About a consistent implementation of physics into the ICON heat equation:

- The reason, why isochoric T-tendencies need to be applied for the ICON system.
- The confusion, whether T-tendencies from sub-grid transport need to be multiplied by the factor c_p^d / c_v^d.
- The setting and consequences of a hidden wrong implementation.

Matthias Raschendorfer (DWD)

Thermo-Dynamic budget equations:

specific source term for air component $k \in \{d, v, l, i\}$

barycentric velocity

$$\rho \cdot d_t q^k = I^k - \nabla \cdot \underline{J}^k \qquad q^k := \frac{\rho^k}{\rho}, \quad \rho = \sum_k \rho^k, \quad \sum_k q^k = 0 = \sum_k I^k, \qquad \underline{v} = \frac{1}{\rho} \sum_k \rho^k \underline{v}^k$$

specific relative flux density due to molecular diffusion and microscopic sedimentation

any variable being proportional to a mass concentration

mean velocity of component k

$$d_t \rho = -\rho \nabla \cdot \underline{v}$$

$$d_t \phi := \partial_t \phi + \underline{v} \cdot \nabla \phi = \frac{1}{\rho} \left[\partial_t (\rho \cdot \phi) + \nabla \cdot (\rho \phi \underline{v}) \right]$$

$$\rho \begin{pmatrix} c_p \\ c_v \end{pmatrix} d_t T = \begin{pmatrix} d_t p \\ -p \nabla \cdot \underline{v} \end{pmatrix} + \begin{pmatrix} Q_p^{diab} \\ Q_v^{diab} \end{pmatrix}$$

$$\begin{pmatrix} c_p \\ c_v \\ R \end{pmatrix} = \sum_k q^k \cdot \begin{pmatrix} c_p^k \\ c_v^k \\ R^k \end{pmatrix}$$

specif. heat capacity $c_p^k = c_v^k + R^k$

specif. gas constant

$R^c \approx 0$ for $c \in \{l, i\}$ (quasi-microscopic approximation)

$$p = \rho R T$$

molecular heat flux $\underline{H}$

(neglected … outside lam. lay. generally)

net radiation flux

dissipation

latent heat of isobaric condensation at temperature T_0

(neglected reduction of heat capacity

$$\begin{pmatrix} Q_p^{diab} \\ Q_v^{diab} \end{pmatrix} + \nabla \cdot \left(\underline{S} - \lambda^H \nabla T + \sum_k c_v^k T \underline{J}^k \right) \approx -\varepsilon + \sum_{c \in \{l,i\}} I^c \begin{pmatrix} L_0^c \\ L_0^c - R^v T_0 \end{pmatrix} + \sum_k \begin{pmatrix} c_p^k \\ c_v^k \end{pmatrix} T \nabla \cdot \underline{J}^k =: \begin{pmatrix} Q_p^{conv} \\ Q_v^{conv} \end{pmatrix}$$

$$\Delta Q^{diab} := Q_v^{diab} - Q_p^{diab} = \rho T d_t R$$

neglecting $\left(c_{v,p}^v - c_v^c\right) \cdot (T - T_0)$ compared to L_0^c or $R^v T_0$

$$\left| \Delta Q^{diab} \right| < 5\% \left| Q_p^{diab} \right|$$

The deeper problem:

The approximated gird-scale T-equation:

molecular heat flux

net-radiation flux

heat source by conversions (**not** expressible in flux-divergence form)

$$\begin{pmatrix} c_p^d \\ c_v^d \end{pmatrix} \cdot \left[\bar{\rho}\, \hat{d}_t \hat{T} + \nabla \cdot \left(\underbrace{\overline{\rho T'' \underline{v}''}}_{\approx \pi \overline{\rho \theta'' \underline{v}''}} \right) \right] \approx \underbrace{-\nabla \cdot \left(\underline{\bar{H}} + \underline{\bar{S}} \right) + \bar{Q}^{conv}}_{\bar{Q}^{diab}} + \underbrace{\begin{pmatrix} \hat{d}_t \bar{p} & + \overline{\underline{v}'' \cdot \nabla p} \\ -\left(\bar{p} \nabla \cdot \underline{\hat{v}} & + \overline{p \nabla \cdot \underline{v}''} \right) \end{pmatrix}}_{\bar{Q}^{adia}}$$

Exner-pressur: $\left(\frac{\bar{p}}{p_{ref}} \right)^{\frac{R_d}{c_{pd}}}$

a **sub-grid** contribution to **pure transport**

$$\bar{Q}^{conv}_{isobar} \approx \bar{Q}^{conv} \approx \bar{Q}^{conv}_{isochor} \qquad c_{p,v} \approx c^d_{p,v}$$

$$\partial_t \hat{T} \Big|^{diab}_{isochor} \approx \frac{c_p^d}{c_v^d} \partial_t \hat{T} \Big|^{diab}_{isobar}$$

