An update on the Hierarchical Bayes Ensemble Kalman Filter (HBEF)

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Outline

1. The HBEF design.
2. Performance of the HBEF for a model of truth on the circle.
3. Possibility of an application with the LETKF.

The HBEF design

HBEF: overview

We propose to introduce a secondary filter in which background-error covariances are **cycled**.

1. $\mathbf{B}$ is a random matrix.
2. Ensemble members are assimilated as generalized observations on $\mathbf{B}$.
3. The HBEF treats the extended control vector $(\mathbf{x}, \mathbf{B})$ and produces cycled estimates of both $\mathbf{x}$ and $\mathbf{B}$:
   $\mathbf{x}^f$ and $\mathbf{B}^f$
   $\mathbf{x}^a$ and $\mathbf{B}^a$

HBEF forecast step

Given the analysis of the state $\mathbf{x}^a$, the forecast $\mathbf{x}^f$ is computed in the traditional way.

Given the analysis of the covariance matrix $\mathbf{B}^a$, the forecast $\mathbf{B}^f$ is computed using persistence.

HBEF analysis step

Secondary filter

Analysis of **B**

$$\mathbf{B}^a = w\mathbf{B}^f + (1-w)\mathbf{S}$$

Primary filter

Analysis of **x**: The standard EnKF analysis with $\mathbf{B} = \mathbf{B}^a$.

Secondary filter: Analysis

$$\mathbf{B}^a = w\mathbf{B}^f + (1 - w)\mathbf{S}$$

Note that the analysis update equations for the covariances in the HBEF are exactly the same as used in EnVar – with the exception that the HBEF makes use of the **accumulated recent past covariances** instead of climatological covariances.

Numerical experiments with an advection-diffusion model of the “truth” (on the circle)

The doubly stochastic advection-diffusion-decay model

$$\frac{\partial x}{\partial t} + U\frac{\partial x}{\partial s} + \rho\, x - \nu\frac{\partial^2 x}{\partial s^2} = 0,$$

where x is the "true" spatio-temporal field in question, t is time, and s is the spatial coordinate.

$$\frac{\partial x}{\partial t} + U\frac{\partial x}{\partial s} + \rho\, x - \nu\frac{\partial^2 x}{\partial s^2} = \sigma \cdot \alpha(t,s), \qquad (1)$$

where α is the driving noise.

$$\boxed{\frac{\partial x}{\partial t} + U(t,s)\frac{\partial x}{\partial s} + \rho(t,s)\, x - \nu(t,s)\frac{\partial^2 x}{\partial s^2} = \mathrm{e}^{\Sigma(t,s)}\,\alpha(t,s),} \qquad (2)$$

