New explicit sedimentation scheme for the 2-moment scheme

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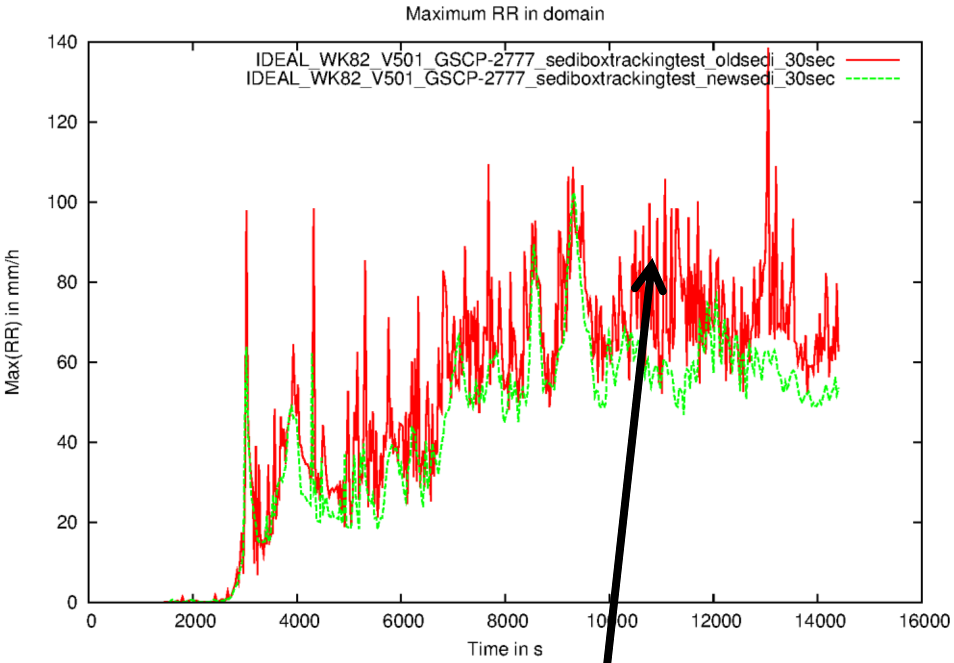
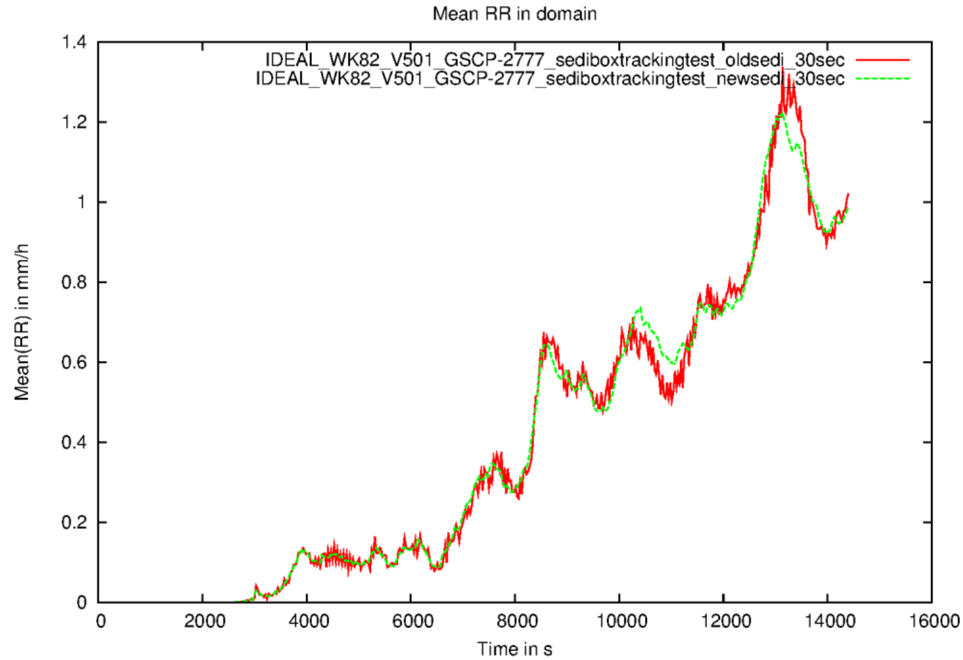
Motivation

➔ Weisman-Klemp supercell simulation (2-mom)





