Status report of

WG 2 – Numerical aspects

COSMO General meeting, Rome, Italy
09-12 Sept. 2019

Michael Baldauf, Daniel Reinert, Günther Zängl (DWD)

- PP EX-CELO → Zbigniew Piotrowski
- PT CCE final report → Damian Wójcik

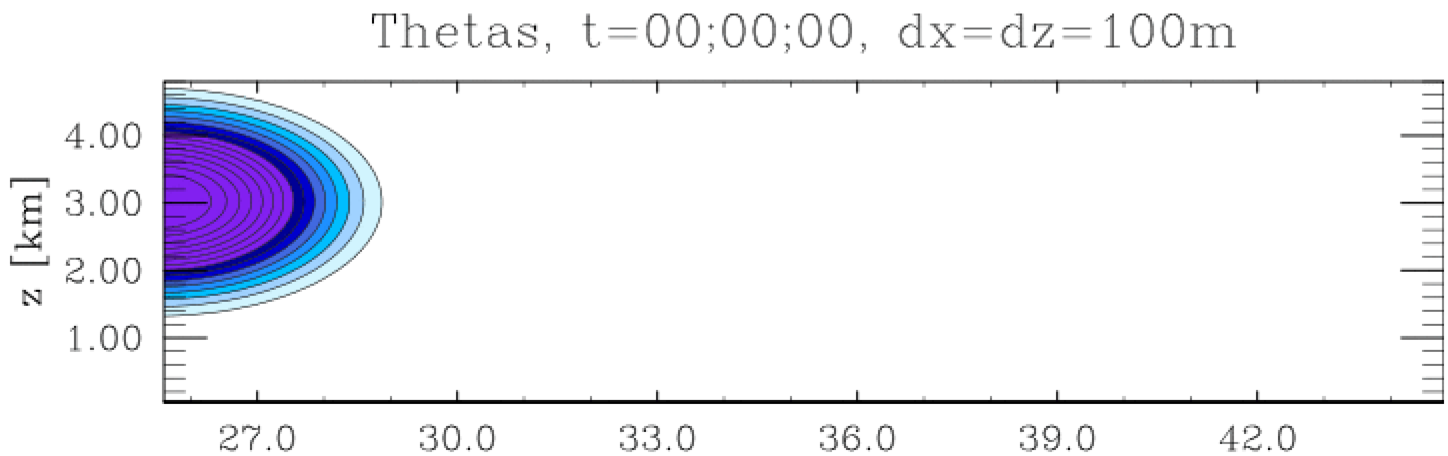
Addendum to the Priority Project ‚Comparison between the dynamical cores of COSMO and ICON' (CDIC)

- PP CDIC officially finished in Aug. 2018
- however, the Straka et al. case did not work correctly
  this is now solved (thanks to D. Reinert (DWD))
  → all relevant idealized test cases are working correctly with ICON
- The code for all the new test cases is available in the `icon-nwp-dev`-branch
- Final report is still overdue
  (until now, only contribution by D. Wójcik, (thank you!) received)

Test case 3: cold bubble

R. Dumitrache, A. Iriza (NMA), M. Baldauf (DWD)

Testsetup by *Straka et al (1993)*



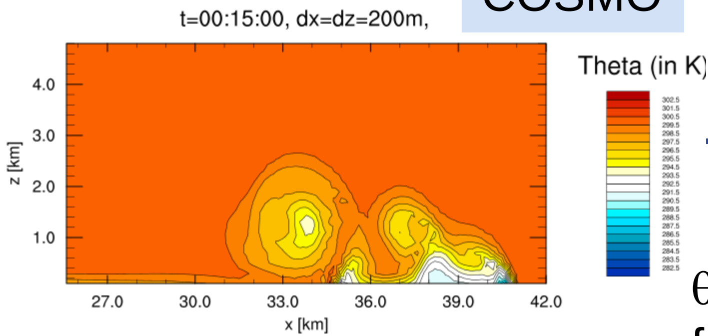
Test properties:

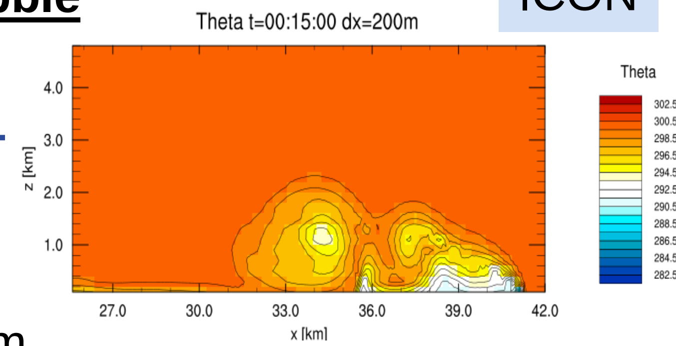
- test of dry Euler equations (without Coriolis force)
- unstationary
- strongly nonlinear
- comparison with reference solution from paper

Test case 3: cold bubble

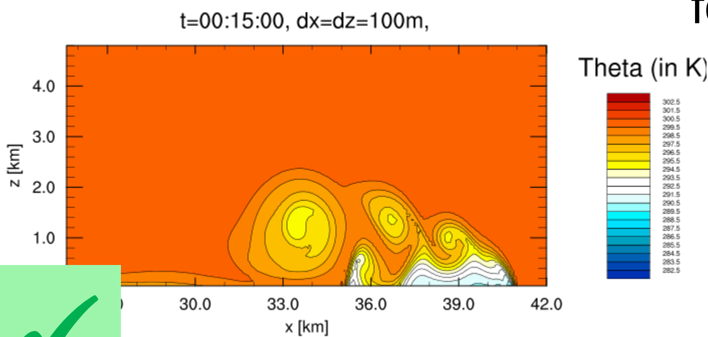
COSMO

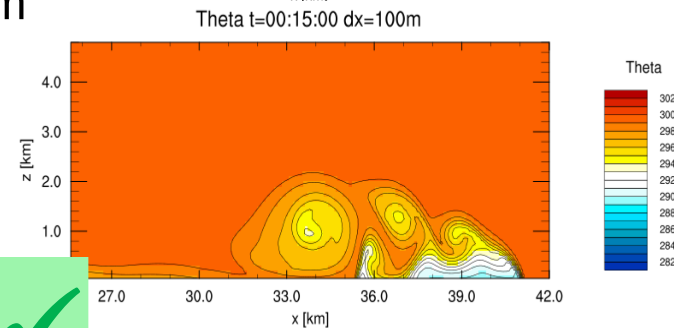
ICON

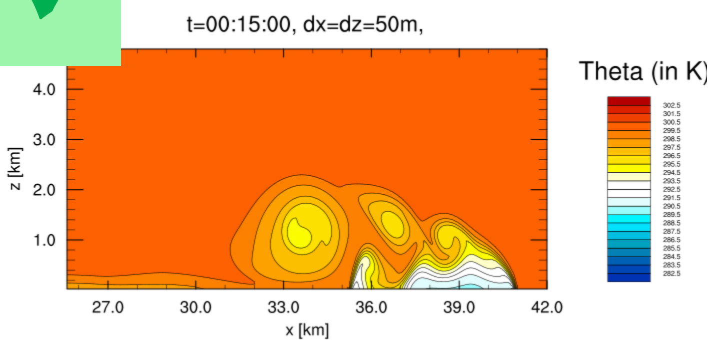


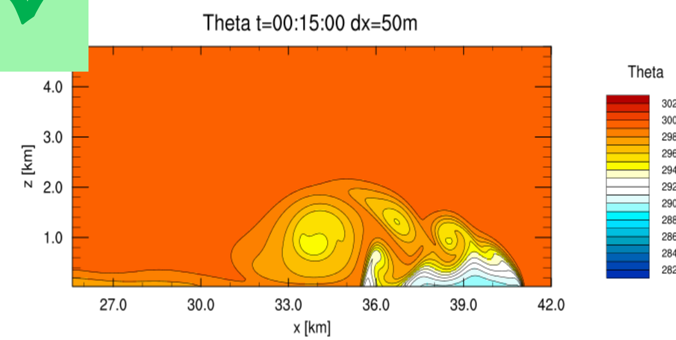


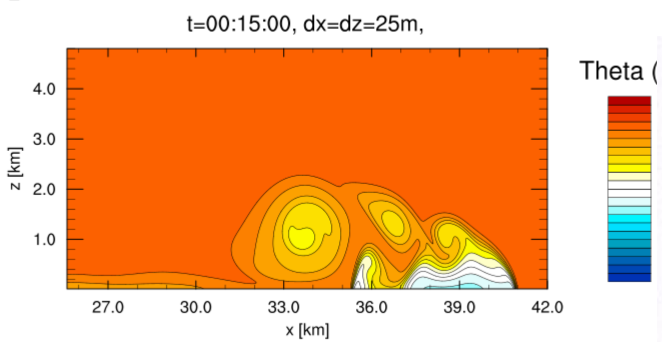
θ at t=15 min.
for Δx= 200, 100, 50, 25m



