A computationally efficient stochastic modelling of the cloud-base mass flux

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Outline

- Introduction: the Plant & Craig and Sakradzija & Seifert schemes
- Reduction of these schemes to efficient ones
- Summary and outlook

Plant-Craig and Sakradzija & Seifert Schemes

The usual assumption in the conventional convection parameterization – **convective equilibrium**:

the number of clouds in a grid box is always sufficient to represent an ensemble of clouds.

↔ the tendency due to convection = convective forcing

But: the grid box has a finite →
the number of clouds is also finite, the total mass flux can fluctuate.

Plant-Craig Scheme

The assumptions made in the Plant-Craig scheme

- scale separation
- a priori equal probabilities
- no interaction between clouds
- the mean mass flux per cloud is independent on the large-scale situation

Clouds appear according to a Poisson process (with the parameter λ dependent on the large-scale situation).

Then

the distribution of the mass fluxes of new-born clouds
as well as instantaneous distribution of the mass fluxes
is exponential.

The Problem

To track each cloud is expensive.

The information about each cloud or their distribution is not needed.

Only the total mass flux M (and, probably, total number of clouds N) is needed.

The question:

is it possible to average, or coarse-grain, the Plant & Craig and Sakradzija & Seifert schemes, so that a reduced model consists of only 2 stochastic equations for N und M?

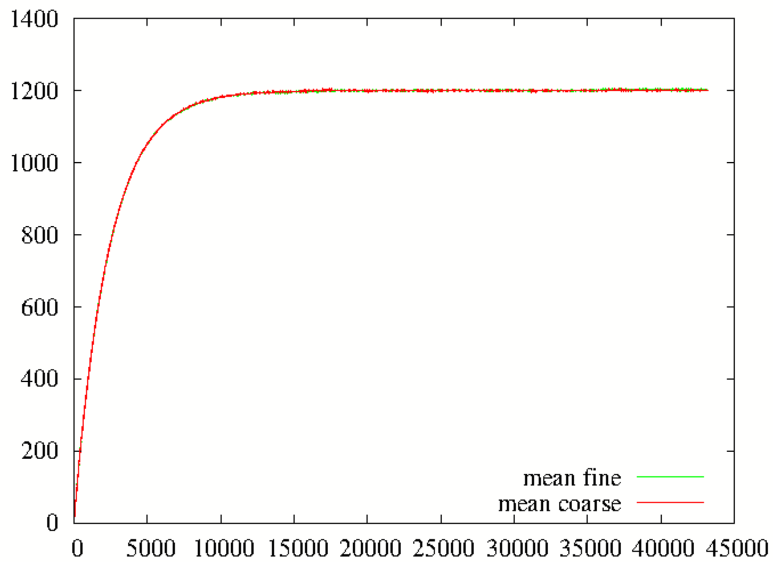
Solution: the Simplest Case

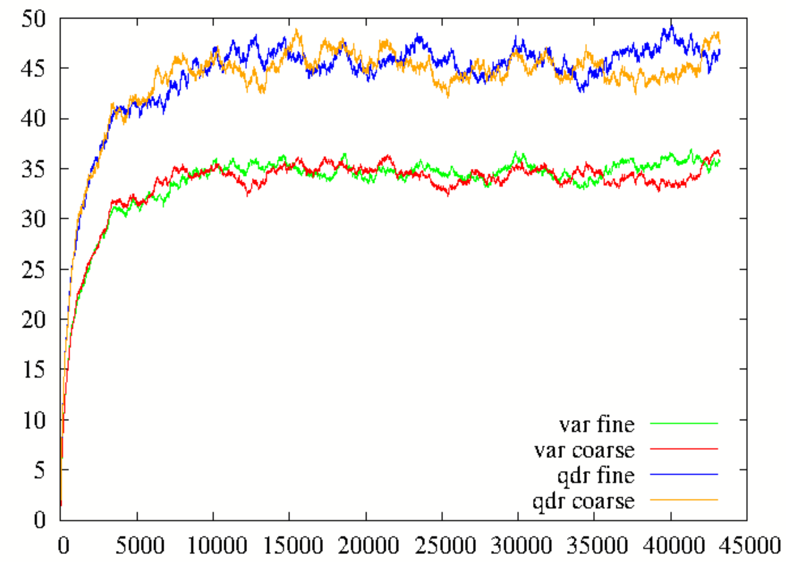
1. The mass flux of all the clouds is the same ($= \langle m_b \rangle = m$),

