# Task 2.3 Extreme Value Theory (EVT) approach - Fitting precipitation object characteristics to different distributions

## Verification of large contiguous precipitation areas using Generalized Pareto distribution

## Results

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**Finished. Report under preparation**

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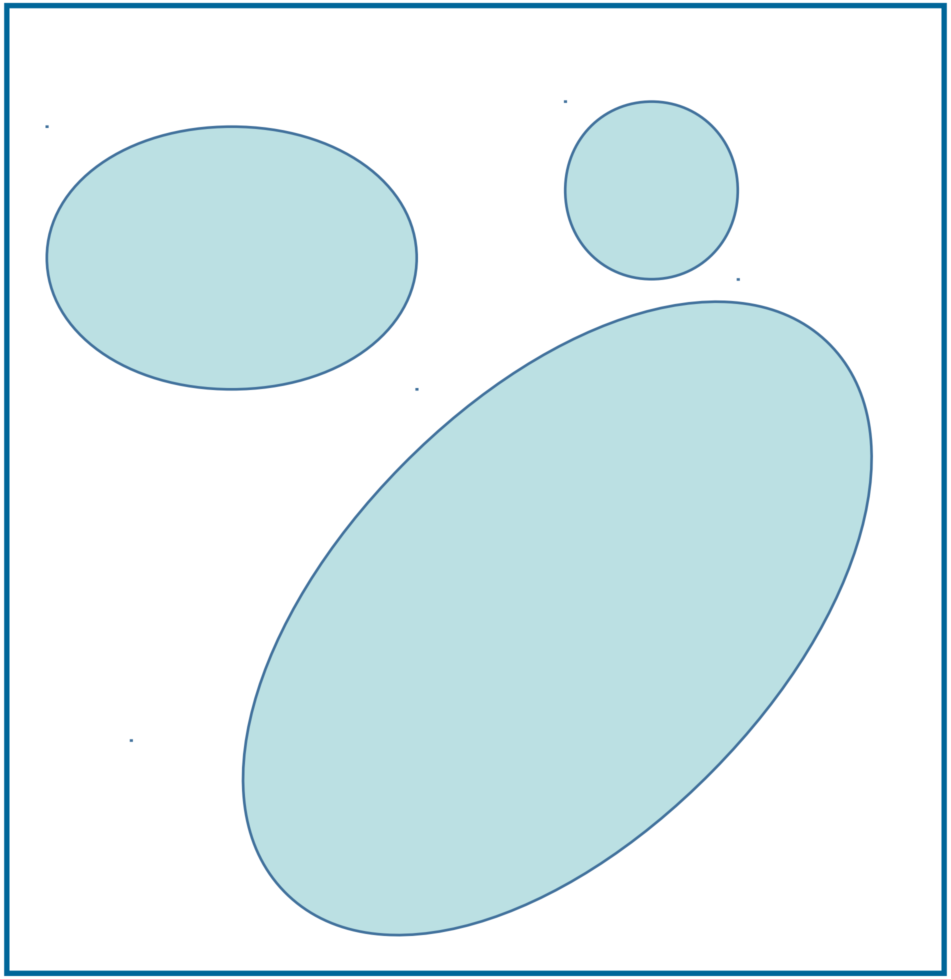
# Analysis setup

- The core of the system is the statistical STEPS scheme (Short Term Ensemble Prediction System) (*Seed 2003, Seed 2004, Bowler N. et al., 2006*)
- Verification period: Warm: May-September 2017

  Cold: November 2017-March 2018
- 9 radars in Central Russia used as observations
- 10 min time step until 3 h
- Grid size of about 2 km, 256x256 grid points domain
- R SpatialVx was used to identify objects (FeatureFinder)