Rain rate spikes

Current situation

- Explicit first order flux form advection scheme
- In principle, Courant number independent; in practice up to CFL ≈ 4
- Scheme is very diffusive, which „helps“ somehow the physics
- Problem: By design, the lower „front“ of precipitation cannot move more than 1 vertical level per timestep. Close to the ground, where the vertical resolution is high, this can lead to spurious drastic overshoots and corresponding unrealistic very high temporal peaks in the precipitation rate. The problem is, e.g., visible in our COSMO-DE level configuration with time steps of 20 -30 s. It vanishes for time steps < 10 s.
- Possible mitigations:
  - smaller time step for sedimentation → **very expensive**
  - re-formulation of the sedimentation scheme → ☑

General scheme

Deutscher Wetterdienst
Wetter und Klima aus einer Hand

- Sedimenation equation:

$$\frac{\partial \rho q}{\partial t} = -\frac{\partial}{\partial z}\left[v_{(\rho q)}\, \rho q\right]$$

- Integration over finite time step:

$$\varphi_{(t_{i+1})} = \varphi_{(t_i)} - \frac{\partial}{\partial z}\left[\int_{t_i}^{t_{i+1}} v_{(\varphi_{(t)})}\, \varphi(t)\, dt\right] = \varphi_{(t_i)} - \frac{\partial}{\partial z}\left[\, \overline{P}\Delta t\, \right]$$

- Centered difference discretization:

$$\varphi_k^{(i+1)} = \varphi_k^{(i)} - \frac{\left(\overline{P}_{k-1/2} - \overline{P}_{k+1/2}\right)\, \Delta t}{\Delta z_k}$$

- This formulation is for 1 moment (ρq), but applies without loss of generality to an arbitrary number of moments. Just add one similar equation per additional moment, and let v depend on $(\rho q)_1$, $(\rho q)_2$, …

General scheme

➔ Time integrated flux:

$$\overline{P}_{(z_f)}\Delta t = \int_{t_i}^{t_{i+1}} v_{(\varphi_{(z_f,t)})}\, \varphi_{(z,t)}\, dt \approx$$

„time frozen“
v, φ from time t_i

$$\int_{t_i}^{t_{i+1}} v_{(\varphi_{(z_f+|v_{(\varphi_{(z,t_i)})}|(t-t_i),t_i)})}\, \varphi_{(z_f+|v_{(\varphi_{(z,t_i)})}|(t-t_i),t_i)}\, dt =$$

$$\int_{t_i}^{t_{i+1}} \int_{z_f}^{\infty} v_{(\varphi_{(z,t_i)})}\, \varphi_{(z,t_i)}\, \delta_{[z-z_f-|v_{(\varphi_{(z,t_i)})}|(t-t_i)]}\, dz\, dt$$

expand integrand by help of a δ-function to connect time $(t-t_i)$ with fall distance $v(t-t_i)$, from which φ contributions start to fall to reach z_f at time t:

General scheme

- The variable transformation $\zeta = z_f + |v_{(\varphi_{(z,t_i)})}|(t - t_i) \qquad d\zeta = |v|\, dt$

  $t = t_i \rightarrow \zeta = z_f \qquad t = t_{i+1} \rightarrow \zeta = z_f + |v|\Delta t$

- and a change of integration order leads finally to:

$$\overline{P}_{(z_f)}\Delta t \approx$$

$$-\int_{z_f}^{\infty}\int_{z_f}^{z_f+v\Delta t} \varphi_{(z,t_i)}\, \delta_{(z-\zeta)}\, d\zeta\, dz = -\int_{z_f}^{\infty} \varphi_{(z,t_i)} \int_{z_f}^{z_f+v\Delta t} \delta_{(z-\zeta)}\, d\zeta\, dz =$$

$$-\int_{z_f}^{\infty} \varphi_{(z,t_i)}\, F_{(z,v_{(z,t_i)})}\, dz$$

with a masking function F defined as

$$F = \begin{cases} 1 & \text{for } 0 < z - z_f < |v_{(z,t_i)}|\Delta t \\ 0 & \text{else} \quad . \end{cases}$$