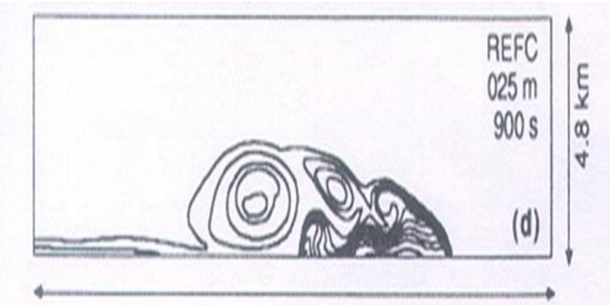


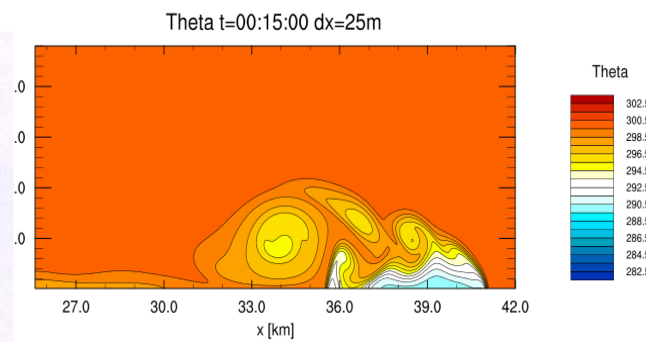






Reference solution
from Straka et al.:





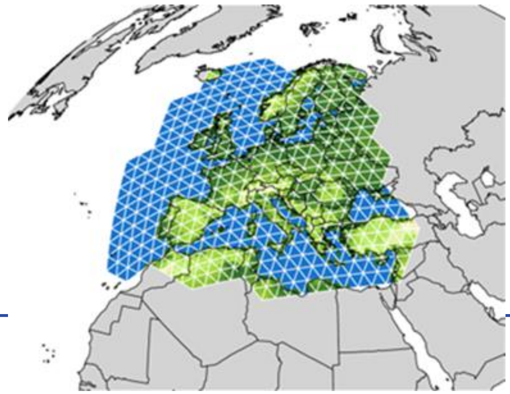
Explore a more advanced vertical discretization for ρ , θ_v , and π

D. Reinert (DWD)

- Replace the 2nd order (linear)
  - vertical interpolation operator (cell to face)
  - vertical advective flux operator

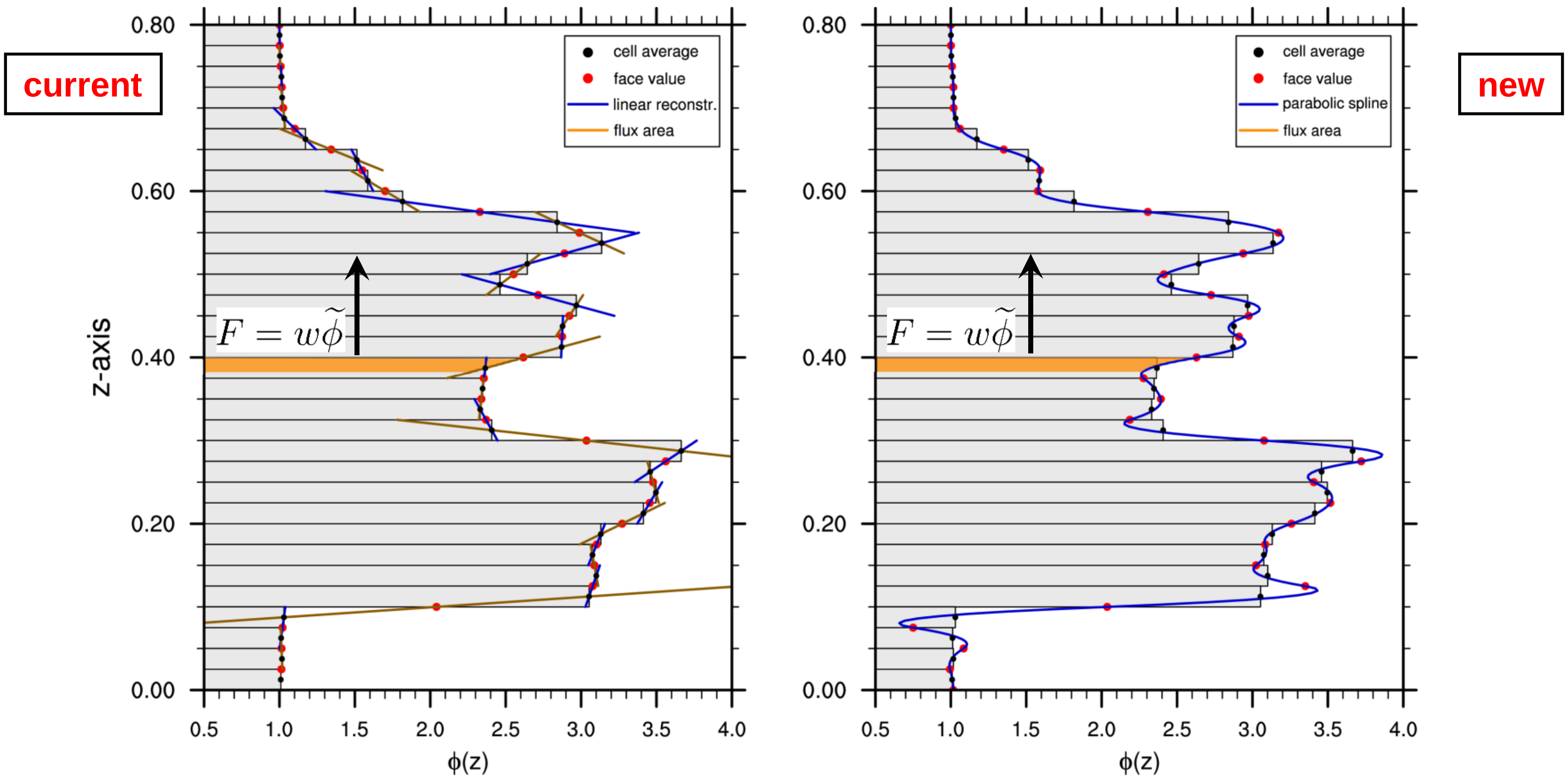
  by 3rd order operators based on reconstructed parabolic splines (Zerroukat et al., 2006)

- Why parabolic splines?
  - successfully tested and used for tracer transport in ICON (vertical PSM-scheme)
  - Operators are already available in ICON and ‚only‘ have to be applied within the dynamical core



Old vs. new vertical discretization

Example: dycore-like reconstruction of an irregular 1D test signal *D. Reinert (DWD)*