- **Is there a reason to treat the diffusion term** $\frac{1}{\bar{\rho}} \nabla \cdot \left(\overline{\rho T'' \underline{v}''} \right)$ **accordingly?**

➢ **Theoretical investigation with respect to the sub-gird contribution by the adiabatic terms:**

The general grid –scale budget equation:

grid-scale filter average

$$\overline{\rho\phi} = \overline{\rho}\hat{\phi} \qquad \left.\begin{aligned} \phi' &= \phi - \overline{\phi} \\ \phi'' &= \phi - \hat{\phi} \end{aligned}\right\} \text{related fluctuation}$$

mass-weighted grid-scale filter average

$$\overline{\rho\, d_t \phi} = \underbrace{\partial_t\left(\overline{\rho}\cdot\hat{\phi}\right) + \nabla\cdot\left(\overline{\rho}\,\hat{\phi}\underline{\hat{v}}\right)}_{=:\,\overline{\rho}\,\hat{d}_t\hat{\phi}} + \nabla\cdot\underbrace{\left(\overline{\rho\,\phi''\underline{v}''}\right)}_{\text{sub-grid scale flux density}} = \overline{Q^{\phi}}$$

filtered source term,

(which can also be split into a **grid scale** and a **sub-grid scale** contribution)

The crudely approximated grid-scale heat equation in a system with a pressure equation as in COSMO:

$$\overline{\rho}\,\hat{d}_t\hat{T} + \nabla\cdot\underbrace{\left(\overline{\rho\,T''\underline{v}''}\right)}_{\approx\,\pi\overline{\rho\,\theta''\underline{v}''}} = \overline{\left[\frac{d_t p + Q_p^{diab}}{c_p}\right]} \approx \frac{\overline{d_t p} + \overline{Q_p^{diab}}}{c_p^d}$$

employing $c_p \approx c_p^d$

employing: $\pi := \left(\frac{p}{p_{ref}}\right)^{\frac{R_d}{c_{pd}}} \approx \overline{\pi} \approx \left(\frac{\overline{p}}{p_{ref}}\right)^{\frac{R_d}{c_{pd}}}$ Exner pressure (factor)

$\theta := \frac{T}{\pi}$ potential temperature

employing $\overline{p\nabla\cdot\underline{v}} \approx \overline{p}\nabla\cdot\hat{\underline{v}}$ and $\frac{R}{c_v} \approx \frac{R^d}{c_v^d}$

$$\hat{d}_t\overline{p} \approx \overline{d_t p} = \overline{\frac{R}{c_v}Q_p^{diab} - \frac{c_p}{c_v}\left(p\nabla\cdot\underline{v} - \Delta Q^{diab}\right)} \approx \frac{R^d}{c_v^d}\overline{Q_p^{diab}} - \frac{c_p^d}{c_v^d}\left(\overline{p}\nabla\cdot\hat{\underline{v}} \underbrace{- \overline{\Delta Q^{diab}}}_{\approx 0}\right)$$

The ICON system with its approximate θ_v -equation :

$$\theta_v := \frac{R}{R_d}\theta$$ virtual potential temperature

$$\pi \rho d_t \theta_v \approx \frac{Q_p^{diab}}{c_p^d} + \frac{\Delta Q^{diab}}{R^d}$$

- **getting rid of adiabatic terms (θ- effect)**
- **heat-capacity for $k \neq d$ needs not to be considered (R/R_d - effect)**

employing $\frac{R}{c_v} \approx \frac{R^d}{c_v^d}$

contains not explicitly treated sub-grid scale correlation terms

$$\bar{\rho}\hat{d}_t\hat{\theta}_v = \partial_t\left(\bar{\rho}\cdot\hat{\phi}\right) + \nabla\cdot\left(\bar{\rho}\,\hat{\theta}_v\,\underline{\hat{v}}\right) \approx -\nabla\cdot\left(\underbrace{\overline{\rho\,\theta_v''\,\underline{v}''}}\right) + \frac{1}{\pi}\cdot\left[\overbrace{\frac{\overline{Q_p^{diab}}}{c_p^d} + \frac{\overline{\Delta Q^{diab}}}{R^d}}\right]$$

employing $\bar{\pi} \approx \pi$ sub-grid scale θ_v-flux density

$$\hat{d}_t\bar{\rho} = -\bar{\rho}\,\nabla\cdot\underline{\hat{v}}$$

$$\bar{\rho}\cdot\hat{d}_t\hat{q}^k = -\nabla\cdot\left(\overline{\underline{J}^k} + \overline{\rho\,q^{k''}\underline{v}''}\right) + \overline{I^k} \qquad k \neq d$$

The current physic-dynamic coupling in ICON (for "fast-processes"):