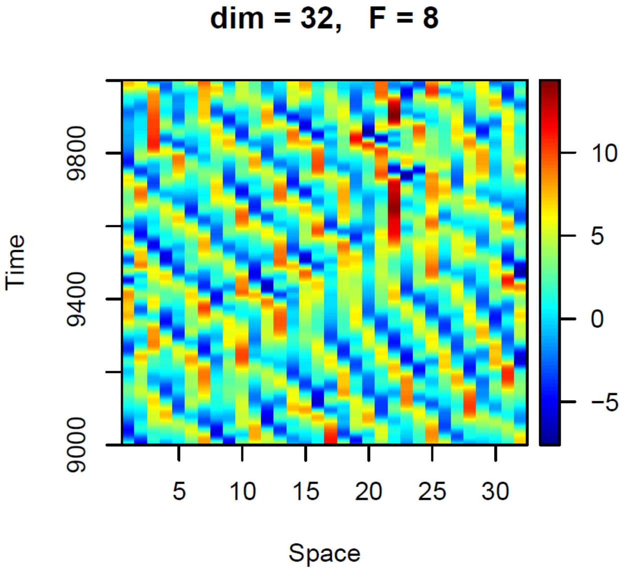
where the coefficients,

$U(t,s)$, $\rho(t,s)$, $\nu(t,s)$, and $\Sigma(t,s)$ (or some of them),

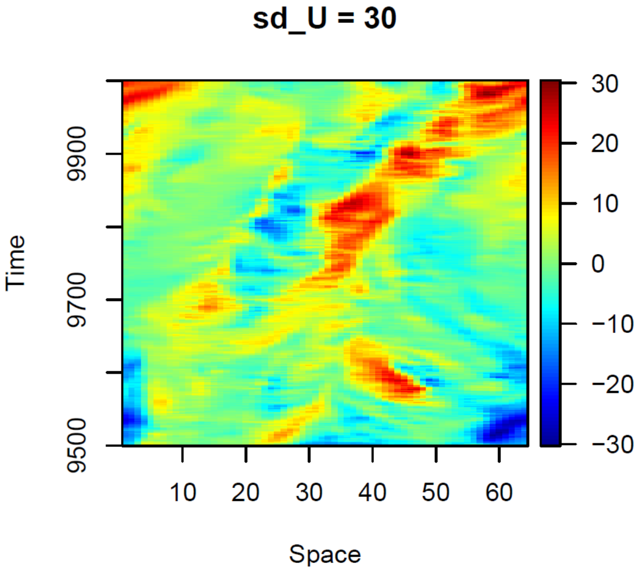
are **spatio-temporal random fields** by themselves postulated to satisfy the singly stochastic advection-diffusion-decay model (1).

Our model can be used instead of the Lorenz-96 model.

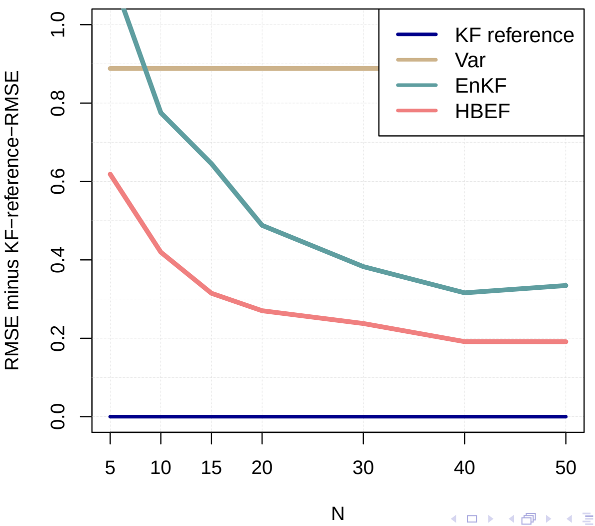
An x-t plot for the Lorenz-96 model



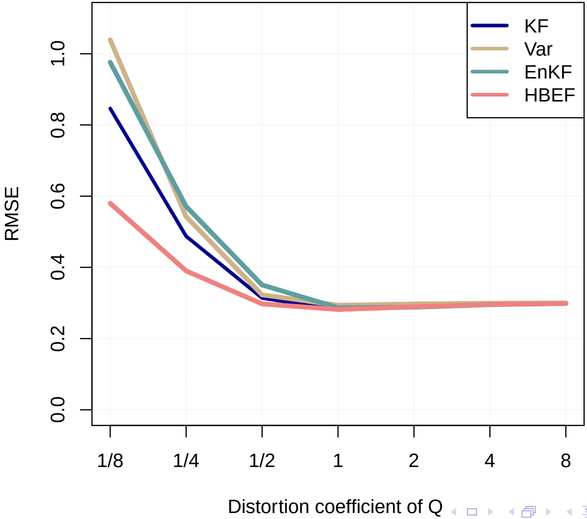
An x-t plot for our model of truth



Analysis RMSEs as functions of ensemble size. $n_{grid} = 32$, $n_{obs} = 8$



Analysis RMSEs when the model-error variance is distorted



LETKF + HBEF?

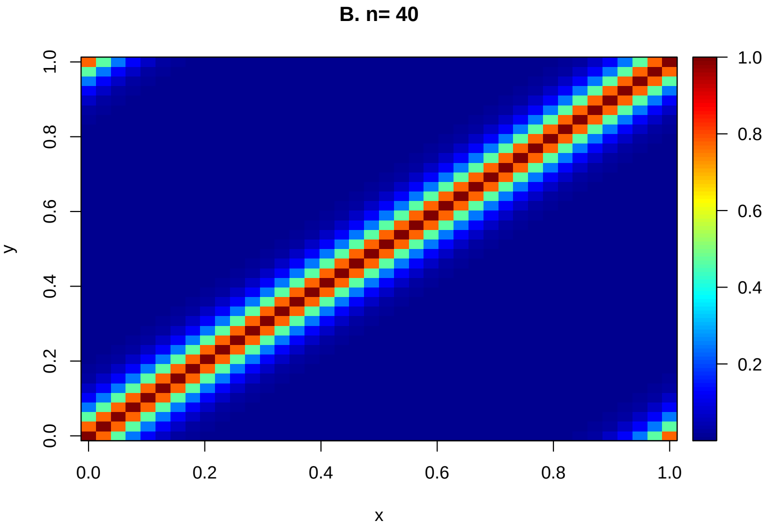
Possibility of an application with the LETKF