General scheme

- Generally: explicit first order upwind scheme, where v(z) is assumed to be constant within grid boxes and is taken as the velocity at the box center:

$$v_{k+1/2}^{(i)} = v_k^{(i)} \;=\; \mathrm{fct}(\varphi_k^{(i)})$$

Current scheme

- v(z) is approximated as $v_{k+1/2}$ for all heights above $z_{k+1/2}$. The masking function F simplifies to a boxcar function in the interval [$z_{k+1/2}$, $z_{k+1/2}$ + $v_{k+1/2}$ Δt], and the time averaged flux becomes:

$$\overline{P}_{k+1/2}\Delta t \approx - \int_{z_{k+1/2}}^{z_{k+1/2}+v_{k+1/2}\Delta t} \varphi_{(z)}\, dz \approx - \left[\sum_{l=0}^{N-1} \varphi_{k-l}\Delta z_{k-l} + \varphi_{k-N}\left(z_o - z_{k+1/2-N}\right) \right]$$

- N = number of height levels with cell faces entirely within the maximum transport range of $z_{k+1/2}$ + |$v_{k+1/2}$| Δt
- An efficient computation of the above sum is implemented in the code
- To use this scheme, during compilation do not use a special preproc. flag or set –DSEDI_VECTORIZED (vector implementation for the NEC SX)

New scheme

- The new scheme directly discretizes the time-averaged flux without approximation of v(z) by $v_{k+1/2}$:

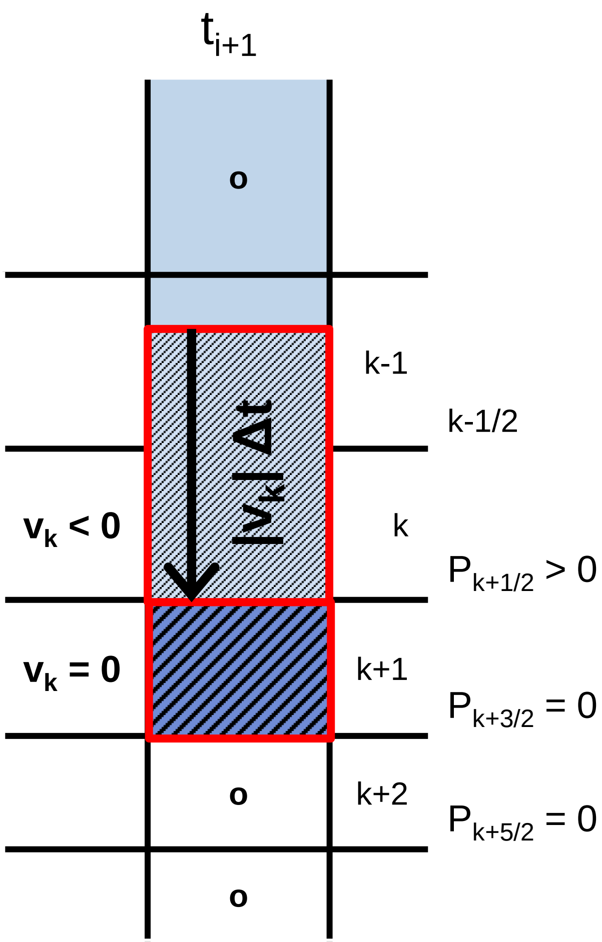
$$\overline{P}_{k+1/2}\Delta t \approx -\sum_{l=0}^{k-1} \varphi_{k-l} \left(z_{up} - z_{low} \right)$$
$$z_{up} = \min \left[z_{k+1/2-l-1} \; , \; z_{k+1/2} + |v_{k-l}|\Delta t \right]$$
$$z_{low} = \min \left[z_{k+1/2-l} \; , \; z_{up} \right] \quad .$$

- Note: loop runs from level k to the model top for each grid point (costly!)
- However, reorganizing the computations in a clever way and avoiding „voids“ leads to nearly the same efficiency as in the current scheme. In algorithmic notation:

$\overline{P}_{1/2:k_{end}+1/2} = 0$;
do $k = 1, k_{end}$
  $l = 0$;
  $\Delta_{sum} = 0$;
  while $\Delta_{sum} < |v_k|\Delta t$ **and** $k + l \leq k_{end}$ **do**
    $z_{low} = 0$;
    $z_{up} = \min\left[\Delta z_k \; , \; -\Delta_{sum} + |v_k|\Delta t\right]$;
    $\overline{P}_{k+1/2+l}\,\Delta t \; = \; \overline{P}_{k+1/2+l}\,\Delta t \; - \; \varphi_k \left(z_{up} - z_{low}\right)$;
    $l = l + 1$;
    $\Delta_{sum} = \Delta_{sum} + \Delta z_{k+l}$;
  end while
end for