- **Dynamics:**
  - **Grid-scale transport of:** $\overbrace{\bar{\rho},}^{\text{full } \bar{\rho}\text{- incrementation}}$ $\hat{\theta}_v$ **and** $\hat{q}^k \quad (k \neq d)$
  - **Diagnostic of updated** $\hat{T} \approx \frac{R^d \hat{\theta}_v}{\hat{R}} \cdot \left(\frac{\bar{\rho} R^d \hat{\theta}_v}{p_{ref}} \right)^{\frac{R^d}{c_v^d}}$ **and** $\bar{p} \approx \bar{\rho} \hat{R} \hat{T}$

    neglecting 3-rd order correlation terms

- **Physics: Source-terms and (so far considered) sub-grid scale correlation terms:**
  - **Incrementation of** $\hat{q}^k \quad (k \neq d)$
  - **Isochoric incrementation of** $\hat{T}$ **according to** $\hat{T}$**- equation in** c_v**- form**

    as $\bar{\rho}$ is already fully updated and **must not further be modified** during the current time step
  - **Diagnostic of updated** $\hat{\theta}_v \approx \frac{\hat{R}\hat{T}}{R^d} \cdot \left(\frac{p_{ref}}{\bar{\rho}\hat{R}\hat{T}} \right)^{\frac{R^d}{c_p^d}}$

T-tendencies according the (crude) COSMO-approximation :

$$\bar{\rho}\,\hat{d}_t\hat{T} + \nabla\cdot\underbrace{\left(\overline{\rho\,T''\underline{v}''}\right)}_{\approx\,\pi\overline{\rho\,\theta''\underline{v}''}} \approx \begin{cases} \dfrac{\overline{Q_p^{diab}} + \hat{d}_t\bar{p}}{c_p^d} \\[2ex] \dfrac{\overline{Q_p^{diab}} - \bar{p}\nabla\cdot\hat{\underline{v}}}{c_v^d} \end{cases}$$

a **sub-grid** contribution to **<u>pure</u> transport**

$$\bar{\rho}\,\partial_t\hat{T}\Big|^{diab}_{isobar}$$

$$\boxed{\bar{\rho}\,\partial_t\hat{T}\Big|^{phys}_{isochor} + \nabla\cdot\left(\pi\overline{\rho\,\theta''\underline{v}''}\right) \approx \frac{\overline{Q_p^{diab}}}{c_v^d} \approx \frac{c_p^d}{c_v^d}\left[\bar{\rho}\,\partial_t\hat{T}\Big|^{conv}_{isobar} - \nabla\cdot\left(\underline{\bar{H}} + \underline{\bar{S}}\right)\right]}$$

Attention:

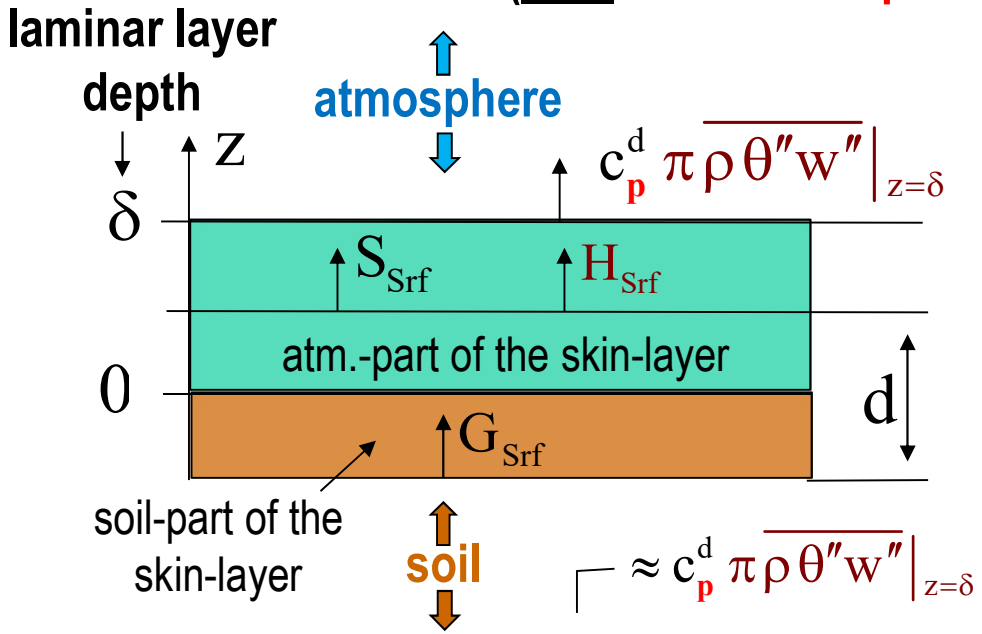
- In this approx., **sub-grid diabatic sources** need to be <u>separated</u> from **sub-grid transport**.
- $H_z\big|_{z=Srf} \approx -\overline{\lambda^H\partial_z T} \quad \neq \quad c_v^d\pi\overline{\rho\,\theta''w''}\big|_{z=Srf} = 0$ ($w \equiv 0$ at the rigid surface)

  $\approx c_p^d\pi\overline{\rho\,\theta''w''}\big|_{z=\delta}$ (or is it rather $c_v^d\pi\overline{\rho\,\theta''w''}\big|_{z=\delta}$ **??**)