1. M. Kretschmer, B.R. Hunt, and E. Ott *(Data assimilation using a climatologically augmented local ensemble transform Kalman filter, Tellus A, 2015, v.67)* proposed to enrich the LETKF ensemble with the eigen-vectors of the **"climatological"** covariance matrix.

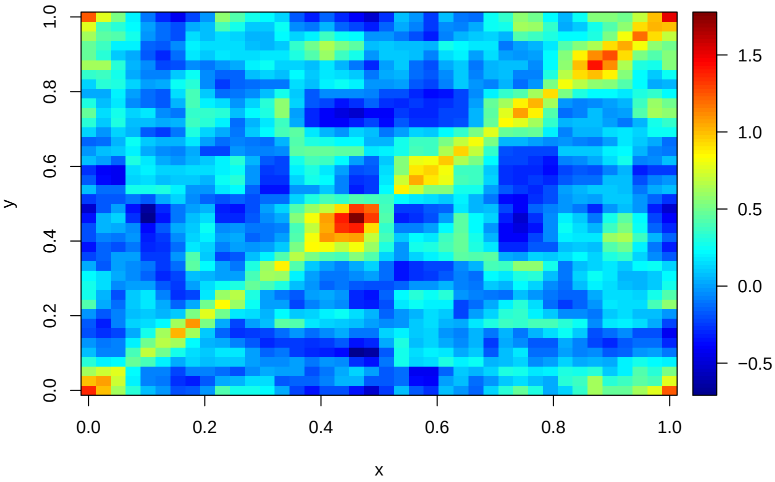
2. We may use this approach combined with the HBEF methodology through replacing the "climatological" covariances with the respective accumulated by the **secondary filter** (i.e. recent-past) ones.

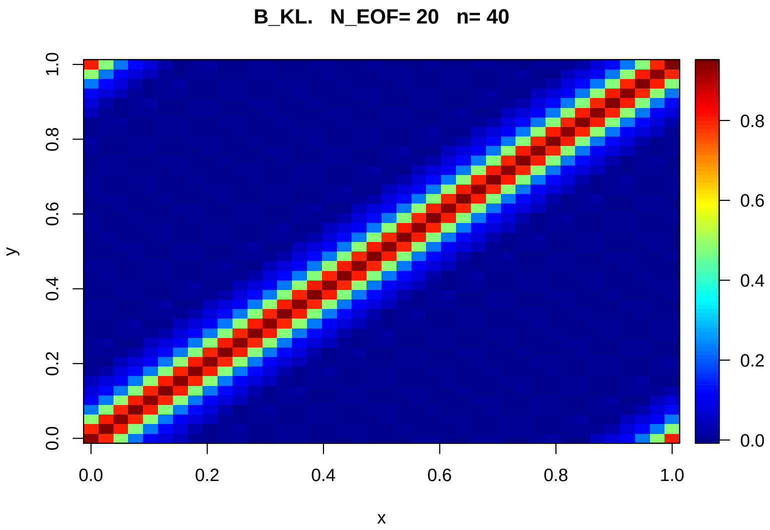
3. In the secondary filter, the covariances are propagated forward in time using persistence, possibly accompanied with (i) spatial smoothing and (ii) mixing with "climatological" covariances.