The cloud lifetime does not depend on the cloud mass flux and is distributed exponentially (with the mean $= \langle \tau \rangle$).

$$\frac{\partial N}{\partial t} = B - D, \qquad B \sim Poisson(\lambda), \qquad D \sim Poisson\left(\frac{N}{\langle \tau \rangle}\right)$$

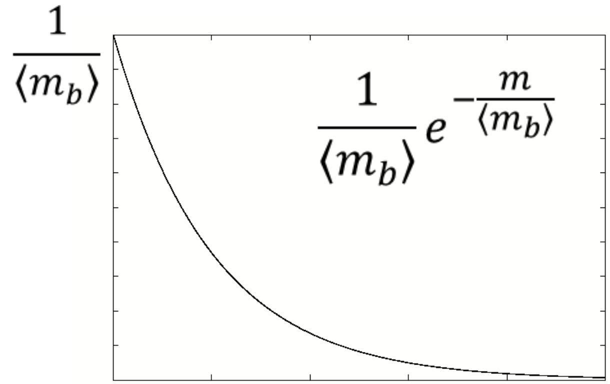
$$M = mN$$





Solution: Intermediate Case (≈Plant-Craig scheme)

2. The mass flux of the newly generated clouds is distributed exponentially (with the parameter $\frac{1}{\langle m_b \rangle}$, *i.e.*, mean = $\langle m_b \rangle$),



the cloud lifetime does not depend on the cloud mass flux and is distributed exponentially.

$$\frac{\partial N}{\partial t} = B - D, \qquad B \sim Poisson(\lambda), \qquad D \sim \ Poisson\left(\frac{N}{\langle \tau \rangle}\right)$$

$$\frac{\partial M}{\partial t} = S_+ - S_-,$$

$$\Pr(S_+ = \Delta M) = \Pr(a\ cloud\ emerges) \cdot \Pr(it\ has\ \Delta M) = \lambda \cdot \frac{1}{\langle m_b \rangle} e^{-\frac{\Delta M}{\langle m_b \rangle}},$$

$$\Pr(S_- = \Delta M) = \frac{instantaneous\ distribution\ p(\Delta M)}{\langle \tau \rangle} = \frac{1}{\langle \tau \rangle} \frac{1}{\langle m_b \rangle} e^{-\frac{\Delta M}{\langle m_b \rangle}}$$

independent

Solution: Advanced Case

3. The mass flux of the newly generated clouds is distributed exponentially (with the parameter $\frac{1}{\langle m_b \rangle}$, *i.e.*, mean = $\langle m_b \rangle$),
the cloud lifetime **does** depend on the cloud mass flux:

$$\tau = \alpha m^{\beta}$$

$$\frac{\partial N}{\partial t} = B - D, \qquad B \sim Poisson(\lambda) \qquad D \sim \ Poisson\left(\frac{N}{instantaneous\ \langle \tau \rangle}\right)$$

$$\frac{\partial M}{\partial t} = S_{+} - S_{-},$$

$$\Pr(S_{+} = \Delta M) = \Pr(a\ cloud\ emerges) \cdot \Pr(it\ has\ \Delta M) = \lambda \cdot \frac{1}{\langle m_b \rangle} e^{-\frac{\Delta M}{\langle m_b \rangle}},$$

$$\Pr(S_{-} = \Delta M) = \frac{instantaneous\ p(\Delta M)}{\tau_{\Delta M}} = \frac{p(\Delta M)}{\alpha (\Delta M)^{\beta}}$$

Solution: Advanced Case

How to find the actual instantaneous distribution $p(m) = \frac{n(m)}{N}$?