The lower boundary condition for a non-evaporating surface:

- **Flux-equilibrium at an idealized surface:**

(**here**: **without evaporation and other water-phase transitions**)



❖ **combined skin-layer heat budget for** $d \rightarrow 0$

➢ **The work against atmospheric pressure for individual sub-grid plumes in the heat budget of the soil can only be eliminated in c_p-form!**

$$\overline{G}_z\big|_{z=Srf} \approx \overline{H}_z\big|_{z=Srf} + \overline{S}_z\big|_{z=Srf}$$

Ground HF G_{Srf} **Sensible HF** H_{Srf} **Net Radiation Flux** S_{Srf} **(each at the surface)**

➢ $\overline{H}_z\big|_{z=Srf}$ **is the laminar heat-flux through the interface (surface) between soil and atmosphere.**

➢ **In a skin-layer heat budget including the constant-flux approx., it is comparable with the turbulent heat flux** $c_p^d \pi \overline{\rho \theta'' w''}|_{z=\delta}$ **and not with** $c_v^d \pi \overline{\rho \theta'' w''}|_{z=\delta}$ **!!**

The history of a misleading implementation:

- My first suggested approximation: of 2012 according to COSMO:

$$\overline{\rho}\,\partial_t \hat{T}\Big|_{\text{isochor}}^{\text{phys}} \approx \frac{c_p^d}{c_v^d}\,\overline{\rho}\,\partial_t \hat{T}\Big|_{\text{isobar}}^{\text{conv}} - \nabla\cdot\left[\pi\overline{\rho\,\theta''\underline{v}''} + \frac{1}{c_v^d}\left(\overline{\underline{H}} + \overline{\underline{S}}\right)\right]$$

- There was a concern, whether the correct form would be:

$$\overline{\rho}\,\partial_t \hat{T}\Big|_{\text{isochor}}^{\text{phys}} \approx \frac{c_p^d}{c_v^d}\,\overline{\rho}\,\partial_t \hat{T}\Big|_{\text{isobar}}^{\text{conv}} - \frac{c_p^d}{c_v^d}\nabla\cdot\left[\pi\overline{\rho\,\theta''\underline{v}''} + \frac{1}{c_p^d}\left(\overline{\underline{H}} + \overline{\underline{S}}\right)\right] \approx \frac{c_p^d}{c_v^d}\,\overline{\rho}\,\partial_t \hat{T}\Big|_{\text{isobar}}^{\text{phys}}$$

- In both versions the sub-grid contribution keeps a divergence-form, and thus is automatically energy-conserving!

- Although the last proposal has been realized in the LES branch, I have initially rejected it, as it implies:

$$\overline{\rho T \underline{v}} = \overline{\rho}\,\hat{T}\hat{\underline{v}} + \frac{c_p^d}{c_v^d}\overline{\rho\,T''\underline{v}''}\ ,$$ what is obviously wrong!

So, I derived a more complete approach, which also considers the (so far neglected) sub-grid scale contribution by the adiabatic source term:

The extended approximation:

$$\nabla\cdot\left(\pi\overline{\rho\,\theta''\underline{v}''}\right)\approx \pi\nabla\cdot\overline{\rho\,\theta''\underline{v}''}+\frac{\overline{\underline{v}''\cdot\nabla p}}{c_p^d}$$

$$c_{v,p}\approx c_{v,p}^d\,,\ \pi\approx\overline{\pi}$$

$$\approx \frac{c_p^d}{c_v^d}\pi\nabla\cdot\overline{\rho\,\theta''\underline{v}''}-\frac{\overline{p\nabla\cdot\underline{v}''}}{c_v^d}$$

$$\begin{pmatrix}\overline{d_t p}\\ -\overline{p\nabla\cdot\underline{v}}\end{pmatrix}=\begin{pmatrix}\hat{d}_t\overline{p}+\overline{\underline{v}''\cdot\nabla p}\\ -\left(\overline{p}\nabla\cdot\hat{\underline{v}}+\overline{p\nabla\cdot\underline{v}''}\right)\end{pmatrix}$$

$$\overline{\rho}\,\hat{d}_t\hat{T}+\boxed{\pi\nabla\cdot\left(\overline{\rho\,\theta''\underline{v}''}\right)}\approx\frac{\overline{Q_p^{diab}}+\hat{d}_t\overline{p}}{c_p^d}$$

$$\overline{\rho}\,\hat{d}_t\hat{T}+\boxed{\frac{c_p^d}{c_v^d}\pi\nabla\cdot\left(\overline{\rho\,\theta''\underline{v}''}\right)}\approx\frac{\overline{Q_v^{diab}}-\overline{p}\nabla\cdot\hat{\underline{v}}}{c_v^d}\approx\frac{\overline{Q_p^{diab}}-\overline{p}\nabla\cdot\hat{\underline{v}}}{c_v^d}$$

$$\Delta Q^{diab}\approx 0$$

combined **sub-grid** contribution by **transport** <u>and</u> the respective **adiabatic source terms**

and this was intended to be implemented!!