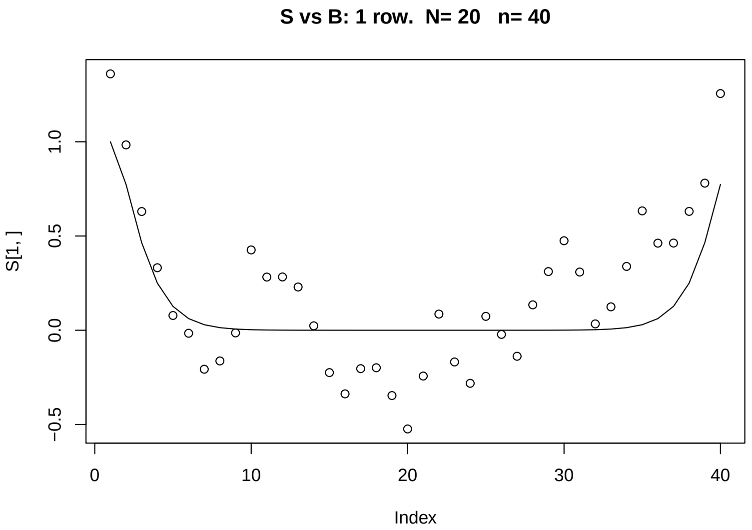
4. Handling the high-dimensional covariance matrices can be feasible if the covariances are defined on a coarse grid and are localized.