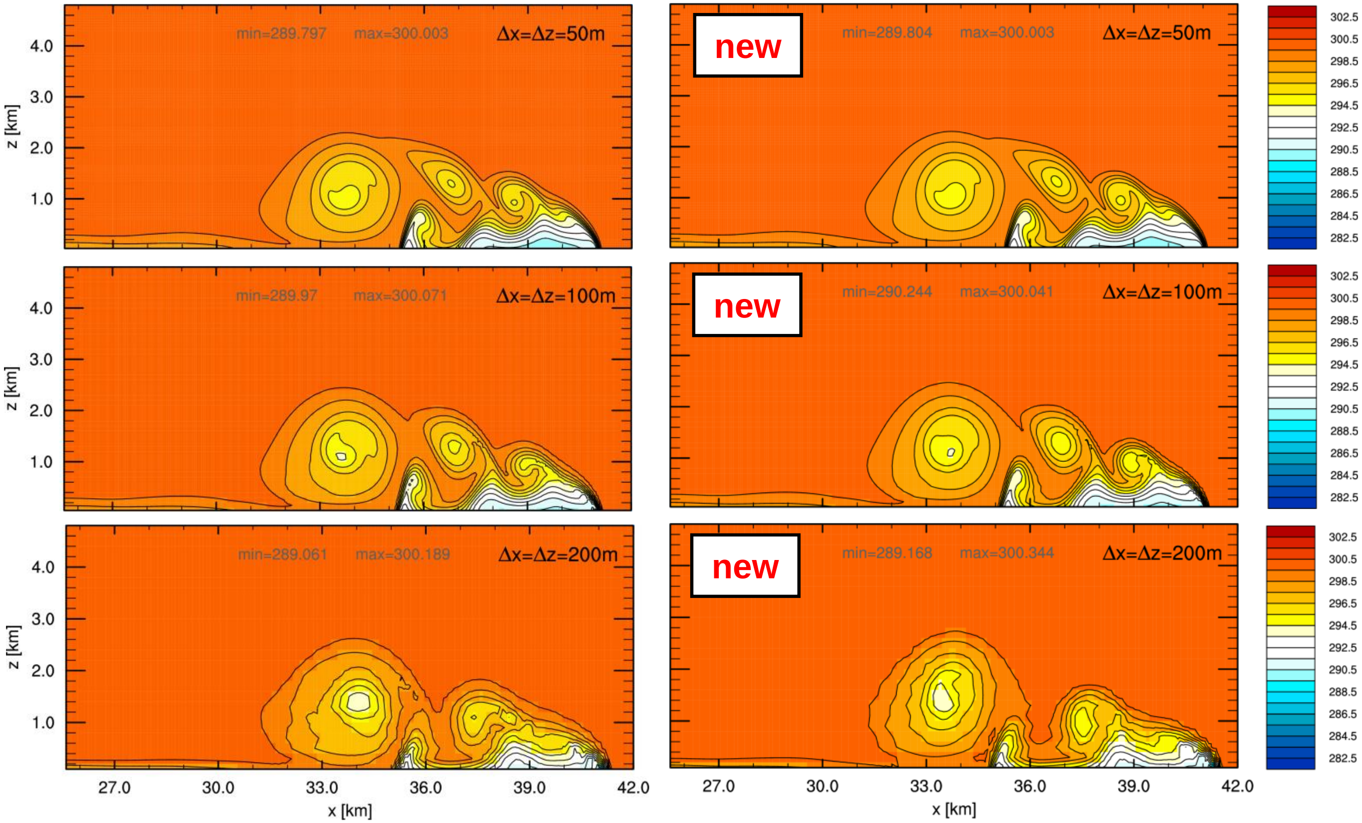


- 2nd order reconstruction
- Continuously differentiable at cell interfaces
- Reconstruction not unique in cell interior

- 3rd order reconstruction
- Continuously differentiable everywhere
- Unique reconstruction

2D nonlinear density current

D. Reinert (DWD)



- similarity of the (almost) converged solution at 50m suggests that the 3rd order operators are implemented correctly.
- Unfortunately, only small improvements in simulation quality (if any) are noticeable (see e.g. middle rotor at Δx=Δz=200m).

Implementation of the supercell detection index (SDI) into ICON

Wicker et al. (2005):

M. Baldauf, G. Zängl (DWD)

$$SDI_1(x,y) = \rho_{w\zeta}(x,y) \cdot \bar{\zeta}(x,y)$$ (SDI_2 similar)

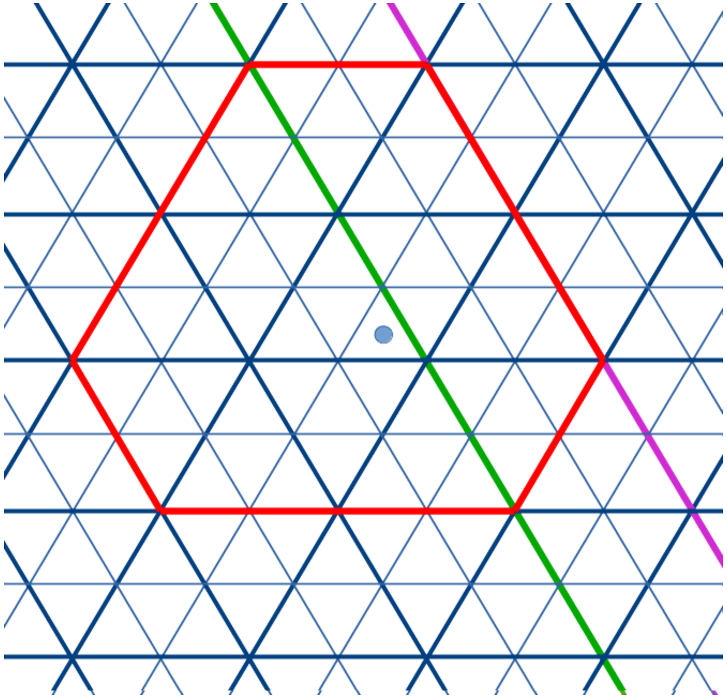
with the velocity-vorticity correlation:

$$\rho_{w\zeta}(x,y) = \frac{< w'\zeta' >_{(x,y)}}{\sqrt{< w'^2 >_{(x,y)}}\sqrt{< \zeta'^2 >_{(x,y)}}}$$

<…> = volume average
= horiz. average of vertical averages

use the parent grid for a larger horiz. averaging area!

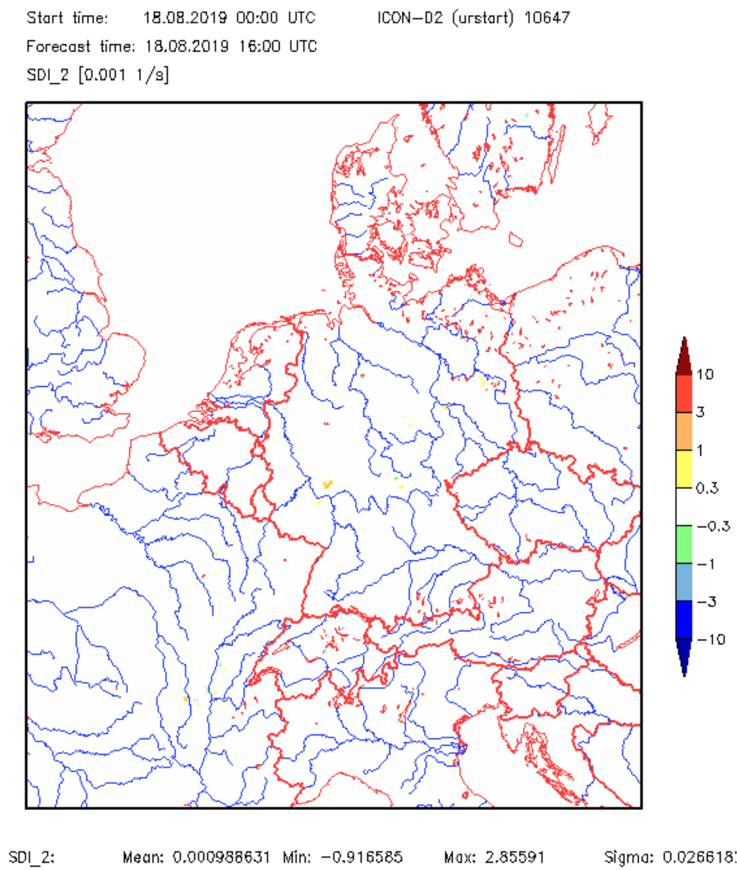
1. calc. values on fine grid
2. average to parent grid
3. exchange parent cells
4. average on parent grid
5. write back to fine grid



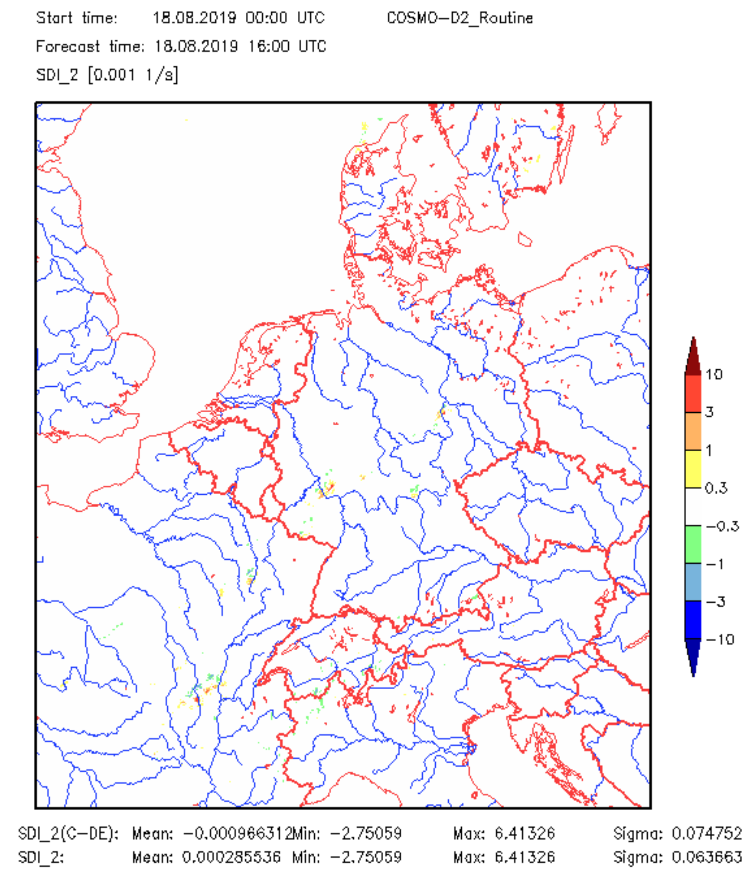
Similar averaging method is used for the lightning potential index (LPI)