$$\overline{\rho}\,\partial_t\hat{T}\Big|_{\text{isochor}}^{\text{phys}}\approx\frac{c_p^d}{c_v^d}\overline{\rho}\,\partial_t\hat{T}\Big|_{\text{isobar}}^{\text{conv}}-\frac{c_p^d}{c_v^d}\left[\pi\nabla\cdot\overline{\rho\,\theta''\underline{v}''}+\frac{1}{c_p^d}\nabla\cdot\left(\underline{\overline{H}}+\underline{\overline{S}}\right)\right]\approx\frac{c_p^d}{c_v^d}\cdot\overline{\rho}\,\partial_t\hat{T}\Big|_{\text{isobar}}^{\text{phys}}$$

A somewhat less accurate approximation:

$$-\frac{\overline{p\nabla\cdot\underline{v}''}}{c_v^d} \approx \nabla\cdot\left(\pi\overline{\rho\,\theta''\underline{v}''}\right) - \frac{c_p^d}{c_v^d}\pi\nabla\cdot\overline{\rho\,\theta''\underline{v}''} \approx -\frac{R^d}{c_v^d}\pi\nabla\cdot\overline{\rho\,\theta''\underline{v}''} + \underbrace{\frac{\overline{\underline{v}''\cdot\nabla p}}{c_p^d}}$$

$$-\frac{g}{c_p^d\,\hat{T}}\pi\overline{\rho\,\theta''w''} \approx \pi\overline{\rho\,\theta''\underline{v}''}\cdot\nabla\ln(\pi) \approx$$

- As $H_{Top} := \frac{c_p^d\,\hat{T}}{g} \approx 30150\text{m}$ is much larger than the ABL-depth,

  $\left|\frac{\overline{\underline{v}''\cdot\nabla p}}{c_p^d}\right|$ is in the order of 1% of $\left|\nabla\cdot\left(\pi\overline{\rho\,\theta''\underline{v}''}\right)\right|$, and

  $\left|\frac{\overline{p\nabla\cdot\underline{v}''}}{c_v^d}\right|$ is in the order of 30% of $\left|\nabla\cdot\left(\pi\overline{\rho\,\theta''\underline{v}''}\right)\right|$.

- Neglect of $\left|\frac{\overline{\underline{v}''\cdot\nabla p}}{c_p^d}\right|$

$$\nabla\cdot\left(\pi\overline{\rho\,\theta''\underline{v}''}\right) \approx \pi\nabla\cdot\overline{\rho\,\theta''\underline{v}''}$$

and this is (correctly) implemented in the LES version

$$\overline{\rho}\,\partial_t\hat{T}\Big|_{\text{isochor}}^{\text{phys}} \approx \frac{c_p^d}{c_v^d}\overline{\rho}\,\partial_t\hat{T}\Big|_{\text{isobar}}^{\text{conv}} - \frac{c_p^d}{c_v^d}\nabla\cdot\left[\pi\overline{\rho\,\theta''\underline{v}''} + \frac{1}{c_p^d}\left(\underline{\overline{H}} + \underline{\overline{S}}\right)\right] \approx \frac{c_p^d}{c_v^d}\overline{\rho}\,\partial_t\hat{T}\Big|_{\text{isobar}}^{\text{phys}}$$

Possible approximations for isochoric T-tendencies:

The crude first "fully incompressible"-approximation,

neglecting $-\dfrac{\overline{p\nabla\cdot\underline{v}''}}{c_v^d}$:

$$\bar{\rho}\,\partial_t\hat{T}\Big|_{\text{isochor}}^{\text{phys}} \approx \frac{c_p^d}{c_v^d}\left(\bar{\rho}\,\partial_t\hat{T}\Big|_{\text{isobar}}^{\text{conv}} - \frac{1}{c_p^d}\nabla\cdot\underline{\bar{S}}\right) - \nabla\cdot\left(\pi\overline{\rho\,\theta''\underline{v}''} + \frac{\underline{\bar{H}}}{c_v^d}\right)$$

$$3\cdot\left|\frac{\overline{p\nabla\cdot\underline{v}''}}{c_v^d}\right| \approx \left|\nabla\cdot\left(\pi\overline{\rho\,\theta''\underline{v}''}\right)\right| \approx 100\cdot\left|\frac{\overline{\underline{v}''\cdot\nabla p}}{c_p^d}\right|$$