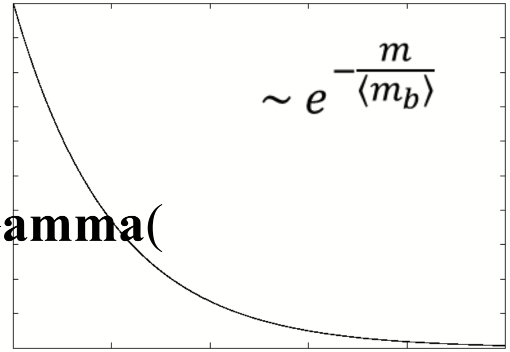
Stationary and averaged $\bar{p}(m)$ can be found from the prognostic equation setting the time rate-of-change to 0

$$\frac{\partial p(m)}{\partial t} = \frac{\partial}{\partial t}\left(\frac{n(m)}{N}\right) = 0,$$

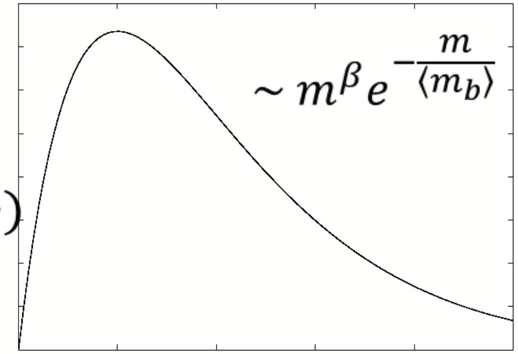
$$\frac{d\bar{N}}{dt} = 0 \implies \frac{\partial}{\partial t}\left(\frac{\bar{n}(m)}{\bar{N}}\right) = \frac{1}{\bar{N}}\frac{\partial \bar{n}(m)}{\partial t} = 0$$

$$\frac{\partial \bar{n}(m)}{\partial t} = \frac{\lambda}{\langle m_b \rangle} e^{-\frac{m}{\langle m_b \rangle}} - \frac{\bar{n}(m)}{\alpha m^{\beta}} = 0 \implies \bar{n}(m) = \frac{\lambda \alpha}{\langle m_b \rangle} m^{\beta} e^{-\frac{m}{\langle m_b \rangle}}$$

Averaged Generation rate: **exponential** Gamma(



$\implies$ Averaged **instantaneous** distribution: **Gamma**$(\beta + 1, \langle m_b \rangle)$



Solution: Advanced Case

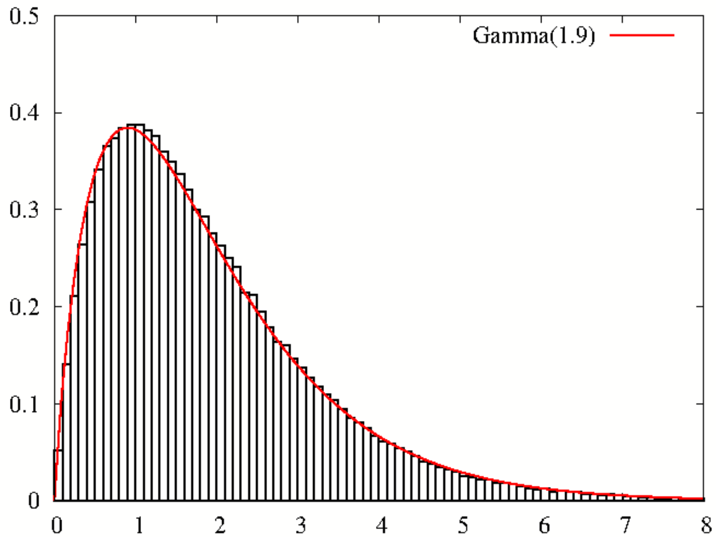
How to find the actual instantaneous distribution $p(m)$? (Continued.)