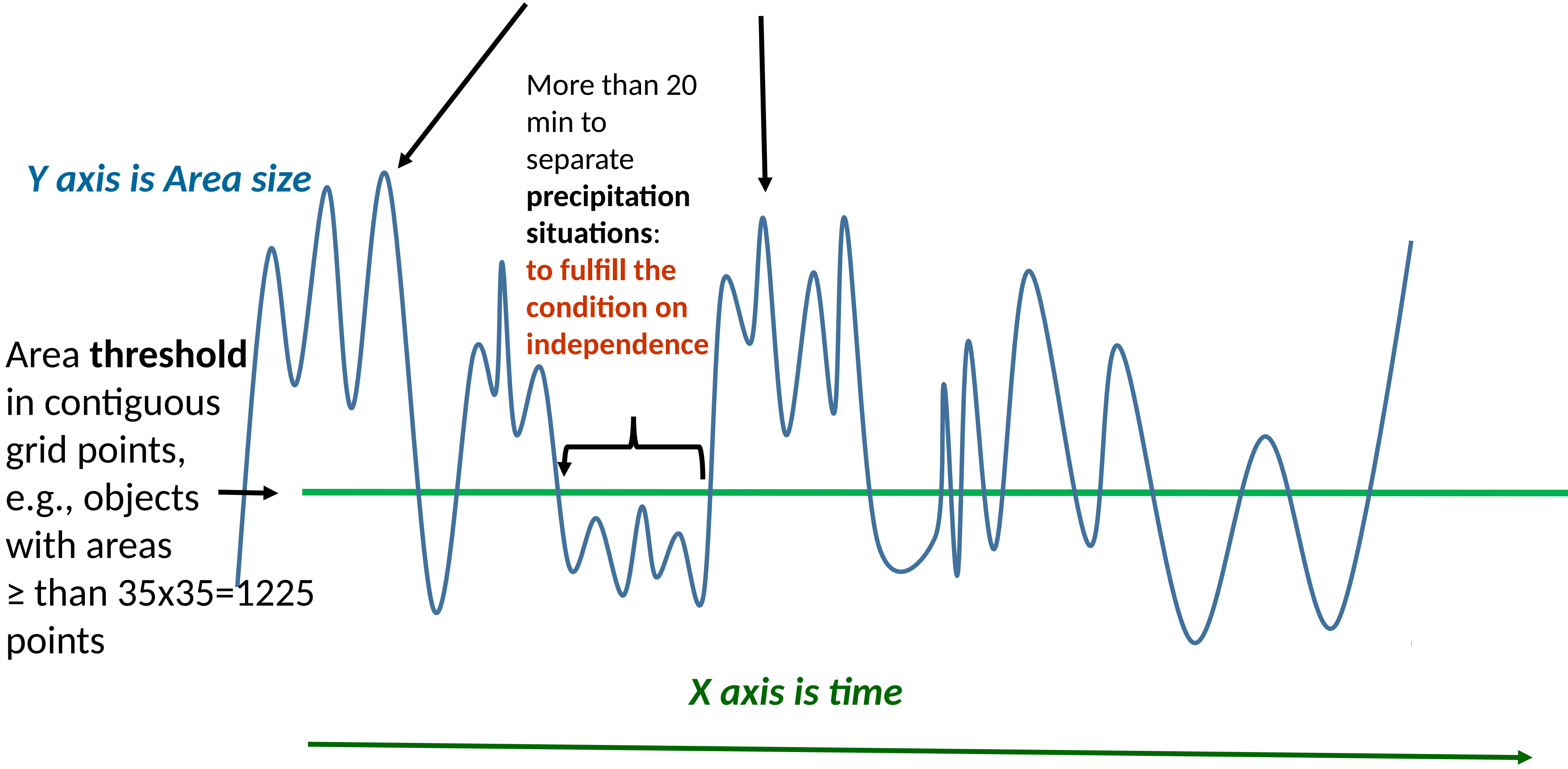
# Object identification: 1h precipitation greater than a certain value, e.g. >1mm/h,



The objects
in the domain are
identified by the number
of contiguous grid points
higher than a threshold,
convolution smoothing
with a radius of 9 grid
points (~18 km)