The better "ICON-LES"-approximation,

neglecting $\dfrac{\overline{\underline{v}''\cdot\nabla p}}{c_p^d}$:

$$\Updownarrow$$

$$\nabla\cdot\left(\pi\overline{\rho\,\theta''\underline{v}''}\right) \approx \pi\nabla\cdot\overline{\rho\,\theta''\underline{v}''}$$

$$\bar{\rho}\,\partial_t\hat{T}\Big|_{\text{isochor}}^{\text{phys}} \approx \frac{c_p^d}{c_v^d}\left[\bar{\rho}\,\partial_t\hat{T}\Big|_{\text{isobar}}^{\text{conv}} - \frac{1}{c_p^d}\nabla\cdot\underline{\bar{S}} - \nabla\cdot\left(\pi\overline{\rho\,\theta''\underline{v}''} + \frac{\underline{\bar{H}}}{c_p^d}\right)\right] \approx \frac{c_p^d}{c_v^d}\bar{\rho}\,\partial_t\hat{T}\Big|_{\text{isobar}}^{\text{phys}}$$

The more complete "ICON-NWP"-approximation,

(being intended since long)

neglecting $\dfrac{\underline{\bar{H}}\cdot\nabla\ln\pi}{c_v^d}$:

$$\bar{\rho}\,\partial_t\hat{T}\Big|_{\text{isochor}}^{\text{phys}} \approx \frac{c_p^d}{c_v^d}\cdot\left[\bar{\rho}\,\partial_t\hat{T}\Big|_{\text{isobar}}^{\text{conv}} - \frac{1}{c_p^d}\nabla\cdot\underline{\bar{S}} - \pi\nabla\cdot\left(\overline{\rho\,\theta''\underline{v}''} + \frac{\underline{\bar{H}}}{\pi c_p^d}\right)\right] \approx \frac{c_p^d}{c_v^d}\cdot\bar{\rho}\,\partial_t\hat{T}\Big|_{\text{isobar}}^{\text{phys}}$$

is provided by turbulence without the factor c_p^d / c_v^d erroneously assuming that it would be multiplied here, that is after collecting the physical tendencies!

The crude first "fully incompressible"-approximation:

$$\bar{\rho}\,\partial_t \hat{T}\Big|_{\substack{\text{phys}\\ \text{isochor}}} - \frac{c_p^d}{c_v^d}\cdot\left[\bar{\rho}\,\partial_t \hat{T}\Big|_{\substack{\text{conv}\\ \text{isobar}}} - \frac{1}{c_p^d}\nabla\cdot\underline{\bar{S}}\right] \approx -\nabla\cdot\left(\pi\rho\,\overline{\theta''\underline{v}''} + \frac{\underline{\bar{H}}}{c_v^d}\right)$$

The wrong implementation of the intended ICON-NWP-solution

but employing <u>here</u> (only for the purpose of a comparison) $\nabla\cdot\left(\pi\overline{\rho\,\theta''\underline{v}''}\right) \approx \pi\nabla\cdot\overline{\rho\,\theta''\underline{v}''}$:

$$\bar{\rho}\,\partial_t \hat{T}\Big|_{\substack{\text{phys}\\ \text{isochor}}} - \frac{c_p^d}{c_v^d}\cdot\left[\bar{\rho}\,\partial_t \hat{T}\Big|_{\substack{\text{conv}\\ \text{isobar}}} - \frac{1}{c_p^d}\nabla\cdot\underline{\bar{S}}\right] \approx -\nabla\cdot\left(\pi\rho\,\overline{\theta''\underline{v}''} + \frac{\underline{\bar{H}}}{c_p^d}\right)$$

- Compared to the complete solution, the "fully incompressible" approximation, although neglecting sub-grid scale compression work, finally would <u>only</u> have reduced the sub-grid scale macroscopic heat flux, which vanishes at the surface. Hence, <u>heat conservation would NOT have been affected</u>!
- With the <u>wrong implementation</u> of the <u>more complete</u> ICON-solution, the atmosphere receives a reduced molecular SHF normal to the surface of the earth $c_v^d / c_p^d \cdot H_n\big|_{Srf}$, while the soil <u>still receives full</u> $H_n\big|_{Srf}$!

➢ Violation of heat conservation that can be detected by single-column diagnostics:

Confirming heat-conservation of the corrected implementation (1):

- **Isochoric and dry Single-Column solution:**

$$\overline{Q_v^{diab}} \approx -\partial_z\left(\overline{H}_z + \overline{S}_z\right) \approx \overline{Q_p^{diab}}$$ **no diabatic heat sources except molecular diffusion and radiation**

$$w_{Top} \equiv 0 = \overline{H}_{z=Top}$$ **zero heat-flux condition at Top**

$$w_{Srf} = 0$$ **only molecular flux at Srf**

$$c_v = c_v^d; \quad R = R^d$$ **dry atmosphere**

$$\underline{\hat{v}} \equiv 0$$ **neither compression nor advection at grid scale**

$$\nabla_h \overline{(\quad)} \equiv 0$$ **horizontally homogeneous at grid scale**

$$c_v^d \int_{Srf}^{Top} \overline{\rho}\,\partial_t \hat{T}\,dz \approx \overline{H}_{z=Srf} + \overline{S}_{z=Srf} - \overline{S}_{z=Top}$$