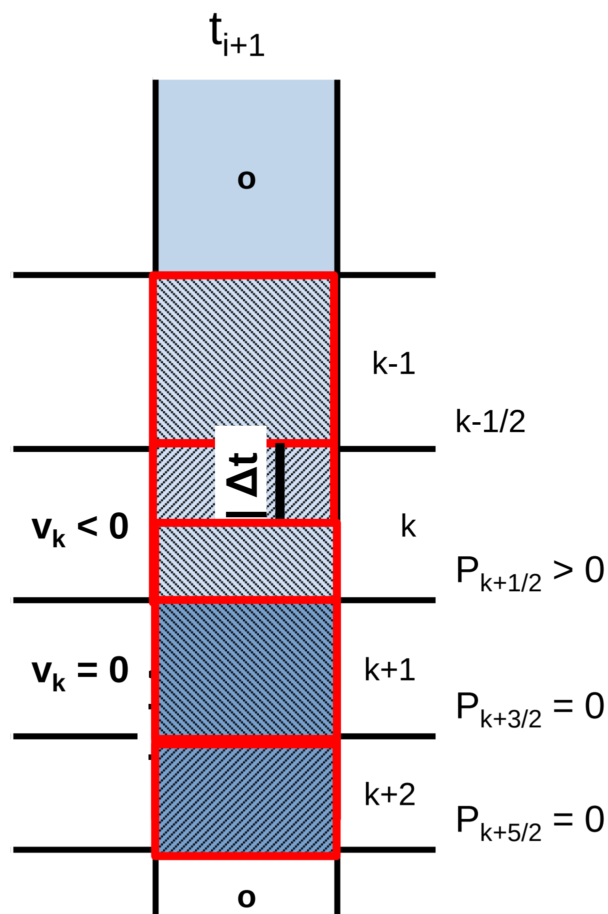
Comparison

Current method



Block „squeezed“ in box k+1 (self amplifying process)

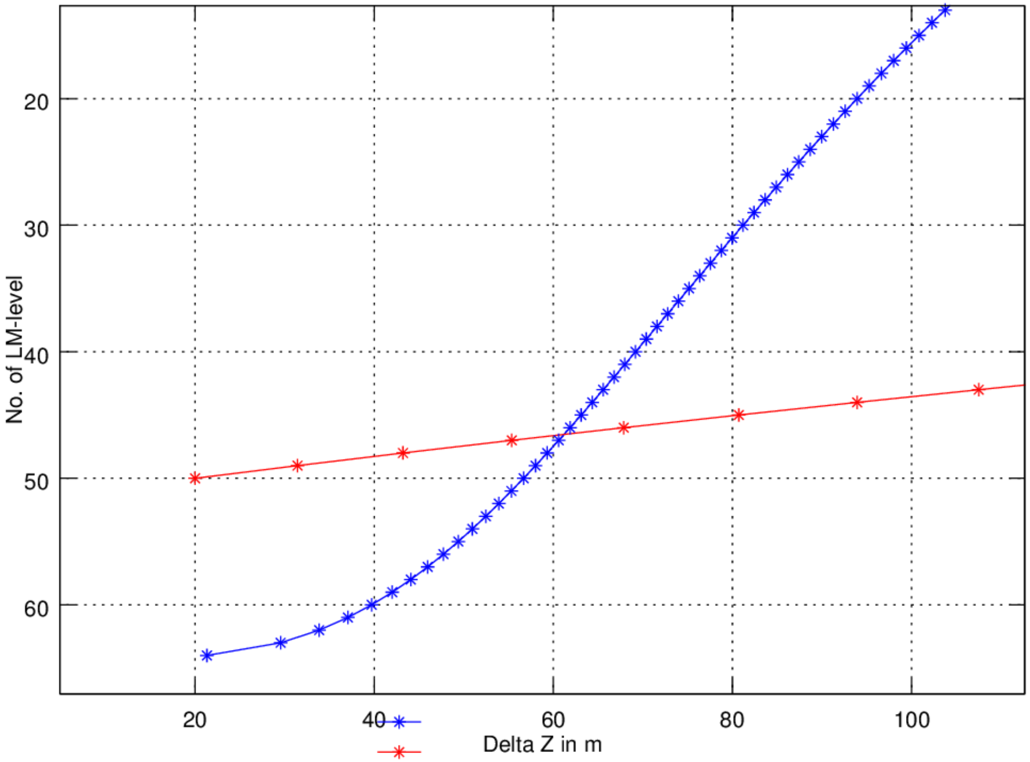
New method



Each box moving at its own speed and contributes to P at all levels which it reaches/traverses during Δt