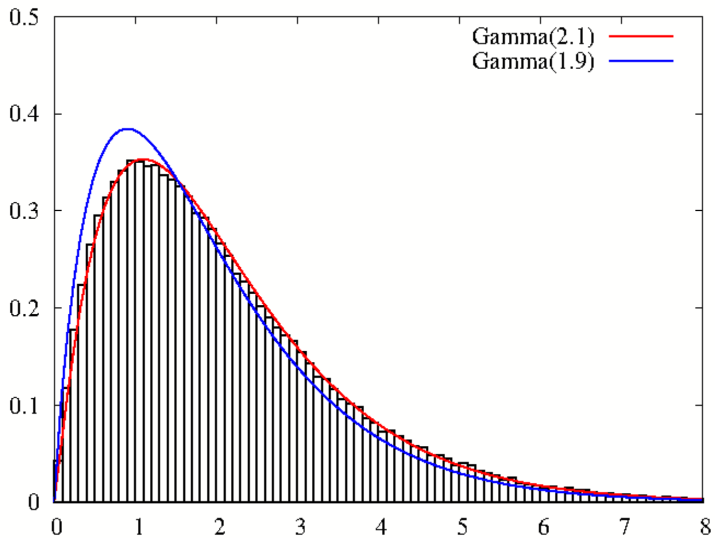
Stationary and averaged $\bar{p}(m) = \frac{1}{\langle m_b \rangle^{\beta+1}\Gamma(\beta+1)} m^{\beta} e^{-\frac{m}{\langle m_b \rangle}}$

The mean of this distribution (mean mass flux per cloud)

$$\langle \bar{m} \rangle = \langle m_b \rangle(\beta + 1) = \frac{\bar{M}}{\bar{N}}$$

Approximate $p(m)$ with the same form as $\bar{p}(m)$ (i.e., gamma distribution), but with the actual parameter $\frac{M}{N}$ instead of the averaged $\frac{\bar{M}}{\bar{N}}$





Solution: Advanced Case

Then the approximated instantaneous distribution

$p(m) \cong \frac{1}{\langle m_b \rangle^a \Gamma(k)} m^{a-1} e^{-\frac{m}{\langle m_b \rangle}}$ (Gamma with the parameters $a, \langle m_b \rangle$.)

where $a = \frac{1}{\langle m_b \rangle} \frac{M}{N}$ – normalized actual mean mass flux per cloud.

Clouds ready for extinction:

$p(m)$ should be weighted with respective lifetimes

$$p_d(m) = \frac{p(m)}{\alpha m^\beta} \sim m^{a-\beta-1} e^{-\frac{m}{\langle m_b \rangle}}$$

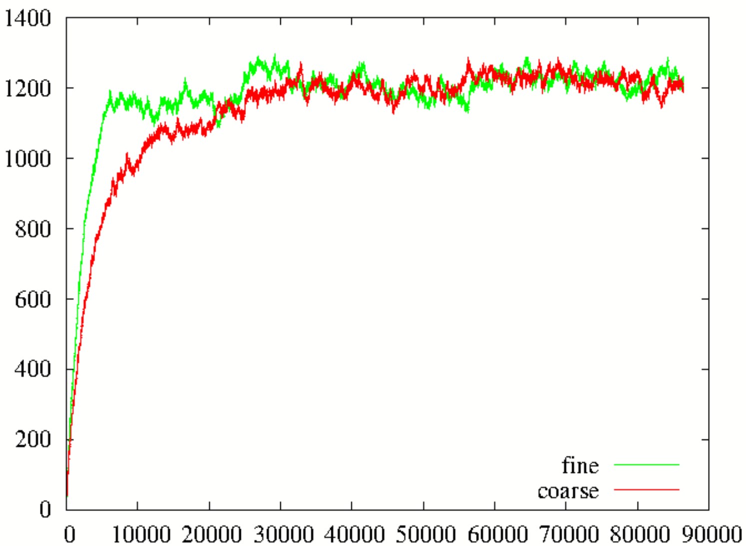
The mean actual lifetime (in the extinction rate $D \sim \ Poisson\left(\frac{N}{actual \ \langle \tau \rangle}\right)$)

$$\langle \tau \rangle = \int_0^\infty \tau(m) p_d(m) dm = \alpha \langle m_b \rangle^\beta \frac{\Gamma(a)}{\Gamma(a-\beta)}$$