Case study 18 Aug. 2019, **SDI2** for a heavy storm event in the vicinity of Frankfurt

ICON-D2 (init. by ICON-EU)



operat. COSMO-D2



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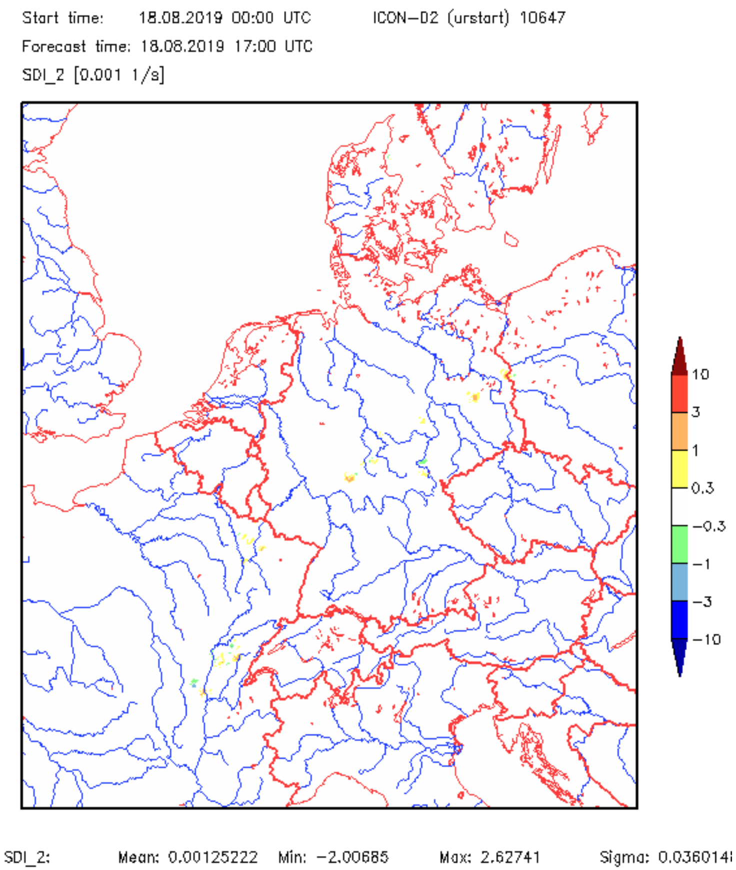

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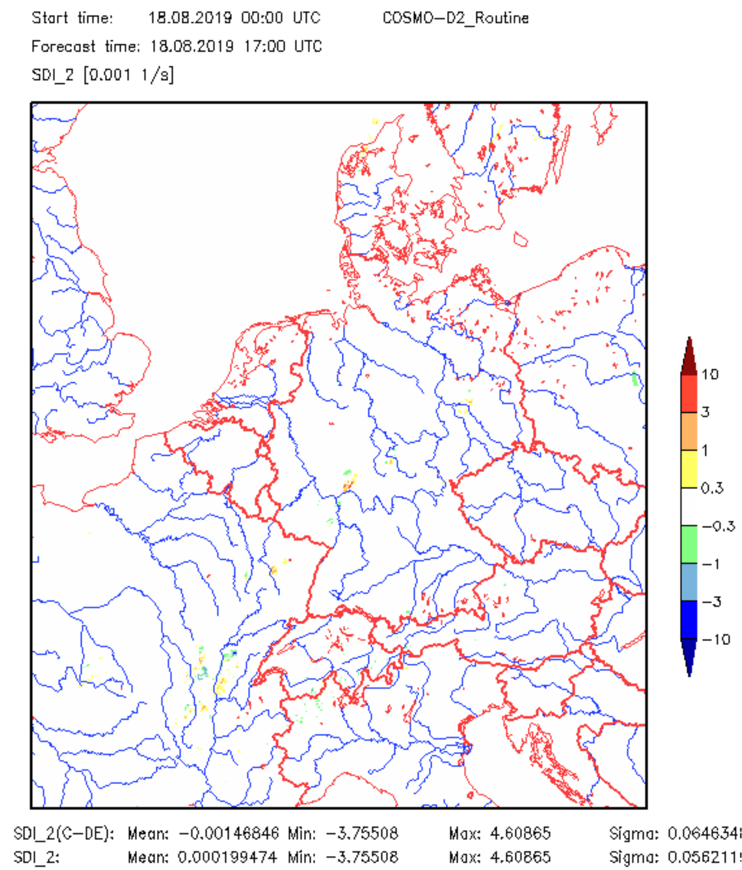


Higher order discretization for COSMO

A. Will (Univ. Cottbus)

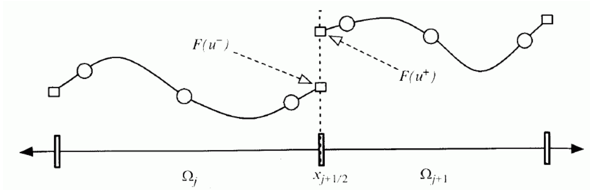
- currently: migration of the code from v5.0 to v5.6
  unfortunately still bugs present in v5.6
  (slow progress due to other tasks at BTU Cottbus)
- testing in hindcast mode and in the NUMEX-system
  for a COSMO-D2 setup (summer case) must be done.
- However, expectations are:
  max w ~ twice as large; more sound wave activity;
  stronger diffusion properties (stronger PBL growth, ...)
  → probably no succesful simulation without adaptation
  at least of the turbulence scheme

A possible alternative dynamical core for ICON based on Discontinuous Galerkin Discretisation

Michael Baldauf (FE13)

Discontinuous Galerkin (DG) methods in a nutshell

$$\frac{\partial q^{(k)}}{\partial t} + \nabla \cdot \mathbf{f}^{(k)}(q) = S^{(k)}(q), \qquad k = 1, ..., K$$



From *Nair et al. (2011)* in *‚Numerical techniques for global atm. models'*

weak formulation $\int_{\Omega_j} dx\, v(\mathbf{x}) \cdot \ldots$

Finite-element ingredient

$$q^{(k)}(x,t) = \sum_{l=0}^{p} q_{j,l}^{(k)}(t)\; p_l(x - x_j)$$

e.g. Legendre-Polynomials

e.g.
Cockburn, Shu (1989) Math. Comput.
Cockburn et al. (1989) JCP
Hesthaven, Warburton (2008): Nodal DG Methods

Finite-volume ingredient

$$\mathbf{f}(q) \to \mathbf{f}^{num}(q^+, q^-) = \frac{1}{2}\left(\mathbf{f}(q^+) + \mathbf{f}(q^-) - \alpha(q^+ - q^-)\right)$$

Lax-Friedrichs flux

Gaussian quadrature for the integrals of the weak formulation

→ ODE-system for $q^{(k)}{}_{jl}$

DG – Pros and Cons

- **local conservation**
- any **order of convergence** possible
- flexible application on **unstructured grids** (also dynamic adaptation is possible, h-/p-adaptivity)
- very good **scalability**
- **explicit** schemes are easy to build and are quite well understood
- higher accuracy helps to **avoid several awkward approaches** of standard 2nd order schemes: staggered grids (on triangles/hexagons, vertically heavily stretched), numerical hydrostatic balancing, grid imprints by pentagon points or along cubed sphere lines, …

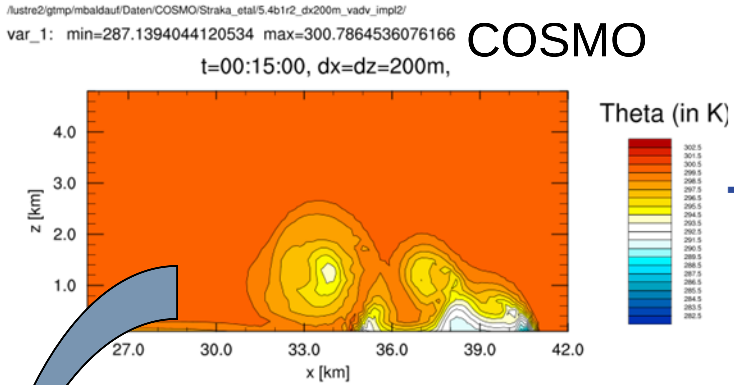
- high computational costs due to
  - (apparently) **small Courant numbers**
  - higher number of DOFs
- **well-balancing** (hydrostatic, perhaps also geostrophic?) in Euler equations is an issue (can be solved!)
- basically ‚only' an **A-grid-method**, however, the ‚spurious pressure mode' is very selectively damped!

