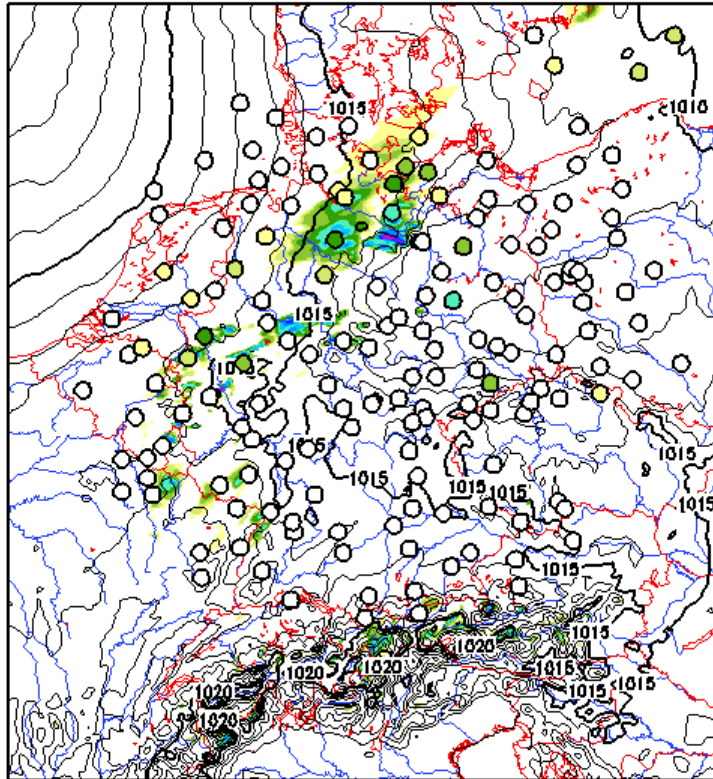
# Deutscher Wetterdienst

## Aktionsprogramm 2003



initial: 25 JUN 2005 00 UTC  
valid: 25 JUN 2005 12 UTC

(1) 1h PRECIPITATION (>0.1mm, MODEL and SYNOP) (2) PMSL



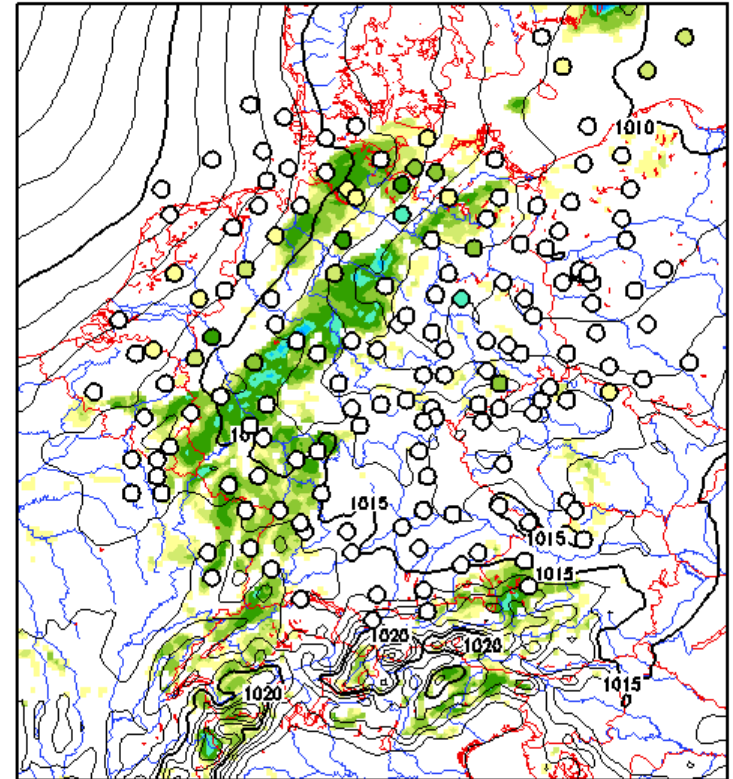
(1) MODEL: Mean: 0.113116 Min: 0 Max: 41.0146  
(1) SYNOP: Mean: 0.228571 Min: 0 Max: 6.2



**LMK**

initial: 25 JUN 2005 00 UTC  
valid: 25 JUN 2005 12 UTC

(1) 1h PRECIPITATION (>0.1mm, MODEL and SYNOP) (2) PMSL



(1) MODEL: Mean: 0.261856 Min: 0 Max: 14.1152  
(1) SYNOP: Mean: 0.228571 Min: 0 Max: 6.2