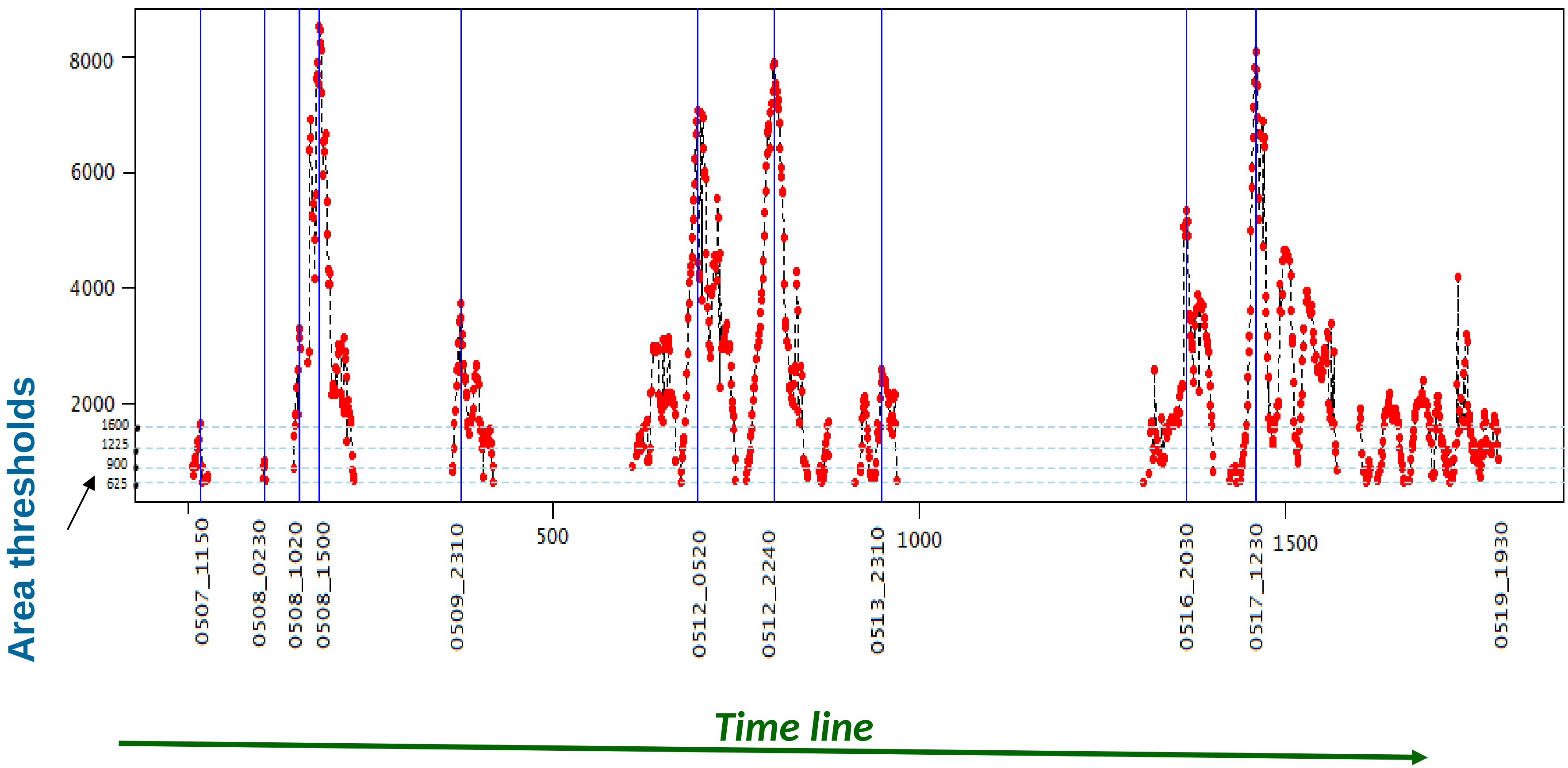
# Peaks over threshold (PoT) model for the area size

**One object larger than a threshold is chosen from each situation:**
**EVT approach, reduces the dimensionality of verification problem**



# Maximum areas of objects in Kursk radar fields from 5 to 19 May 2017.

Blue lines indicate times of maximum areas within a precipitation situation



# Generalized Pareto distribution

- **Generalized Pareto distribution is an approximation of PoT distribution based on the data from Generalized Extreme Value distribution if the following condition is satisfied:**

  **- The moments of peaks in a given time period are the Poisson point process** (a type of random mathematical object that consists of points randomly located on a mathematical space) (Suveges 2010)
- The Poisson point process is defined by the following conditions:

  - stationarity : probability of k events in any time slot of length l depends on k and l only and doesn't depend in the initial time of the slot;

  - occurence of k events in the time slot does not depend on occurence of this event before the given time slot;

  - ordinarity: occurence of not less than two events in the short period is almost impossible
- Let λ to denote the mean number of events in the time unit (process intensity). If the probability of occurence of k events during t is described by equation

  $$P_t(k) = \frac{(\lambda t)^k}{k!} e^{-\lambda t}$$

  these conditions are fulfilled.
- **It was shown that the peak moments in our data satisfy these conditions**

# Generalized Pareto (GP) distribution

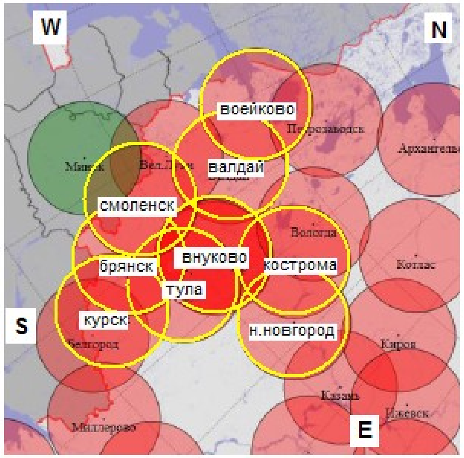
- The objects larger than a threshold (one from each precipitation situation) were fit to GP.
- GP has three parameters: location, scale, and shape.