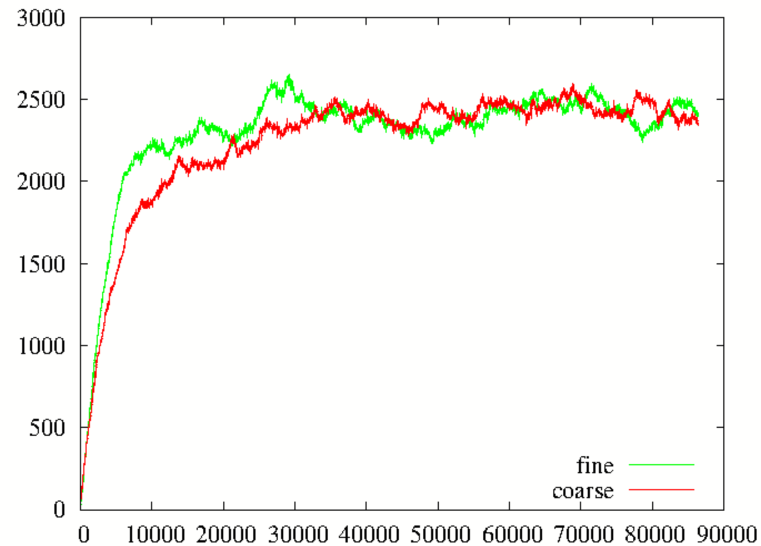
Solution: Advanced Case

Examplesofrealizations Examples of realizations

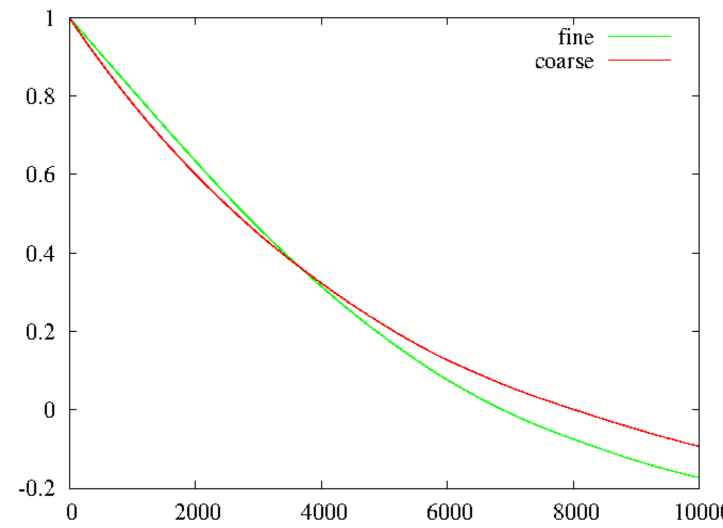
N



M

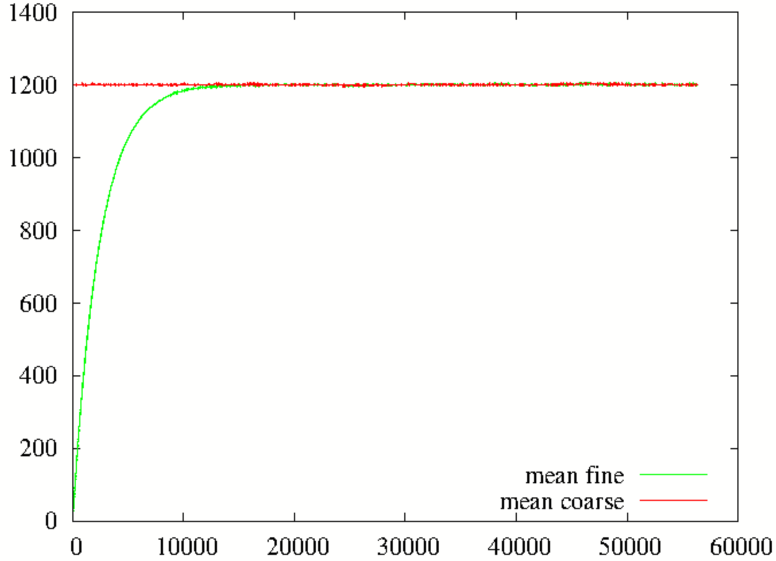


Autocorrelation function for M Autocorrelationfunctionfor

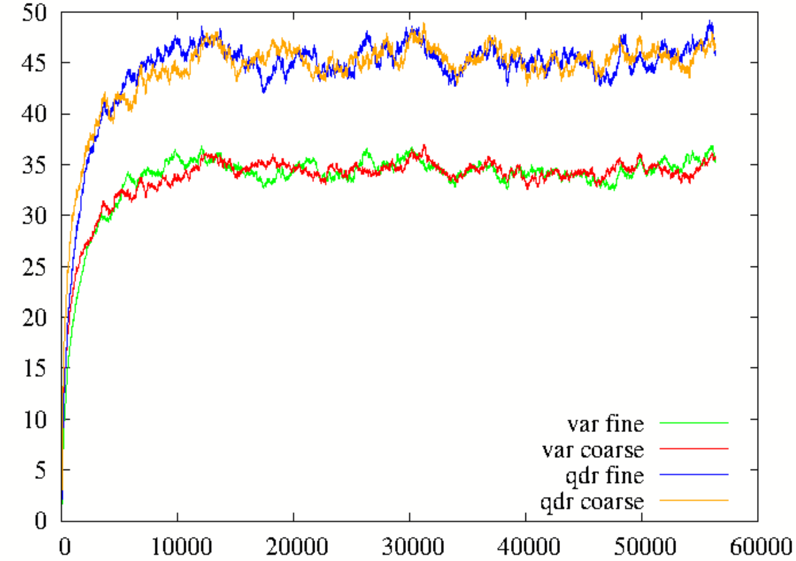


Solution: Advanced Case

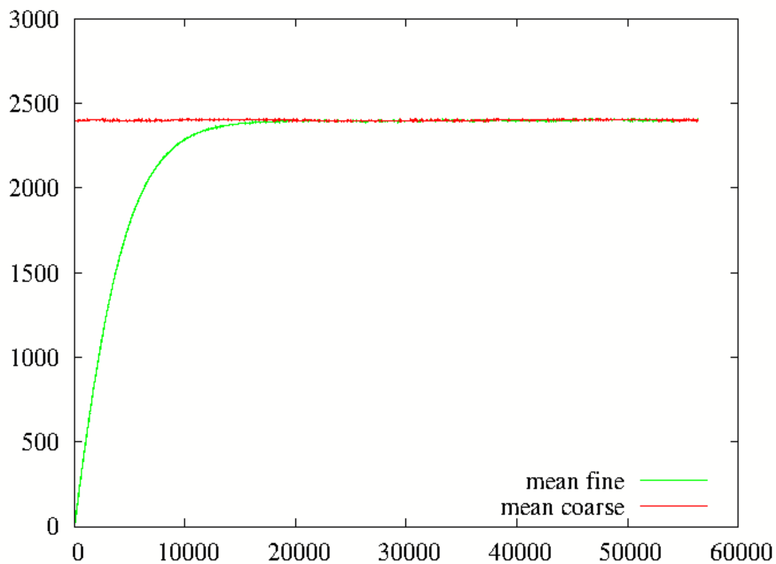
mean N



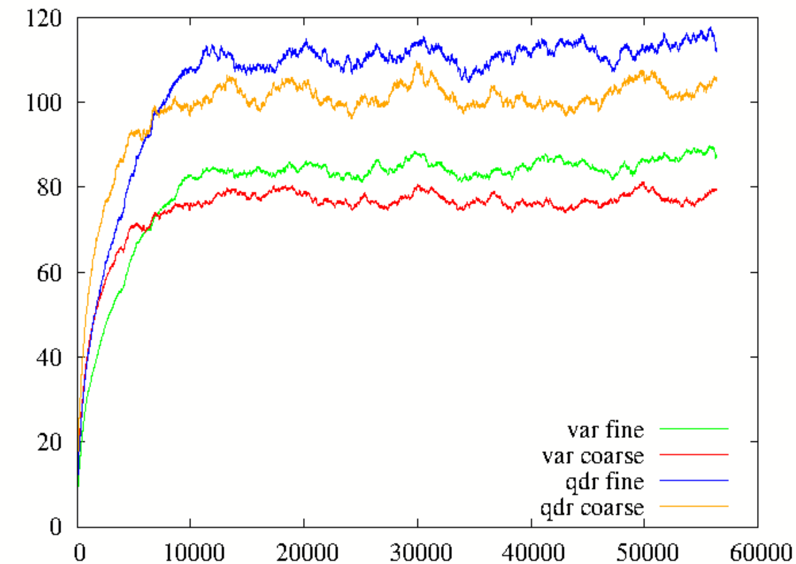
statistics of N



mean M



statistics of M



Solution: the General Recipe

The total number of clouds *N*

$$\frac{\partial N}{\partial t} = B - D, \qquad B \sim Poisson(\lambda) \qquad D \sim Poisson\left(\frac{N}{actual\ \langle\tau\rangle}\right)$$

The total mass flux *M*

Gain: if ΔN_b clouds are new born →

sample their mass fluxes from the distribution for generation (exponential in the PC scheme, Weibull in the SS scheme), sum together