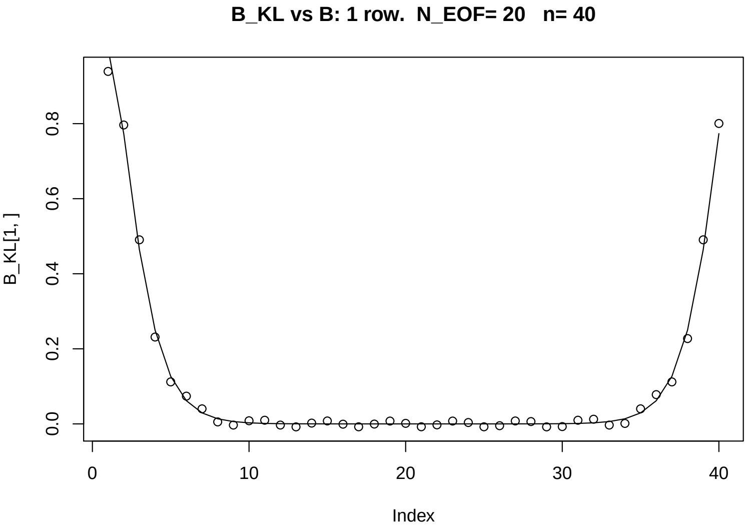

B. n= 40
1.0
0.8
0.6
0.4
0.2
0.0
y
0.0
0.2
0.4
0.6
0.8
1.0
x
1.0
0.8
0.6
0.4
0.2

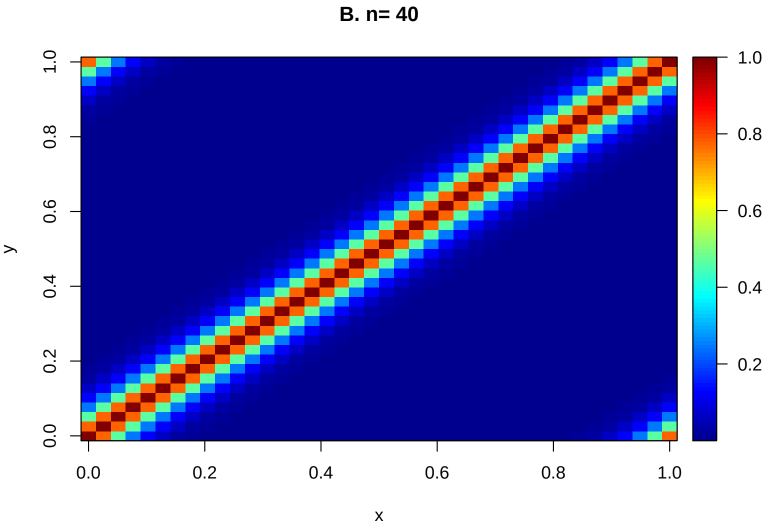
S. N= 20   n= 40

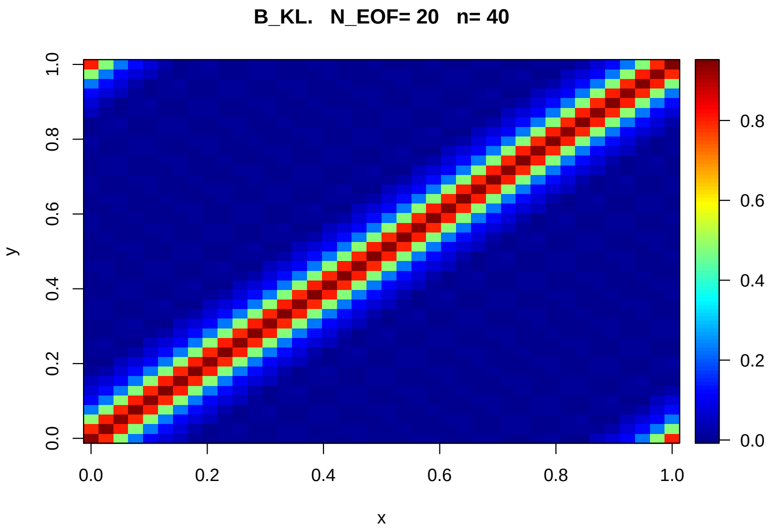


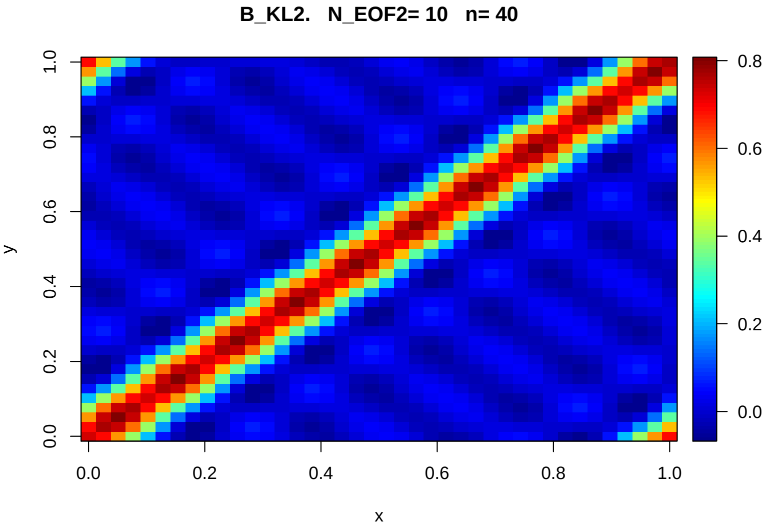

B_KL. N_EOF= 20 n= 40
y
1.0
0.8
0.6
0.4
0.2
0.0
x
0.0
0.2
0.4
0.6
0.8
1.0
0.8
0.6
0.4
0.2
0.0


S vs B: 1 row. N= 20 n= 40
S[1,]
−0.5
0.0
0.5
1.0
0
10
20
30
40
Index


B_KL vs B: 1 row. N_EOF= 20   n= 40
B_KL[1,]
0.0
0.2
0.4
0.6
0.8
0
10
20
30
40
Index


B. n= 40
y
x
0.0
0.2
0.4
0.6
0.8
1.0
0.2
0.4
0.6
0.8
1.0


B_KL. N_EOF= 20 n= 40
y
0.0
0.2
0.4
0.6
0.8
1.0
x
0.0
0.2
0.4
0.6
0.8
1.0
0.0
0.2
0.4
0.6
0.8


B_KL2. N_EOF2= 10 n= 40
y
1.0
0.8
0.6
0.4
0.2
0.0
x
0.0
0.2
0.4
0.6
0.8
1.0
0.8
0.6
0.4
0.2
0.0

Conclusions

1. The HBEF is successfully tested with a doubly stochastic advection-diffusion-decay model on the circle.
2. Cycling the covariances is shown to be beneficial.
3. The feedback from observations to the covariances is demonstrated to significantly improve the performance of the filter – if model error covariances are misspecified.
4. Using the HBEF's paradigm together with the LETKF looks possible.

A paper is in press: Tsyrulnikov M.D. and Rakitko A.S. A Hierarchical Bayes ensemble Kalman Filter. - Physica D (Nonlinear Phenomena), 2016, doi:10.1016/j.physd.2016.07.009.
The paper can be downloaded from arXiv or ReasearchGate.