  **But If location is fixed by the Pareto threshold in the PoT model, GP has only two parameters: scale (σ), and shape (ξ).**
- **Pareto thresholds: 625, 900, 1225 and 1600 grid points were used**
- GP parameters were estimated using 1) generalized maximum likelyhood estimation (GLME), 2) maximum likelyhood,

  3) L-moments, 4) Bayes with Monte Carlo Markov Chain modeling.
- **GMLE was chosen as the basic method**
- The standard errors (se) are calculated, and the confidence intervals for GP parameters estimates are calculated as CI = x±1.96*se
- *R-extRemes, module fevd is used for GP parameters estimation (author: E.Gilleland)*

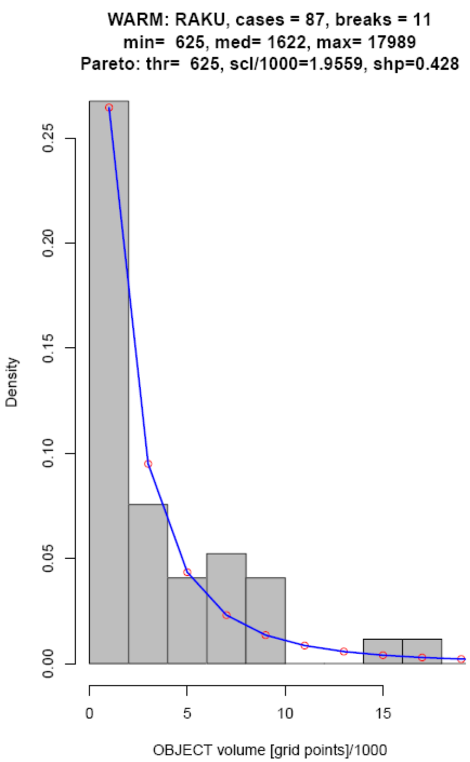
# What does this statistical analysis give?

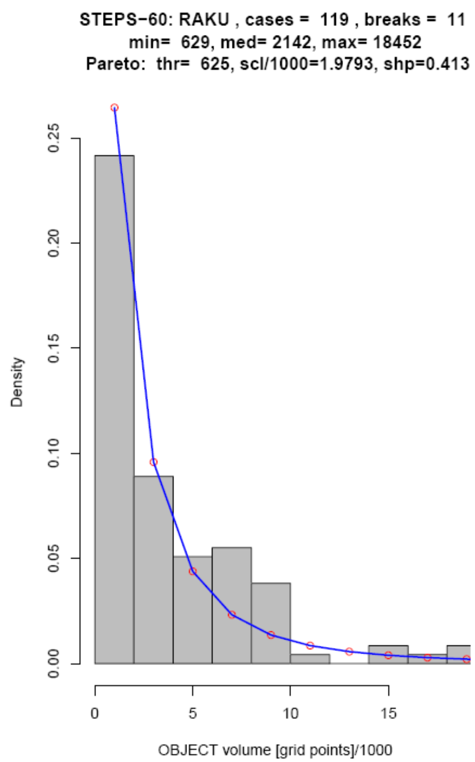
1. General description of large precipitation areas depending on the season and the radar
2. Comparison of parameters of identified objects in the model and in observations in terms of extremes, that is, HIW
3. Estimation of return periods of extremely vast contiguous precipitation areas: predictability of extremes (analysis ongoing).