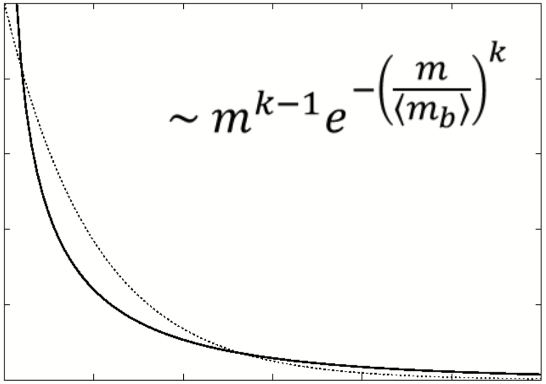
Loss:

- determine the actual $p(m)$ – use the first-order approximation (the shape of $p(m)$ is the same as for $\bar{p}(m)$, the parameters correspond to the actual N and M)
- determine the distribution $p_d(m)$ of mass fluxes at extinction – weight $p(m)$ with the dependence of lifetime on mass flux
- determine the mean lifetime $\langle\tau\rangle$ of the clouds at extinction → death rate D, ΔN_d
- sample their mass fluxes from $p_d(m)$, sum together

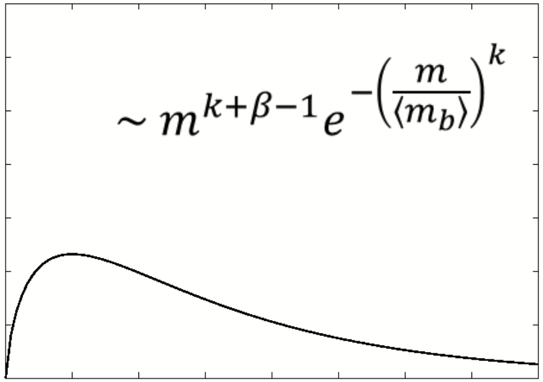
Solution: Even More Advanced Case (the scheme of Sakradzija & Seifert)

Apply the general recipe if the distribution for the cloud generation is Weibull

Generation rate: **Weibull**



Averaged instantaneous distribution: **Generalized Gamma**



$$\langle m \rangle = \frac{\overline{M}}{\overline{N}} = \langle m_b \rangle \frac{\Gamma\left(1+\frac{\beta+1}{k}\right)}{\Gamma\left(1+\frac{\beta}{k}\right)}$$

First-order approximation of the actual distribution

$$p(m) \sim m^{k+a-1} e^{-\frac{m}{\langle m_b \rangle}}$$

where a has to be determined from $\langle m \rangle = \frac{M}{N} = \langle m_b \rangle \frac{\Gamma\left(1+\frac{a+1}{k}\right)}{\Gamma\left(1+\frac{a}{k}\right)}$

Summary and Outlook

- It seems to be possible to replace the expensive "cloud tracking" model of Plant&Craig and Sakradzija&Seifert with two stochastic differential equations for M and N only
- The proposed general strategy and the first results look reasonably, although

- Further work on the formulation is needed
- When ready, the scheme can be coupled to a mass-flux or a subgrid-scale cloud scheme and tested in the COSMO model

Thank you for your attention!

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