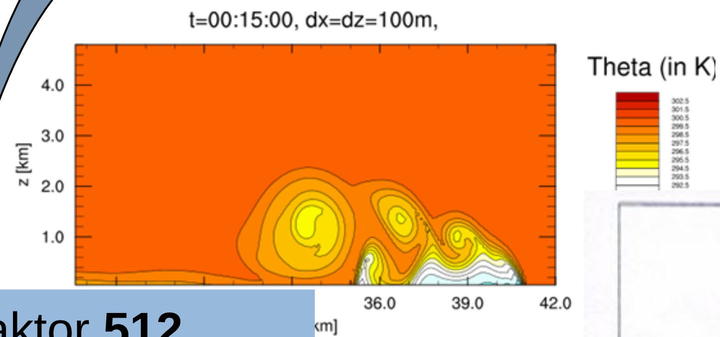
Target system: **ICON model**
(Zängl et al. (2015) QJRMS)
- operational at DWD since Jan. 2015
  (global (13km) and nest over Europe (6.5km))
- convection-permitting (2.2km): Q4/2020

but currently far away from this, only a **toy model for 2D problems exists** with:

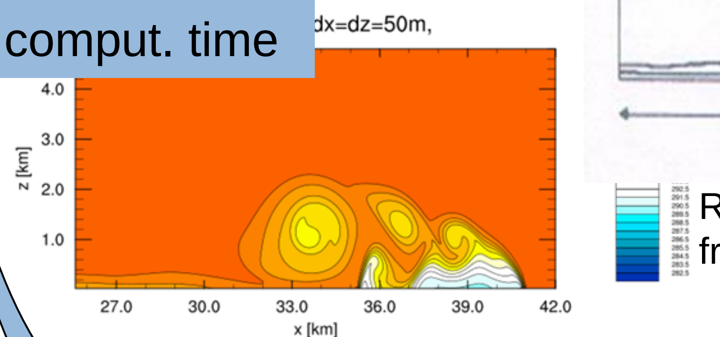
- explicit time integration DG-RK (with Runge-Kutta schemes) or horizontally explicit-vertically implicit (DG-HEVI) (with IMEX-Runge-Kutta)
- ‚local DG' (LDG) option for PDEs with higher spatial derivatives
- use of a triangle grid (also on the sphere) is optional

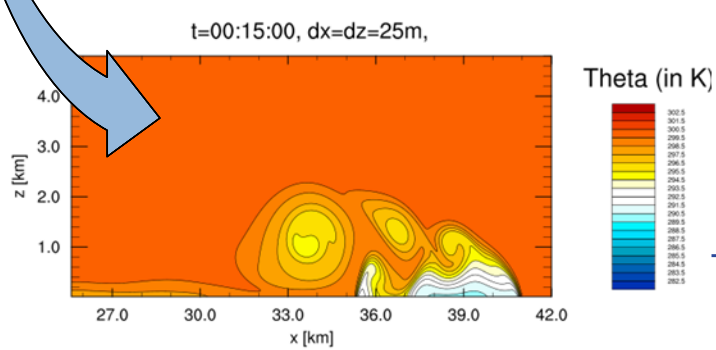
COSMO

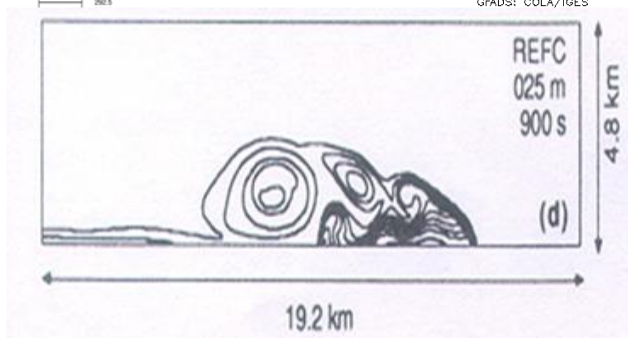




Faktor **512**
in comput. time





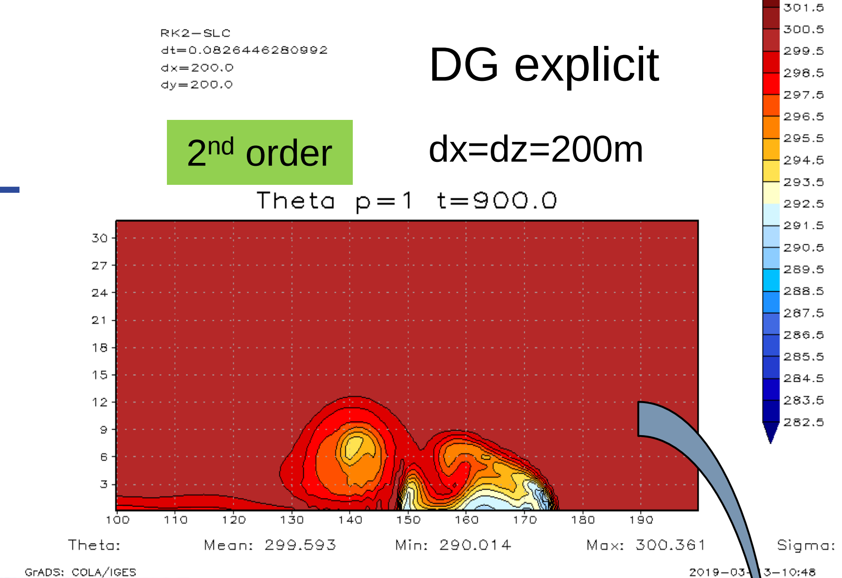


Reference solution
from Straka et al. (1993)