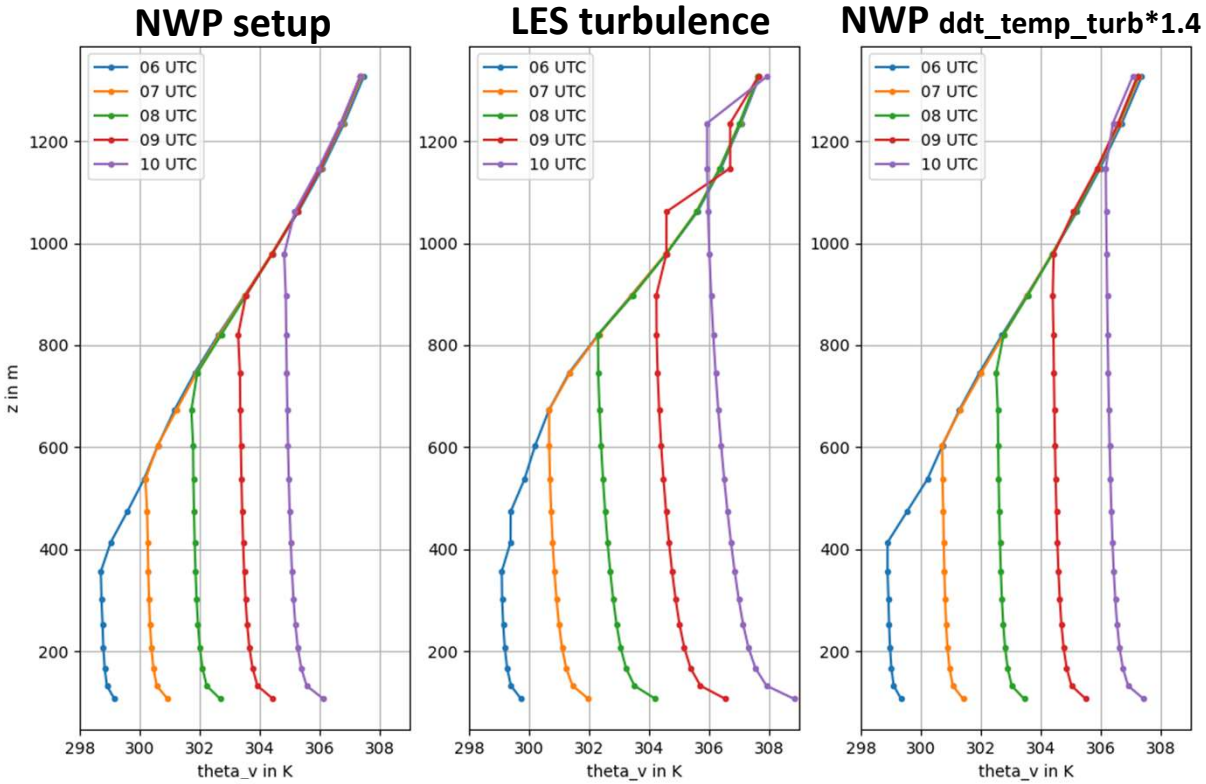
Confirmation heat-conservation of the corrected solution (2):

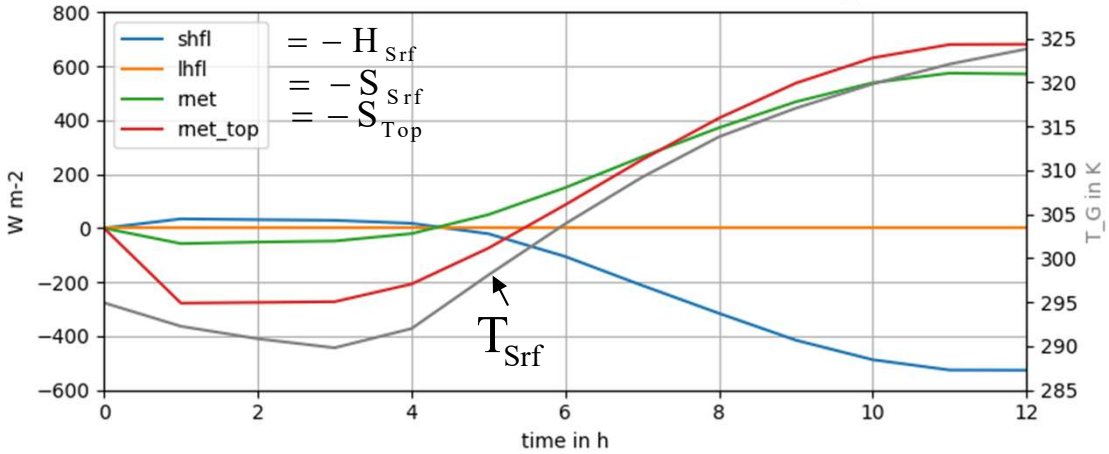
V. Maurer: energy balance calculations with ICON-SCM