Comparison: idealized „boxcar“ test

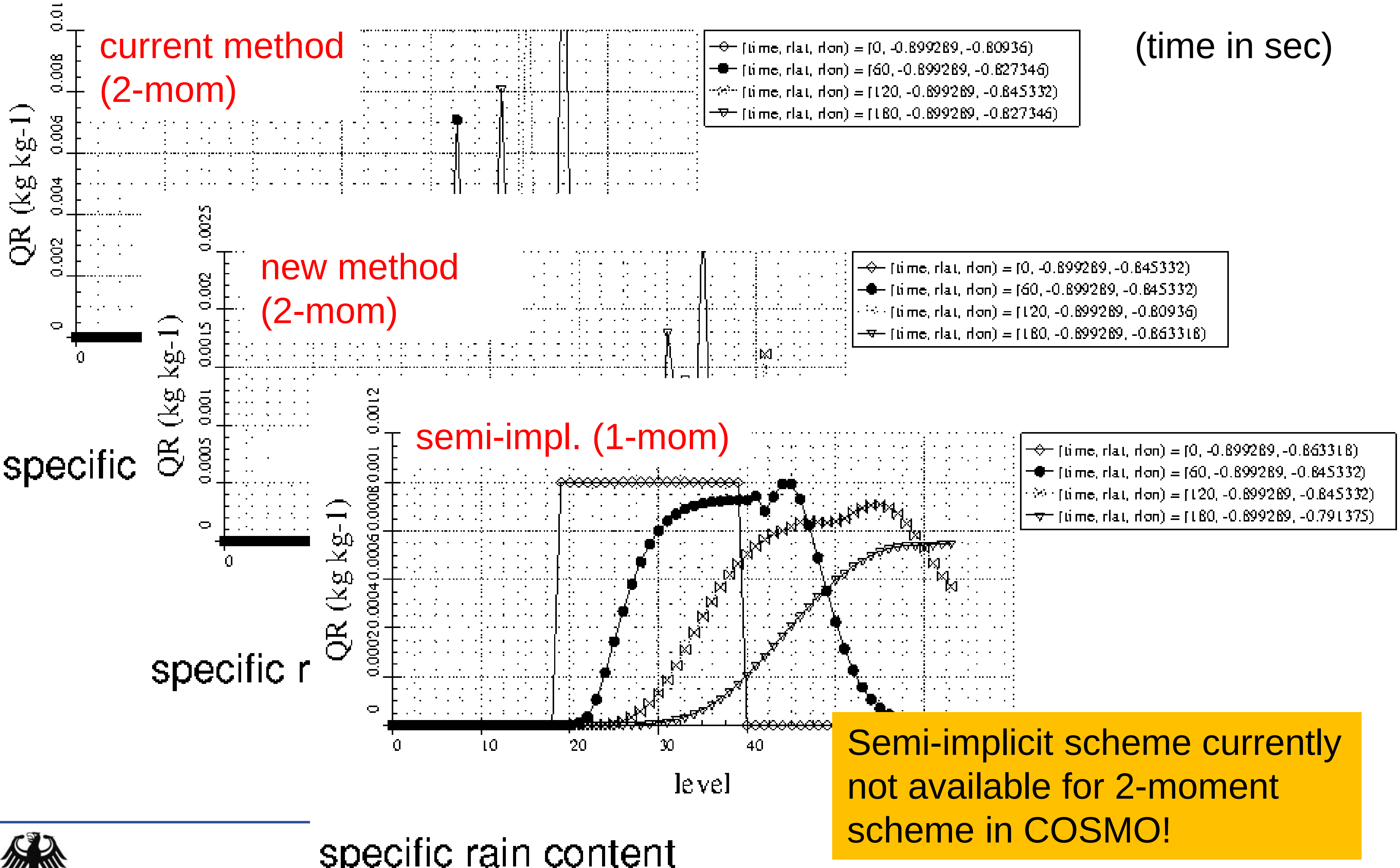
➔ Choice of model levels **(blue)** and COSMO-DE standard levels (red):



➔ extreme test because of very fine vertical resolution!