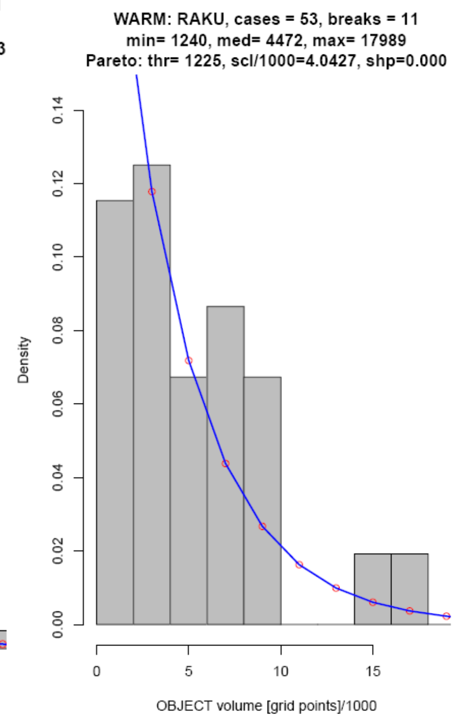


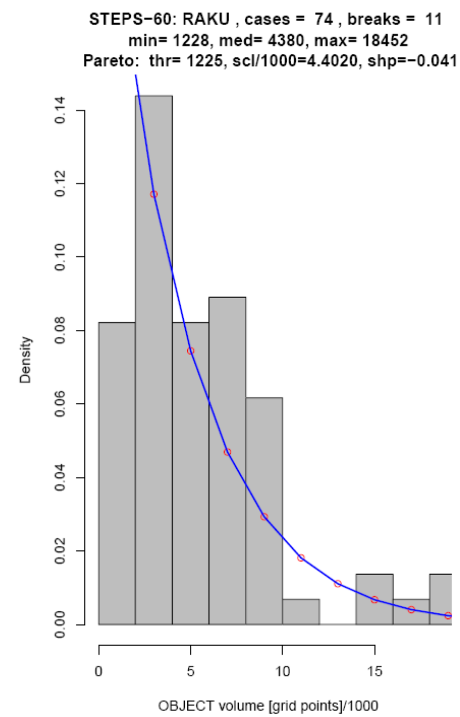

Radars under study: Yellow circles

## Fitting the distribution of precipitation object areas (histograms) to Generalized Pareto distribution (blue line), warm period, Kursk radar, lead time 60 min



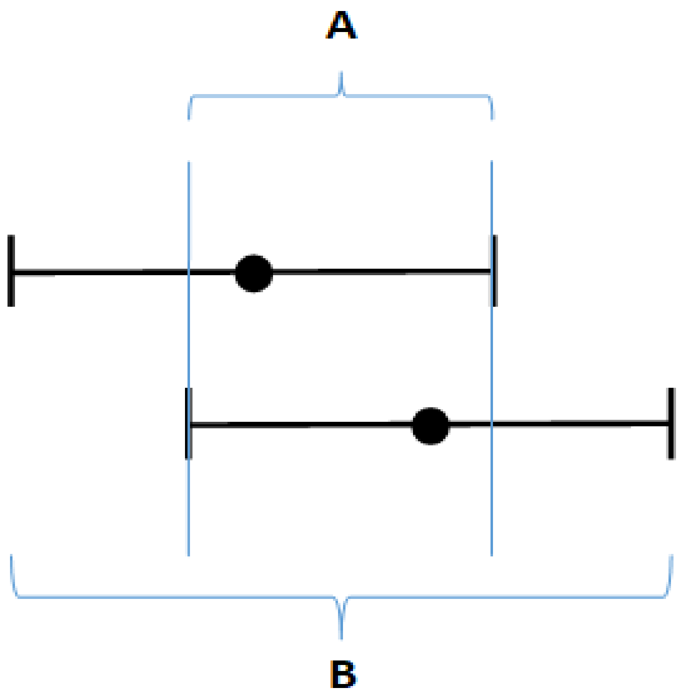




For area threshold of
625 grid points (~50*50 km)

For area threshold of
1225 grid points (~70*70 km)

## A measure of STEPS quality: intersection ratio of confidence intervals of Generalized Pareto parameters estimates (σ and ξ) in STEPS and in observations (radars)



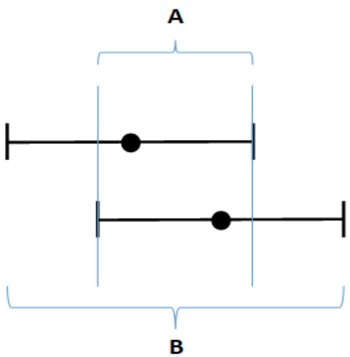

**intersection ratio (IR) = A/B**

**Ideal intersection ratio = 100%**

**IR >= 50% : choosen empirically as a useful skill level**

**The intersection ratio gives a diagnostic estimate of model ability to reproduce vast contiguous precipitation areas (or other extremes)**

# Summary table with scale (σ) parameter intersection ratio (IR), %




**intersection ratio = A/B*100%**

**Scale parameter σ determines PDF at zero point (PDF(0)=1/scale). The higher σ, the smaller PDF at zero, and the smaller the probability of an object with a size close to Pareto threshold.**