DG explicit

2nd order

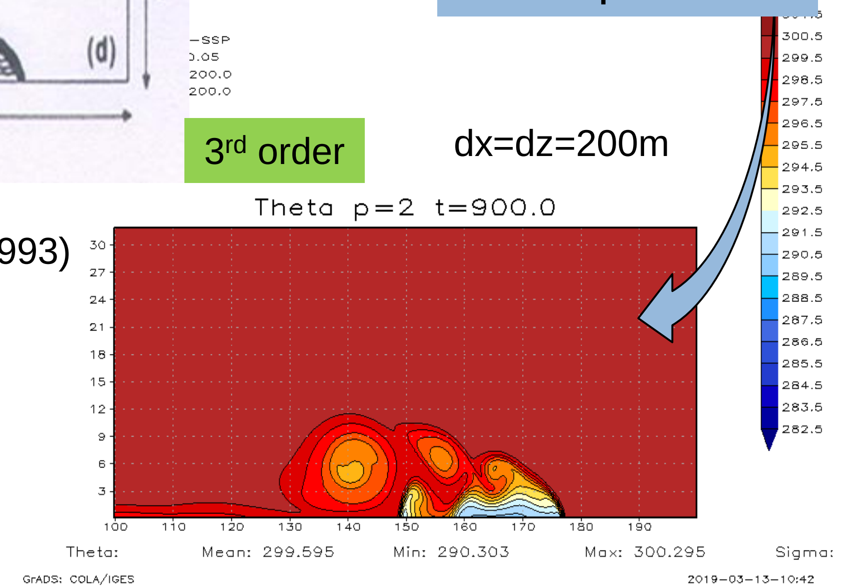
dx=dz=200m



Faktor **4.3**
in comput. time

3rd order

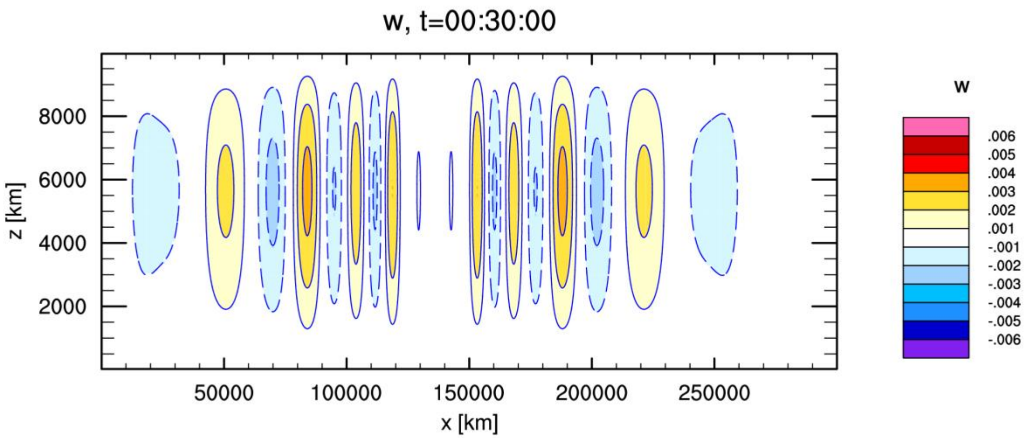
dx=dz=200m

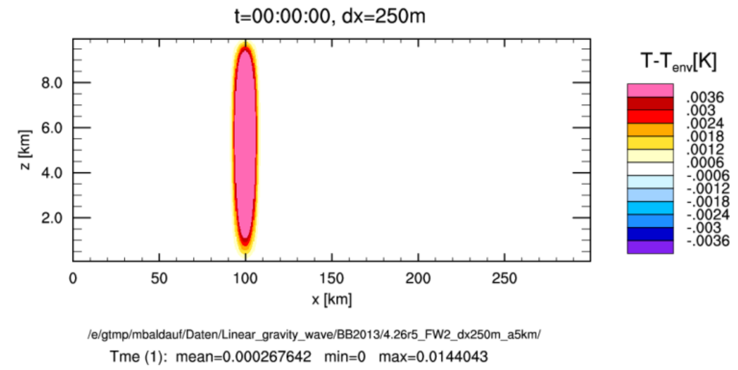


Linear gravity/sound wave expansion in a channel

setup similar to *Skamarock, Klemp (1994) MWR*

Δx=500m, Δz=250m

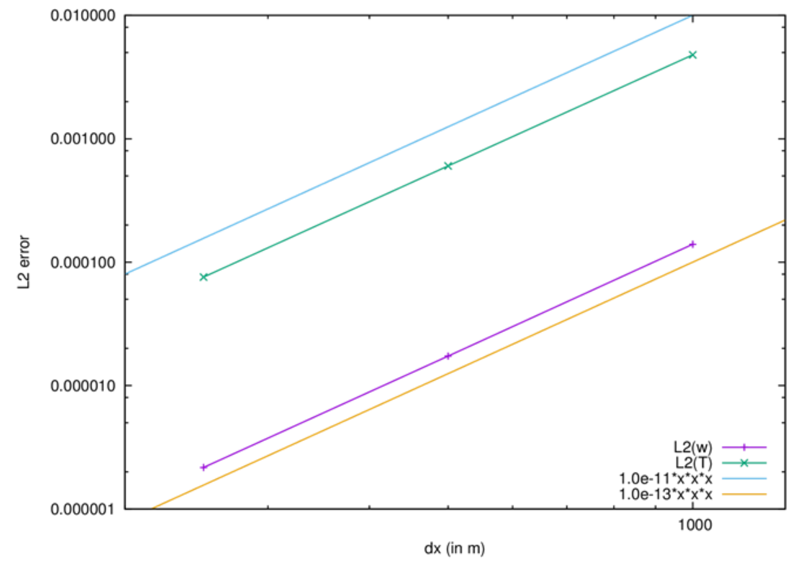




colors : simulation with p=2/RK3-SSP (i.e. 3[rd] order explicit DG)

blue lines: analytic solution for compressible, non-hydrostatic Euler eqns. (*Baldauf, Brdar (2013) QJRMS*)

Exact 3[rd] order convergence for w and T':



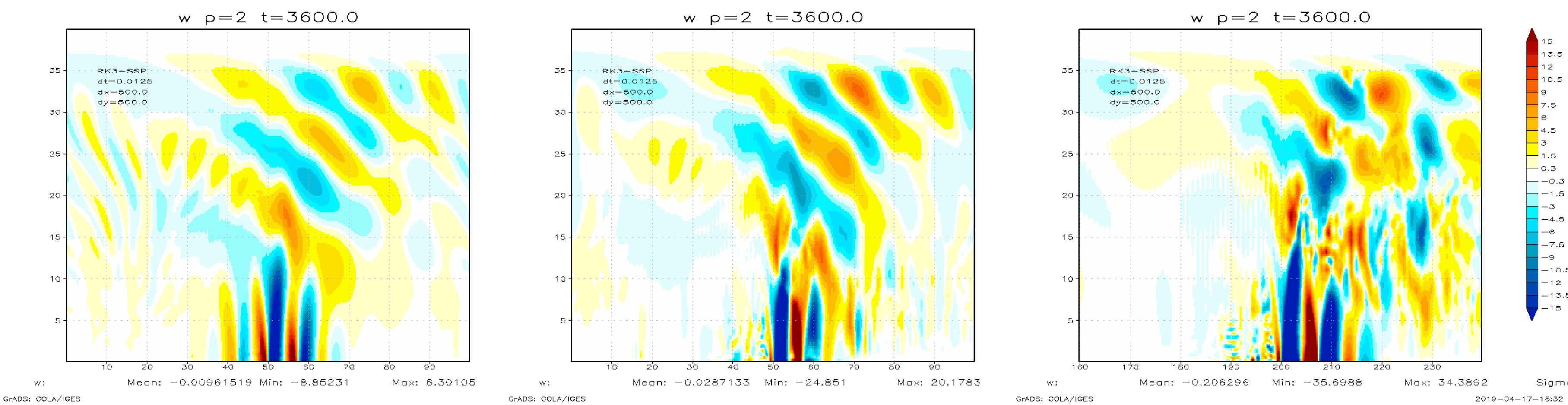
Test case: flow over steep mountains, vertically stretched grid

Schaer et al. (2002) MWR (case 5b: U_0=10m/s, N=0.01 1/s)

H_{oro} = 1000m, α_{max} = 38°

H_{oro}= 2000m, α_{max} = 57°

H_{oro} = 3000m, α_{max} = 61°



with vertical grid stretching ~1:20, Δz_{min}~50m

Explicit DG simulation (3rd order) remains stable even for steeper slopes!
(remark: diffusion switched off → strong gravity wave breaking occurs)

HEVI-scheme with DG

$$\frac{\partial q^{(s)}}{\partial t} + \underbrace{\nabla \cdot \mathbf{f}^{(s)}_{slow}}_{\text{explicit}} + \underbrace{\nabla \cdot \mathbf{f}^{(s)}_{fast}}_{\text{implicit}} = \underbrace{S^{(s)}_{slow}}_{\text{explicit}} + \underbrace{S^{(s)}_{fast}}_{\text{implicit}}$$

$$\mathbf{f}^{(s)}_{fast} = f^{(s)}_{z,fast}\mathbf{e}_z \qquad\qquad f^{(s)}_{z,fast} = \sum_{s'} H^{ss'} q^{(s')}$$

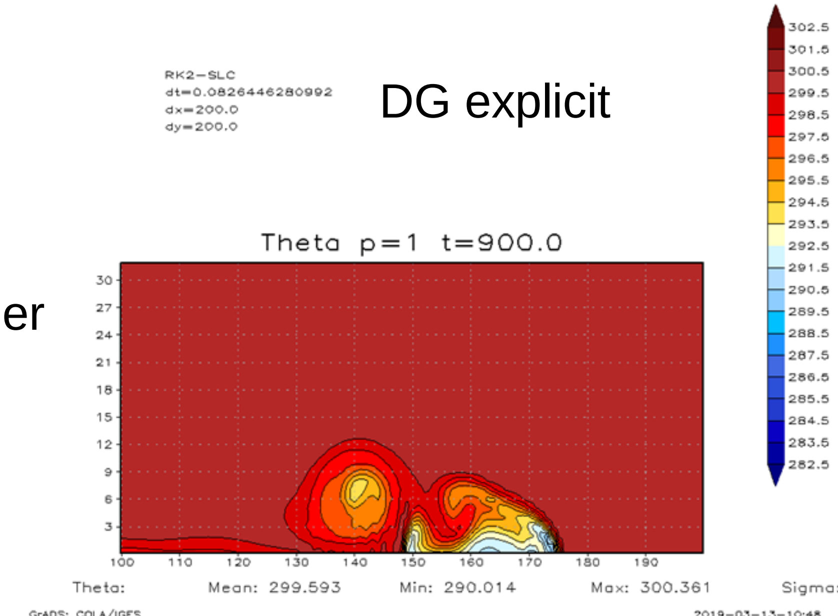
Treatment of numerical diffusion in the local Lax-Friedrich flux:

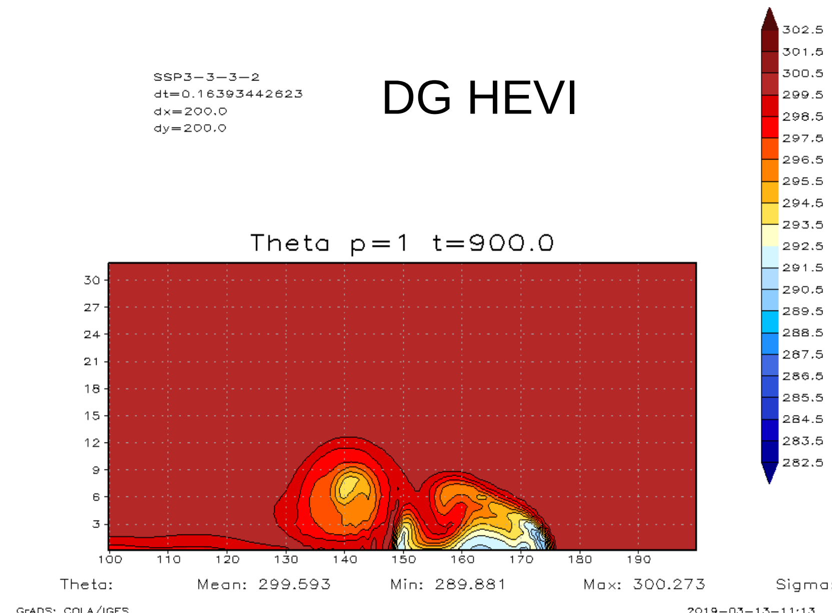
$$\left(\mathbf{f}^{(s)}_{slow} + \mathbf{f}^{(s)}_{fast}\right)^{(num)} \cdot \mathbf{n} = \frac{1}{2}\left(\mathbf{f}^{(s)}_{slow,+} + \mathbf{f}^{(s)}_{slow,-} \overbrace{+ \mathbf{f}^{(s)}_{fast,+} + \mathbf{f}^{(s)}_{fast,-}}^{\text{implicit}}\right) \cdot \mathbf{n}$$
$$- \frac{\lambda_{slow}}{2}\left(q^{(s)}_{+} - q^{(s)}_{-}\right) \underbrace{- \frac{\lambda_{fast}}{2}\left(q^{(s)}_{+} - q^{(s)}_{-}\right)}_{\text{implicit}}$$

Blaise et al. (2016) IJNMF

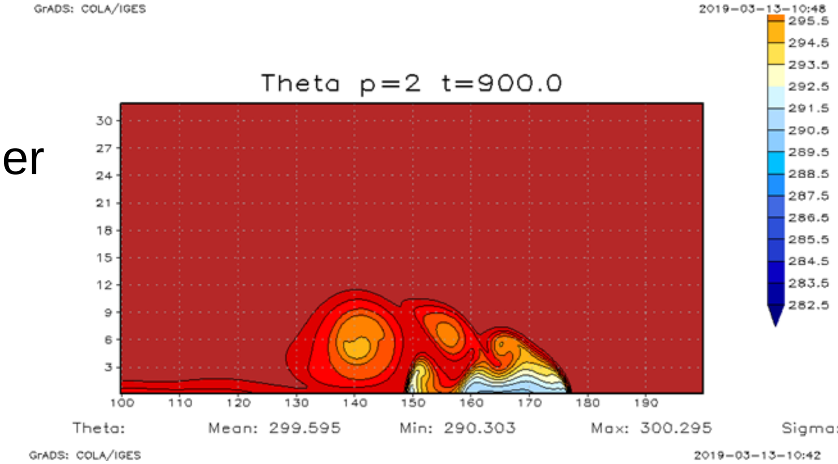
Test case: falling cold bubble (Straka et al. (1993)
Comparison explicit vs. HEVI scheme

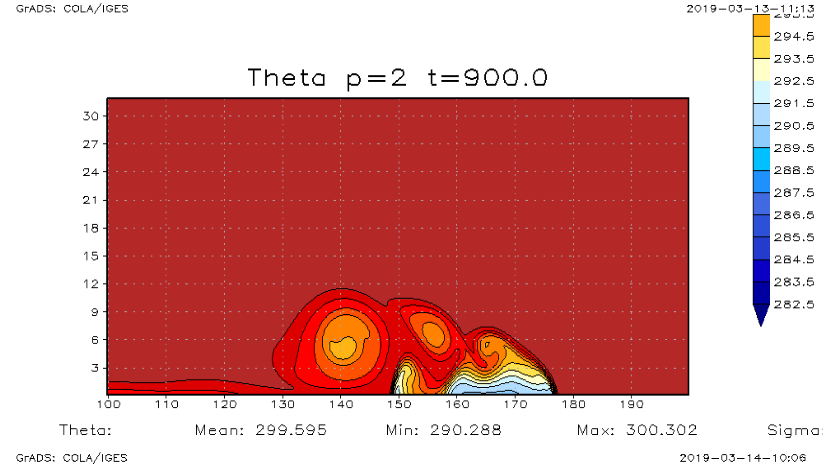
2nd order





3rd order





How to bring DG on the sphere …

Idea to avoid pole problem and to keep high order discretization:
use **local (rotated) coordinates** for every (triangle) grid cell,
i.e. rotate every grid cell towards $\lambda \approx 0$, $\varphi \approx 0$.
→ geometry is treated exactly
→ transform fluxes between neighbouring cells

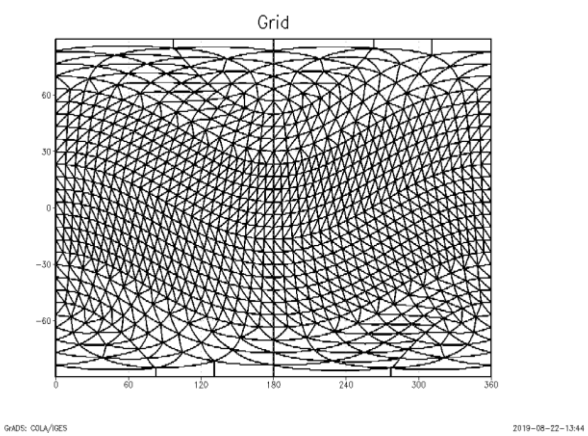
shallow water equations
covariant formulation (here: without bathymetry)