Comparison: idealized „boxcar“ test

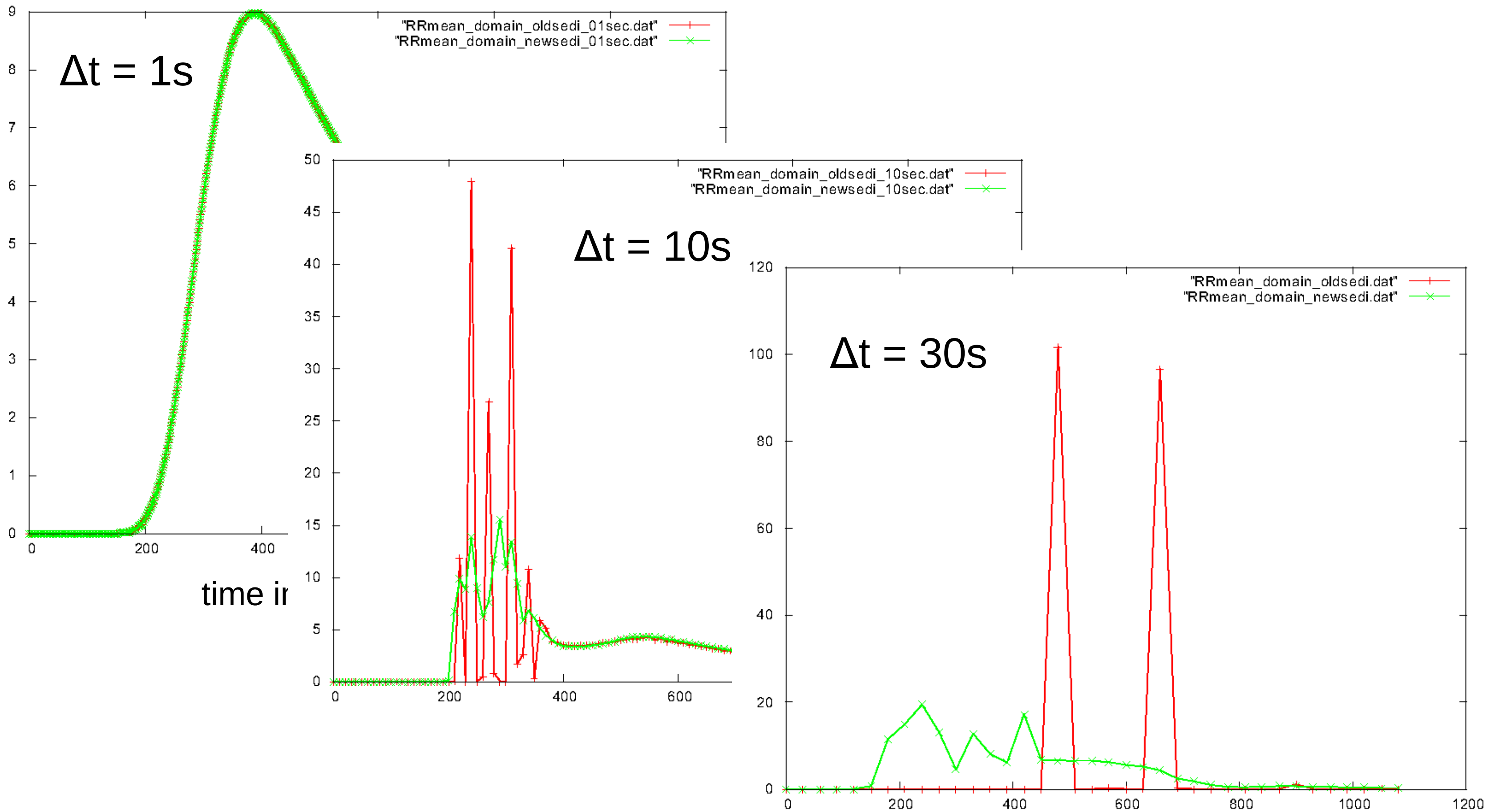
(time in sec)



Semi-implicit scheme currently not available for 2-moment scheme in COSMO!