**Red is IR < 50%**

| Radar | | *Intersection ration (%).* *SCALE PARAMETER* | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | *Warm period* | | | | *Cold period* | | | |
| | Lead\ Thresh | 625 | 900 | 1225 | 1600 | 625 | 900 | 1225 | 1600 |
| RAKU | 30 | 80 | 74 | 61 | 71 | 75 | 68 | 75 | 51 |
| | 60 | 83 | 77 | 80 | 79 | 50 | 63 | 73 | 28 |
| | 90 | 83 | 73 | 84 | 70 | 23 | 38 | 67 | 54 |
| | 120 | 79 | 68 | 79 | 75 | 21 | 20 | 52 | 70 |
| | | | | | | | | | |
| RATL | 30 | 39 | 27 | 65 | 77 | 62 | 78 | 85 | 66 |
| | 60 | 48 | 23 | 75 | 80 | 52 | 55 | 79 | 75 |
| | 90 | 35 | 17 | 35 | 81 | 47 | 53 | 73 | 66 |
| | 120 | 34 | 19 | 26 | 75 | 50 | 62 | 72 | 65 |
| | | | | | | | | | |
| RAVO | 30 | 54 | 38 | 82 | 81 | 70 | 76 | 81 | 75 |
| | 60 | 58 | 37 | 84 | 56 | 64 | 74 | 68 | 76 |
| | 90 | 56 | 46 | 80 | 50 | 61 | 63 | 64 | 73 |
| | 120 | 52 | 35 | 53 | 82 | 50 | 64 | 65 | 64 |
| | | | | | | | | | |
| RUDB | 30 | 92 | 88 | 91 | 73 | 75 | 78 | 71 | 29 |
| | 60 | 87 | 90 | 91 | 68 | 48 | 67 | 65 | 29 |
| | 90 | 81 | 94 | 93 | 70 | 46 | 56 | 57 | 25 |
| | 120 | 88 | 92 | 93 | 67 | 44 | 56 | 53 | 23 |
| | | | | | | | | | |
| RUDK | 30 | 78 | 85 | 41 | 81 | 69 | 73 | 84 | *** |
| | 60 | 76 | 80 | 47 | 76 | 54 | 68 | 67 | *** |
| | 90 | 80 | 82 | 57 | 75 | 50 | 60 | 54 | *** |
| | 120 | 72 | 80 | 59 | 70 | 52 | 50 | 48 | *** |
| | | | | | | | | | |
| RUDL | 30 | 75 | 63 | 62 | 31 | 47 | 52 | 50 | 64 |
| | 60 | 73 | 64 | 59 | 28 | 32 | 47 | 38 | 42 |
| | 90 | 68 | 57 | 55 | 30 | 40 | 45 | 39 | 53 |
| | 120 | 71 | 66 | 52 | 16 | 41 | 42 | 35 | 34 |

# Summary table with shape (ξ) parameter intersection ratio, %

- Negative ξ indicates β-distribution, zero ξ: exponential distribution, and positive ξ: Pareto itself.
- **Positive shape parameter ξ indicates heavy tail in the Pareto distribution, the higher ξ, the heavier the tail (probability of largest precip areas here)**
- (+/-) indicate ξ sign in the observations/forecast distribution pairs,
- 0 indicates ξ in the interval [-0.1, 0.1]
- *** : not enough cases
- **A desirable quality of a forecast system is to reproduce the sign of shape parameter**