**LME**

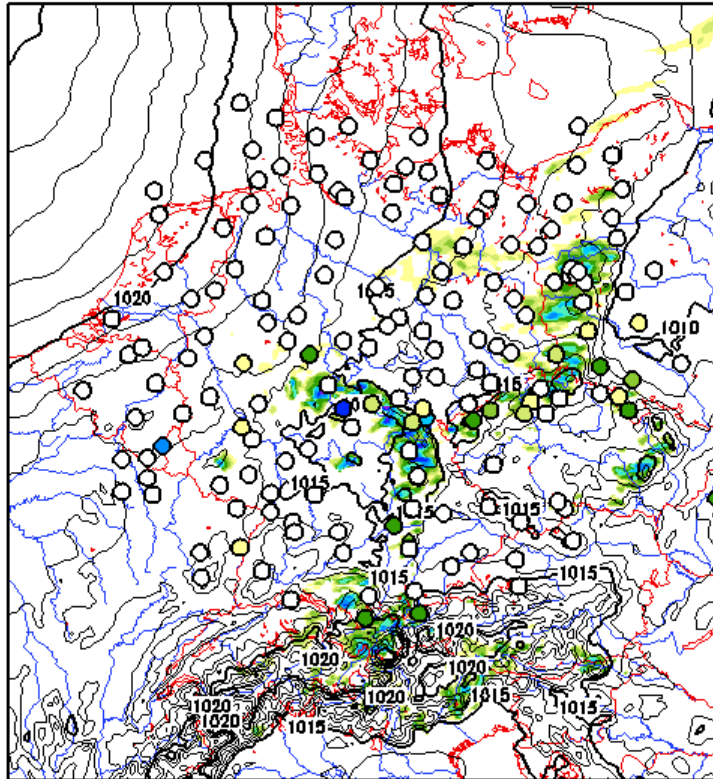
# Deutscher Wetterdienst

## Aktionsprogramm 2003



initial: 25 JUN 2005 00 UTC  
valid: 25 JUN 2005 18 UTC

(1) 1h PRECIPITATION (>0.1mm, MODEL and SYNOP) (2) PMSL



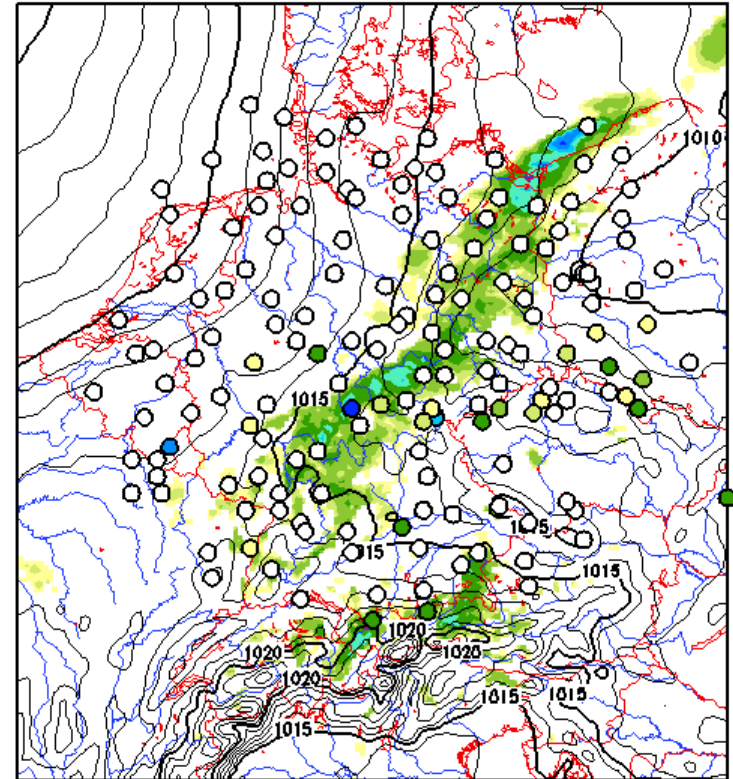
(1) MODEL: Mean: 0.167972 Min: 0 Max: 41.3184  
(1) SYNOP: Mean: 0.47546 Min: 0 Max: 18.8



**LMK**

initial: 25 JUN 2005 00 UTC  
valid: 25 JUN 2005 18 UTC

(1) 1h PRECIPITATION (>0.1mm, MODEL and SYNOP) (2) PMSL



(1) MODEL: Mean: 0.216025 Min: 0 Max: 16.5352  
(1) SYNOP: Mean: 0.47546 Min: 0 Max: 18.8



**LME**



## Summary and Outlook

- Completely Revised Conservative Advection of Moisture Quantities:
  - Clipping of negative values of  $q_x$  (semi-Lagrangian) and different diffusivity of the schemes:  
Model sensitivity with respect to patterns, (too small) mean and (too big) maximum values of precipitation.
  - Monotone Bott schemes (slightly more diffusive than PD) necessary?
- Further Tests of the Various Options in LMK:
  - semi-Lagrangian (tri-cubic) with multiplicative filling / Eulerian advection of  $q_x$ ?
  - New Coupling of Physics and Dynamics:  
Include (complete) diabatic forcing due to latent heat in RK scheme?
  - “Pressure Bias Problem” solved?
    - yes      Corrected “Moisture Convergence” =  $d_t q_{vl}|_{\text{Advection}} + d_t q_{vl}|_{\text{Diffusion}}$
    - no      ... work with Lucio Torrisi in COSMO project
- Merge Different Turbulence Parameterizations  
(work with M. Raschendorfer has started)

... documentation, configuration of vertical layers and filter for orography, conservation properties, divergence damping, convergence tests, tuning, ...