Comparison: idealized „boxcar“ test

➔ Time series of surface rainrate for different time steps (2-mom):



Comparison: idealized „boxcar“ test

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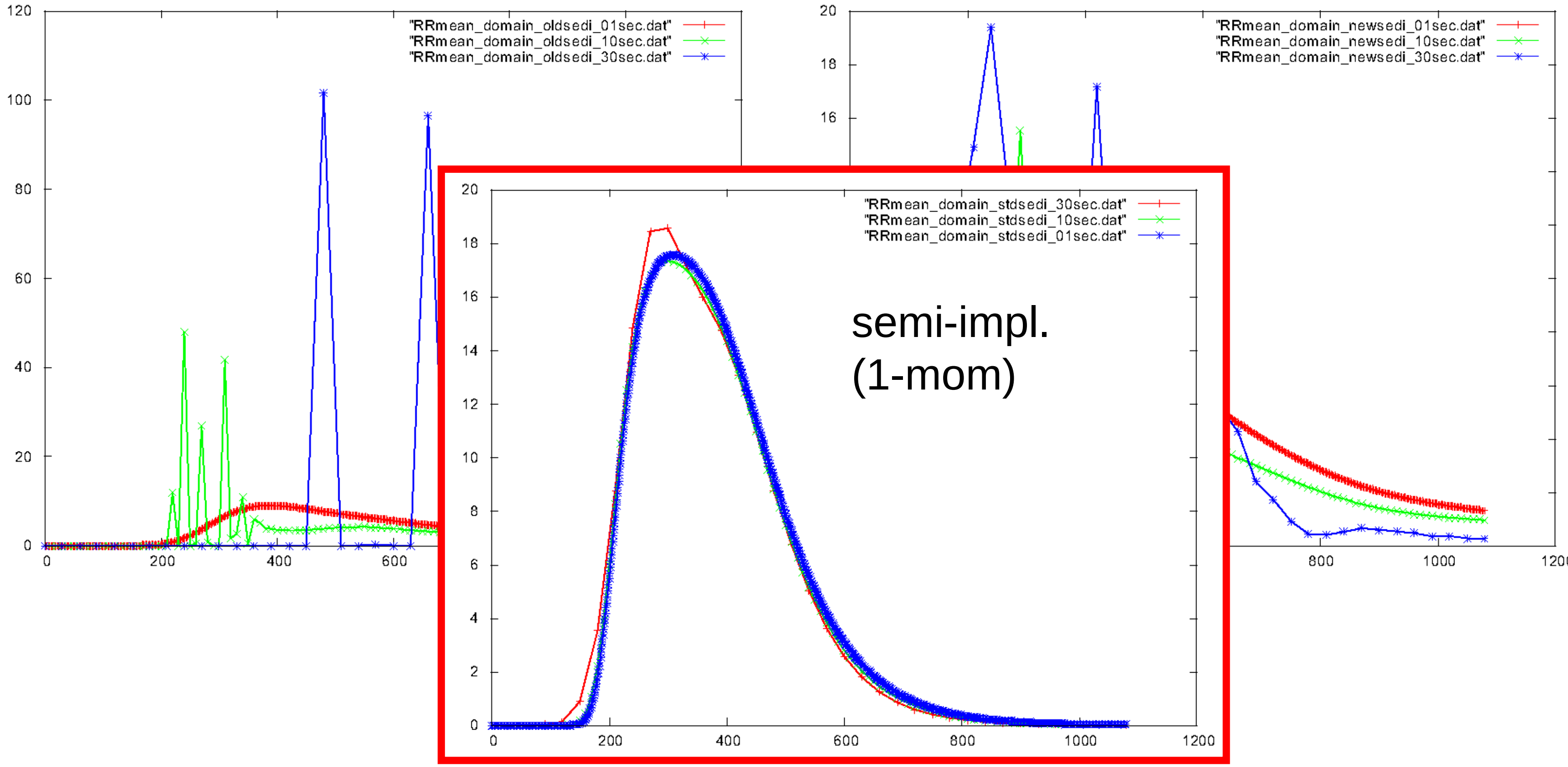
➔ Same curves but re-grouped according to the numerical scheme:

Current | New



Comparison: idealized 3D supercell

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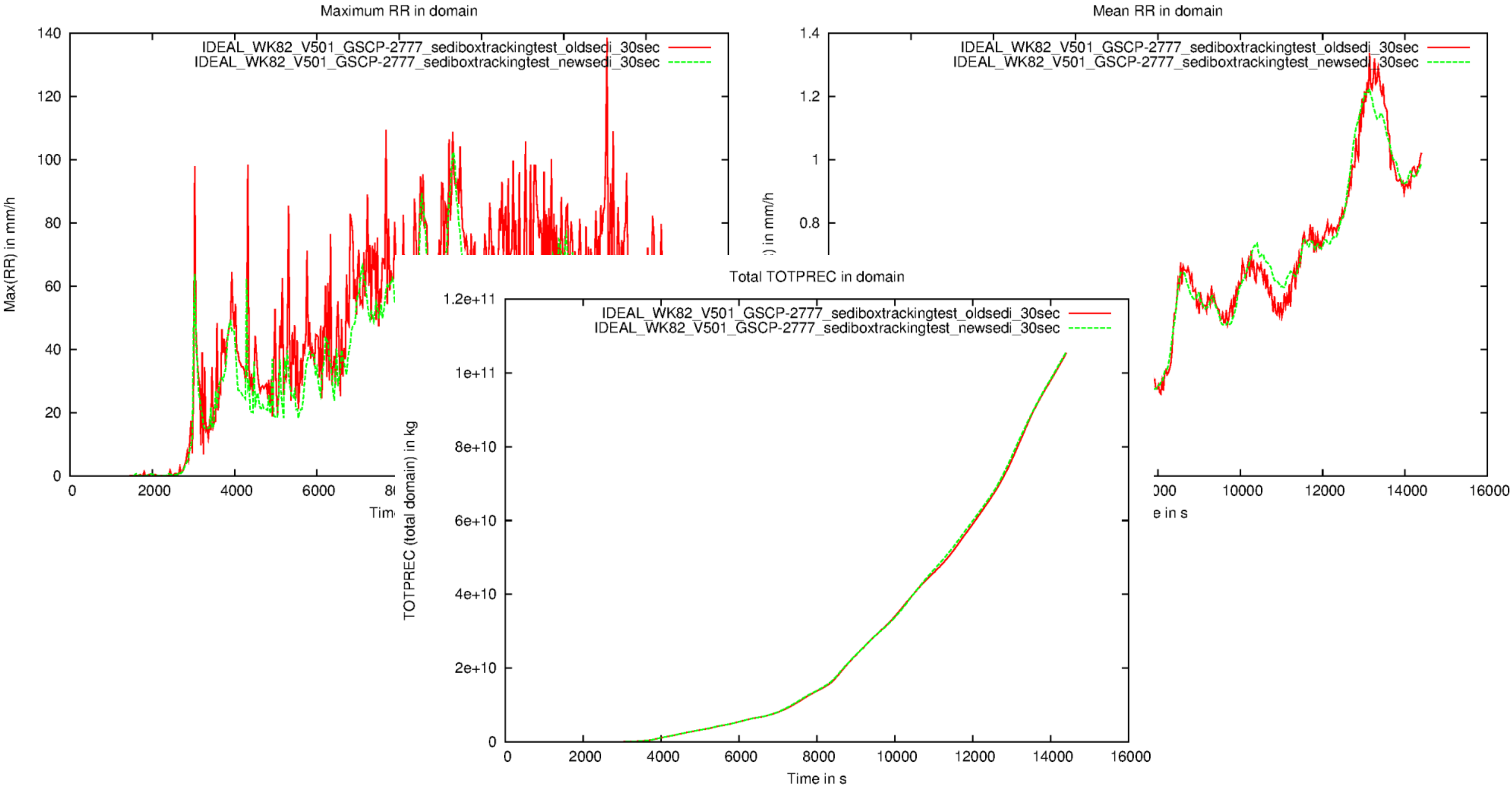
➔ Weisman-Klemp supercell simulation (2-mom)



Summary

The comparison of the current and new explicit scheme shows:

- ➔ Clear advantage of the new explicit method. Mitigates the problem of spurious strong peaks in precipitation rate and in the vertical profiles of hydrometeors.
- ➔ Computationally as efficient as the current method.
- ➔ However, semi-implicit scheme would probably be better, but this is currently not available in COSMO 2-moment scheme. Axel‘s new ICON version? Probably also more costly due to double evaluation of the scheme per time step?
- ➔ Documentation available in the COSMO source code distribution:

  LOCAL/TWOMOM/docu_sedi_twomom.pdf

  or from the author.