| RADAR | | *CI intersection (%).* SHAPE | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | *Warm period* | | | | *Cold period* | | | |
| | **Lead\ Thresh** | 625 | 900 | 1225 | 1600 | 625 | 900 | 1225 | 1600 |
| **RAKU** | 30 | 84 (+ +) | 74 (+ +) | 55 (0 +) | 69 (- -) | 78 (+ +) | 47 (0 +) | 79 (0 0) | 90 (0 0) |
| | 60 | 85 (+ +) | 76 (+ +) | 76 (0 0) | 60 (- -) | 68 (+ +) | 44 (0 +) | 78 (0 0) | 64 (0 -) |
| | 90 | 83 (+ +) | 76 (+ +) | 73 (0 0) | 39 (- -) | 54 (+ +) | 33 (0 +) | 50 (0 +) | 97 (0 0) |
| | 120 | 80 (+ +) | 72 (+ +) | 73 (0 0) | 47 (- -) | 54 (+ +) | 23 (0 +) | 45 (- +) | 82 (0 0) |
| **RATL** | 30 | 74 (+ +) | 41 (0 +) | 78 (0 0) | 48 (+ 0) | 78 (+ +) | 87 (+ +) | 91 (+ +) | 38 (0 +) |
| | 60 | 72 (+ +) | 36 (0 +) | 79 (0 0) | 47 (+ 0) | 69 (+ +) | 71 (+ +) | 87 (+ +) | 81 (0 0) |
| | 90 | 66 (+ +) | 32 (0 +) | 48 (0 +) | 46 (+ 0) | 67 (+ +) | 69 (+ +) | 82 (+ +) | 95 (0 0) |
| | 120 | 65 (+ +) | 34 (0 +) | 44 (0 +) | 43 (+ 0) | 68 (+ +) | 73 (+ +) | 83 (+ +) | 85 (0 0) |
| **RAVO** | 30 | 78 (+ +) | 40 (0 +) | 79 (0 0) | 74 (0 0) | 76 (+ +) | 71 (+ +) | 73 (+ +) | 80 (+ +) |
| | 60 | 76 (+ +) | 36 (0 +) | 78 (0 0) | 69 (0 0) | 72 (+ +) | 67 (+ +) | 65 (+ +) | 71 (+ +) |
| | 90 | 77 (+ +) | 38 (0 +) | 82 (0 0) | 73 (0 0) | 66 (+ +) | 63 (+ +) | 60 (+ +) | 65 (+ +) |
| | 120 | 69 (+ +) | 34 (0 +) | 42 (0 +) | 76 (0 0) | 64 (+ +) | 60 (+ +) | 58 (+ +) | 60 (+ +) |
| **RUDB** | 30 | 93 (+ +) | 91 (+ +) | 94 (+ +) | 90 (+ +) | 72 (+ +) | 72 (+ +) | 65 (+ +) | 37 (0 +) |
| | 60 | 91 (+ +) | 92 (+ +) | 95 (+ +) | 88 (+ +) | 60 (+ +) | 64 (+ +) | 60 (+ +) | 37 (0 +) |
| | 90 | 92 (+ +) | 94 (+ +) | 96 (+ +) | 89 (+ +) | 56 (+ +) | 57 (+ +) | 54 (+ +) | 33 (0 +) |
| | 120 | 89 (+ +) | 92 (+ +) | 94 (+ +) | 89 (+ +) | 55 (+ +) | 56 (+ +) | 52 (+ +) | 32 (0 +) |
| **RUDK** | 30 | 80 (+ +) | 79 (+ +) | 38 (0 +) | 83 (0 0) | 75 (+ +) | 70 (+ +) | 72 (+ +) | *** |
| | 60 | 75 (+ +) | 74 (+ +) | 39 (0 +) | 79 (0 0) | 64 (+ +) | 64 (+ +) | 61 (+ +) | *** |
| | 90 | 76 (+ +) | 74 (+ +) | 43 (0 +) | 72 (0 0) | 60 (+ +) | 57 (+ +) | 52 (+ +) | *** |
| | 120 | 74 (+ +) | 73 (+ +) | 41 (0 +) | 79 (0 0) | 56 (+ +) | 53 (+ +) | 49 (+ +) | *** |
| **RUDL** | 30 | 80 (+ +) | 76 (+ +) | 78 (+ +) | 41 (0 +) | 72 (+ +) | 71 (+ +) | 72 (+ +) | 79 (+ +) |
| | 60 | 76 (+ +) | 73 (+ +) | 73 (+ +) | 36 (0 +) | 64 (+ +) | 63 (+ +) | 60 (+ +) | 63 (+ +) |
| | 90 | 73 (+ +) | 69 (+ +) | 70 (+ +) | 36 (0 +) | 62 (+ +) | 61 (+ +) | 59 (+ +) | 67 (+ +) |
| | 120 | 74 (+ +) | 70 (+ +) | 69 (+ +) | 28 (0 +) | 59 (+ +) | 57 (+ +) | 55 (+ +) | 58 (+ +) |

# Summary table with shape (ξ) parameter intersection ratio, %

- As Pareto threshold rises, ξ has a tendency to change sign from positive to zero, and becomes negative in some cases.
- For Pareto threshold of 625 grid points, only RUDN radar has a zero shape parameter ξ, and no negative ξ. Thus, a conclusion can be made that the threshold of 625 grid points provide the best fit of Pareto distribution
- **Nowcasting system is more informative in forecasting extreme precipitation areas in the cold period: the number of (++) pairs is about 50% of cases in the warm period, and about 75%, in the cold period**