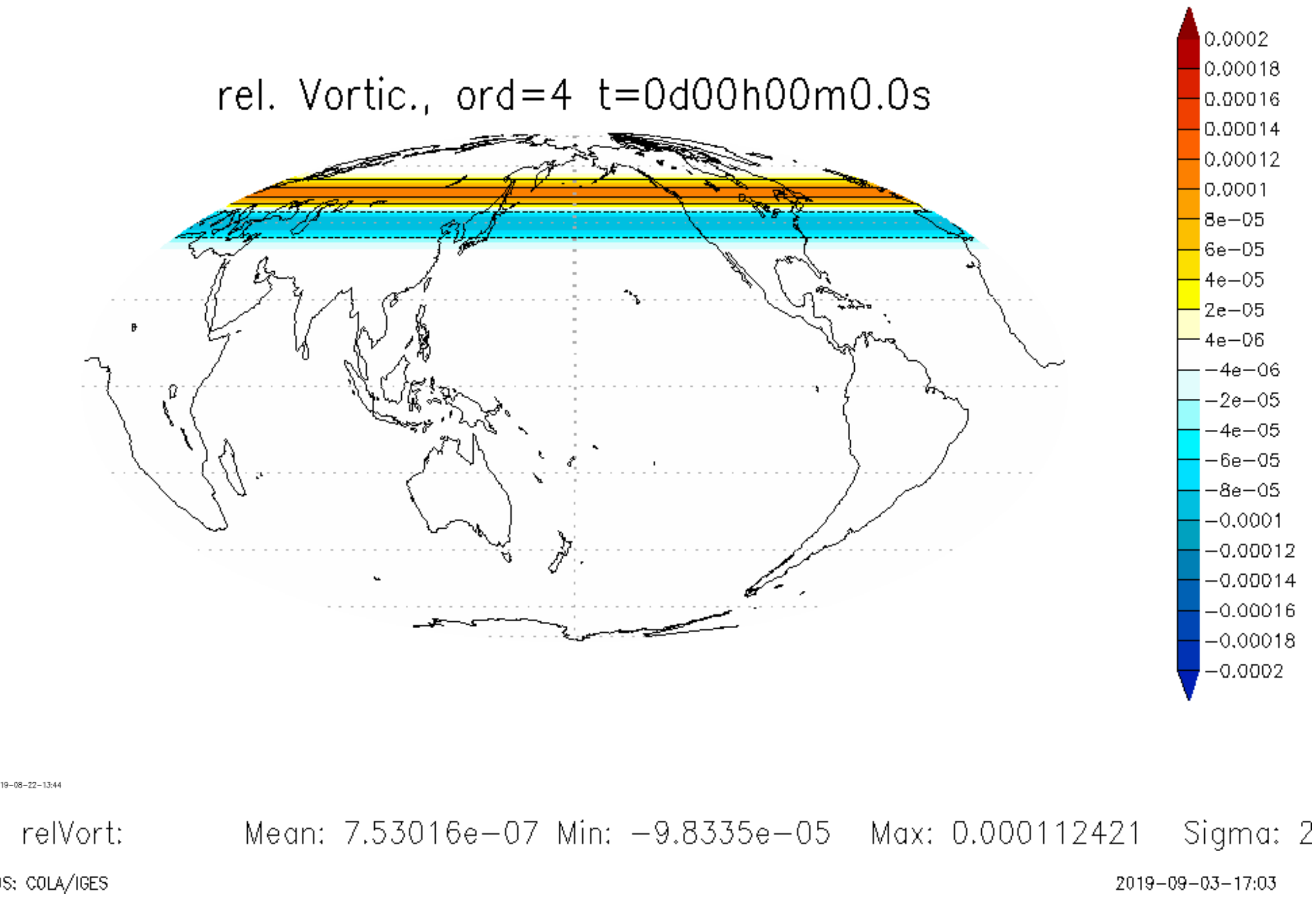
$$
\begin{aligned}
\frac{\partial \sqrt{G} H}{\partial t} + \frac{\partial}{\partial x^i} \sqrt{G} m^i &= 0 \\
\frac{\partial \sqrt{G} m^i}{\partial t} + \frac{\partial}{\partial x^j} \sqrt{G} T^{ij} &= \sqrt{G} (F^i_{Cor} - \Gamma^i_{jk} T^{jk}) \\
T^{ij} &= \frac{m^i m^j}{H} + \frac{1}{2} g^{ij} g_{grav} H^2
\end{aligned}
$$

Barotropic instability test
Galewsky et al. (2004)

4th order DG scheme
without additional diffusion
dx~67 km, dt=15 sec.

simple triangle grid
on the sphere
dx ~ 500km:





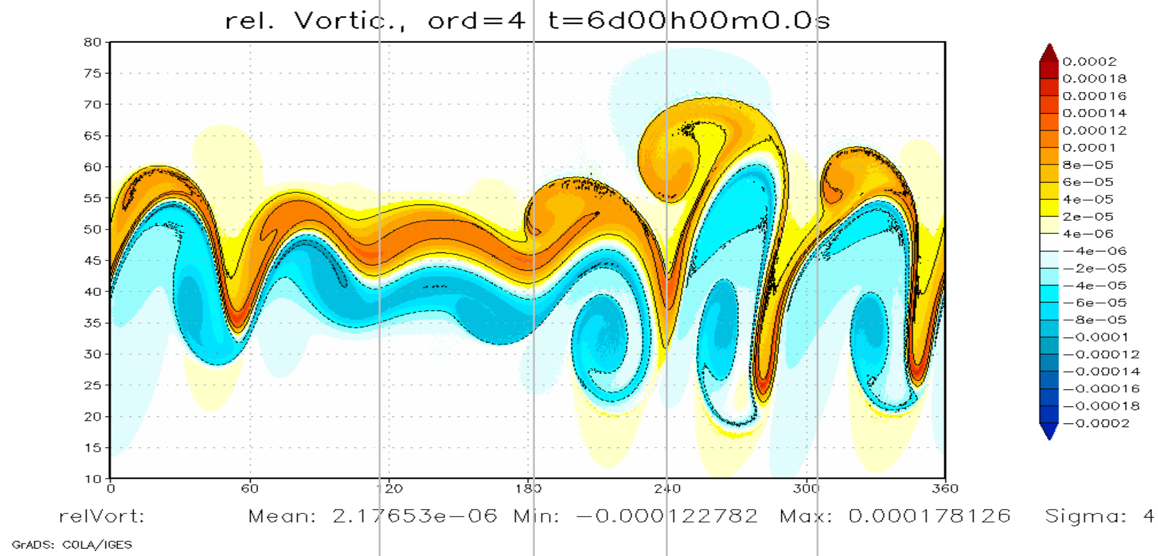
Barotropic instability test
Galewsky et al. (2004)

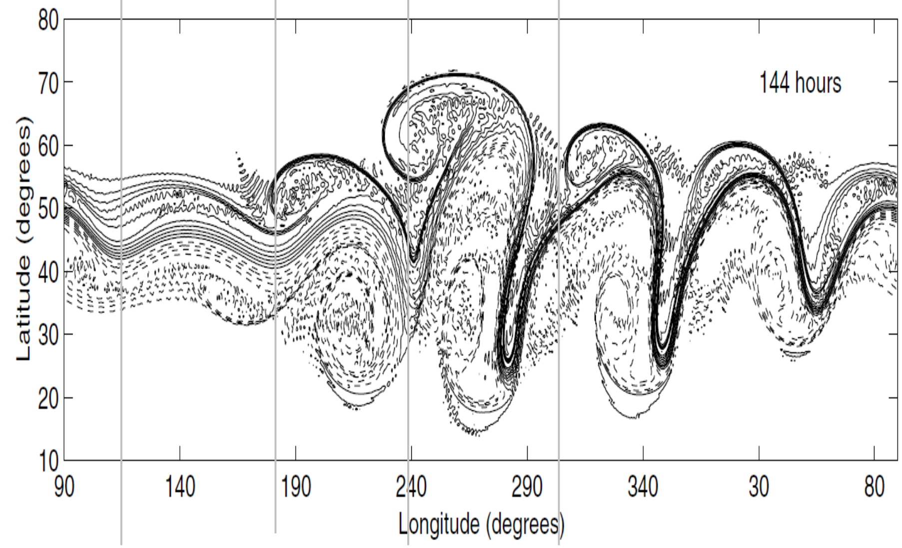
4th order DG scheme
without additional diffusion
dx~67 km, dt=15 sec.

FMS-SWM (Geophys. Fl. Dyn. Lab.)
without additional diffusion
dx~60 km (T341), dt=30 sec.

Fig. 4 from *Galewsky et al. (2004)*

relative vorticity





Summary

- **2D toy model** for
  - explicit DG-RK (on arbitrary unstructured grids with triangle or quadrilateral grid cells) and
  - **HEVI DG-IMEX-RK**

  works for several idealized tests (also Euler equations with terrain-following coordinates), correct convergence behaviour, …
- **DG on the sphere** by use of local (rotated gnomonial) coordinates

Outlook

- **further design decisions**: nodal vs. modal, local DG vs. interior penalty vs. …, …
- coupling of **tracer advection** (mass-consistency)?
- improve **efficiency** in the HEVI direct solver
- further **milestones** (for the next years!)
  - development of a 3D prototype DG-HEVI solver
  - choose optimal convergence order *p* and grid spacing
    estimated: p_{horiz} ~ 3 … 6, p_{vert} ~ 3 … 4 (p_{time} ~ 3…4)

PDEs
THE WORKSHOP ON PARTIAL DIFFERENTIAL EQUATIONS ON THE SPHERE

Announcement:

The next

„Partial differential equations on the sphere“ – workshop

will take place at
DWD, Offenbach, Germany
5-9 October 2020