- ICON-SCM mainly developed by Ivan Bastak (HErZ Frankfurt) and Martin Köhler
  → minimum torus (16 vertices)
  → ldynamics + ltransport = false

- for energy balances, scenario built on „SCM-real“, but
  → no forcing
  → dry, isothermal soil

  → no latent heat flux at the bottom of the soil (no evaporation)





Confirmation heat-conservation of the corrected solution (3):

V. Maurer: energy balance calculations with ICON-SCM

averages for 05-12 UTC

		NWP	LES-turb	ddt_temp_turb*1.4	GME turb.
ATM+SOIL	$\Delta E^{Atm+Soil}$	354.6 W m^{-2}	469.8	460.6	354.6
	$-S_{Top}$	468.0 W m^{-2}	469.0	467.0	472.8
	ratio	**0.76**	**1.00**	0.99	0.75
ATM	ΔE^{Atm}	303.4 W m^{-2}	416.5	405.4	313.3
	$(H+S)_{Srf} - S_{Top}$	414.6 W m^{-2}	421.0	409.7	437.1
	ratio	**0.73**	**0.99**	0.99	0.72
SOIL	ΔE^{soil}	51.2 W m^{-2}	53.3	55.2	41.3
	$-(H+S)_{Srf}$	53.5 W m^{-2}	48.1	57.3	35.6
	ratio	0.96	1.11	0.96	1.16

$$\Delta E^{Dom} := c_v^d \int_{Dom} \overline{\rho}\, \partial_t \hat{T}\, dz$$

Conclusion:

- ICON solves grid-scale 1-st order prognostic equations for density, water phases and virtual potential temperature θ_v .
- Instead of introducing physical tendencies directly into the θ_v-equation, isochoric T-tendencies (converted from the former isobaric ones) are used.
- Although sub-grid T-tendencies, which are generated from heat transport, are <u>not</u> affected by the condition of constant volume or pressure, the combination of sub-grid transport and sub-grid adiabatic sources <u>is actually affected</u>.
- While $\nabla \cdot \left(\pi \overline{\rho \, \theta'' \underline{v}''} \right)$ is a good approximation for the pure sub-grid transport term, $\pi \nabla \cdot \left(\overline{\rho \, \theta'' \underline{v}''} \right)$ can be used, if sub-grid adiabatic sources are included at isobaric conditions and $c_p^d / c_v^d \, \pi \nabla \cdot \left(\overline{\rho \, \theta'' \underline{v}''} \right)$ at isochoric conditions.
- Finally, it holds in general $$\boxed{\overline{\rho} \, \partial_t \hat{T} \Big|_{\text{isochor}}^{\text{phys}} \approx \frac{c_p^d}{c_v^d} \cdot \overline{\rho} \, \partial_t \hat{T} \Big|_{\text{isobar}}^{\text{phys}}},$$ if the sub-grid adiabatic terms are considered.
- Due to a misunderstanding, the factor c_p^d / c_v^d has <u>not</u> been applied to the isobaric T-tendencies from turbulence!

Remarks (1):

- **All the 3 approximations (labelled according to COSMO, LES and ICON) do not violate energy conservation (if correctly implemented) .**
  - **Nevertheless, the "COSMO" one neglects sub-grid scale compression, what reduces turbulent SHF by about 30% compared to the other approximations, provided the same mean state is given.**
    - **This effect is similar to an according reduction of the turbulent length-scale.**
    - **However, due to negative feedback, the atmosphere and the model would, of course, adapt to this SHF-representation with actually rather similar fluxes compared to the other approximations!**
- **The implementation bug of the "ICON" approximation does violate energy conservation, destroying heat for upward SHF at the surface (and vice versa)**
- **The correct ICON approximation is the most accurate one, but it includes an additional sub-grid contribution in non-divergence form with an impact in the order of 1%.**

Remarks (2):

- A direct introduction of $\nabla \cdot \left(\overline{\rho \, \theta_v'' \underline{v}''} \right)$ and diabatic source terms into the θ_v- equation would be more accurate and less intricate than employing an isochoric T-equation.

- Sub-grid T-tendencies from convection are to be treated accordingly.

- There are further issues related to
  - flux-equilibrium at the surface in the case of evaporation
  - general neglect of relative mass fluxes of water components in the heat equation for the atmosphere and the soil (e.g.: SHF related to precipitation is neglected)
  - general destruction of source terms by sub-grid phase transitions due to the application of a grid-scale saturation adjustment

Consequence of the coding-bug:

- **Magnitude of T-tendencies from turbulence have been about 29% too small.**
- **The error has been almost compensated by tuning and the effect of SMA.**
- **Better scores with the corrected version seem only to be feasible with a quite extensive retuning (Günther Zängl, DWD) and using ecRAD as a radiation scheme.**

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- **Thanks to Günther Zängl (DWD), who performed an extensive retuning of the model, where he already employed ecRAD as a radiation scheme (cf. his plenary talk about ICON-news)!**