| RADAR | | *CI intersection (%).* SHAPE | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | *Warm period* | | | | *Cold period* | | | |
| | Lead\ Thresh | 625 | 900 | 1225 | 1600 | 625 | 900 | 1225 | 1600 |
| RAKU | 30 | 84 (+ +) | 74 (+ +) | 55 (0 +) | 69 (- -) | 78 (+ +) | 47 (0 +) | 79 (0 0) | 90 (0 0) |
| | 60 | 85 (+ +) | 76 (+ +) | 76 (0 0) | 60 (- -) | 68 (+ +) | 44 (0 +) | 78 (0 0) | 64 (0 -) |
| | 90 | 83 (+ +) | 76 (+ +) | 73 (0 0) | 39 (- -) | 54 (+ +) | 33 (0 +) | 50 (0 +) | 97 (0 0) |
| | 120 | 80 (+ +) | 72 (+ +) | 73 (0 0) | 47 (- -) | 54 (+ +) | 23 (0 +) | 45 (- +) | 82 (0 0) |
| RATL | 30 | 74 (+ +) | 41 (0 +) | 78 (0 0) | 48 (+ 0) | 78 (+ +) | 87 (+ +) | 91 (+ +) | 38 (0 +) |
| | 60 | 72 (+ +) | 36 (0 +) | 79 (0 0) | 47 (+ 0) | 69 (+ +) | 71 (+ +) | 87 (+ +) | 81 (0 0) |
| | 90 | 66 (+ +) | 32 (0 +) | 48 (0 +) | 46 (+ 0) | 67 (+ +) | 69 (+ +) | 82 (+ +) | 95 (0 0) |
| | 120 | 65 (+ +) | 34 (0 +) | 44 (0 +) | 43 (+ 0) | 68 (+ +) | 73 (+ +) | 83 (+ +) | 85 (0 0) |
| RAVO | 30 | 78 (+ +) | 40 (0 +) | 79 (0 0) | 74 (0 0) | 76 (+ +) | 71 (+ +) | 73 (+ +) | 80 (+ +) |
| | 60 | 76 (+ +) | 36 (0 +) | 78 (0 0) | 69 (0 0) | 72 (+ +) | 67 (+ +) | 65 (+ +) | 71 (+ +) |
| | 90 | 77 (+ +) | 38 (0 +) | 82 (0 0) | 73 (0 0) | 66 (+ +) | 63 (+ +) | 60 (+ +) | 65 (+ +) |
| | 120 | 69 (+ +) | 34 (0 +) | 42 (0 +) | 76 (0 0) | 64 (+ +) | 60 (+ +) | 58 (+ +) | 60 (+ +) |
| RUDB | 30 | 93 (+ +) | 91 (+ +) | 94 (+ +) | 90 (+ +) | 72 (+ +) | 72 (+ +) | 65 (+ +) | 37 (0 +) |
| | 60 | 91 (+ +) | 92 (+ +) | 95 (+ +) | 88 (+ +) | 60 (+ +) | 64 (+ +) | 60 (+ +) | 37 (0 +) |
| | 90 | 92 (+ +) | 94 (+ +) | 96 (+ +) | 89 (+ +) | 56 (+ +) | 57 (+ +) | 54 (+ +) | 33 (0 +) |
| | 120 | 89 (+ +) | 92 (+ +) | 94 (+ +) | 89 (+ +) | 55 (+ +) | 56 (+ +) | 52 (+ +) | 32 (0 +) |
| RUDK | 30 | 80 (+ +) | 79 (+ +) | 38 (0 +) | 83 (0 0) | 75 (+ +) | 70 (+ +) | 72 (+ +) | *** |
| | 60 | 75 (+ +) | 74 (+ +) | 39 (0 +) | 79 (0 0) | 64 (+ +) | 64 (+ +) | 61 (+ +) | *** |
| | 90 | 76 (+ +) | 74 (+ +) | 43 (0 +) | 72 (0 0) | 60 (+ +) | 57 (+ +) | 52 (+ +) | *** |
| | 120 | 74 (+ +) | 73 (+ +) | 41 (0 +) | 79 (0 0) | 56 (+ +) | 53 (+ +) | 49 (+ +) | *** |
| RUDL | 30 | 80 (+ +) | 76 (+ +) | 78 (+ +) | 41 (0 +) | 72 (+ +) | 71 (+ +) | 72 (+ +) | 79 (+ +) |
| | 60 | 76 (+ +) | 73 (+ +) | 73 (+ +) | 36 (0 +) | 64 (+ +) | 63 (+ +) | 60 (+ +) | 63 (+ +) |
| | 90 | 73 (+ +) | 69 (+ +) | 70 (+ +) | 36 (0 +) | 62 (+ +) | 61 (+ +) | 59 (+ +) | 67 (+ +) |
| | 120 | 74 (+ +) | 70 (+ +) | 69 (+ +) | 28 (0 +) | 59 (+ +) | 57 (+ +) | 55 (+ +) | 58 (+ +) |

# Discussion

The importance of compromise between the strict Extreme Value theorems (the asymptotic Fisher-Tippett-Gnedenko theorem and Pickands-Balkema-de Haan theorem) with simple heuristic methods

Thus, the importance of correctly constructing the sample:

1. The model choice (block maxima, PoT, taking into account the season, ...) ,

2. The threshold choice (according to asymptotic theorems, histogram forms, empirically, ...),

3. Method of estimation of distribution parameters  (generalized maximum likelyhood, maximum likelyhood, L-moments, Bayes with Monte Carlo Markov Chain modeling... )

# Summary

- Object-based verification of RHM nowcasting system is performed. The verification period is May-Sep 2017 and Nov-March2017-2018, for seven radars in Central Russia.
- The ability of the system to forecast contiguous precipitation areas greater than a certain threshold (peaks over threshold method) is assessed. Several thresholds were studied.
- Three-parameter Generalized Pareto distribution is used to assess precipitation areas in distribution tails according to the shape parameter. The best fit of Pareto distribution corresponds to the area threshold of 625 points (~50*50 km).
- A measure of STEPS quality is introduced: intersection ratio of confidence intervals of Generalized Pareto parameters ($\sigma$ and $\xi$) estimates in STEPS and in observations (radars). It gives a diagnostic estimate of model ability to reproduce vast contiguous precipitation areas (or other extremes).
- **A paper under preparation**