Stochastic Pattern Generator for use in ensemble assimilation and forecasting

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Pattern Generator (PG) Setup

- Goal
  The PG is designed to be used in a *model-error generator* in the COSMO model — both in the additive model-error and multiplicative model-error models (or mixed).
- Requirements
  1. The PG should produce (on-line) 4-D homogeneous pseudo-random spatio-temporal fields with tunable variance and spatial and temporal length scales.
  2. The pseudo-random realizations should be reproducible and the PG should be fast enough.
  3. The spatio-temporal interactions should be 'meaningful'.

BPG: the proposed solution

Approach: Generated field is a numerical solution of a stochastic partial differential equation.

The basic equation

$$\left(\frac{\partial}{\partial t} + A\right)^p \xi = \sigma\alpha, \tag{1}$$

where A is a linear operator. The evolution domain of ξ is 3D-Torus $S_1 \times S_1 \times S_1$.

We use a less general (but still flexible) form of (1)

$$\left(\frac{\partial}{\partial t} + \mu(1 - \lambda^2\Delta)^q\right)^p \xi = \sigma\alpha. \tag{2}$$

Solving the equation

Fourier decomposition

We consuder the Fourier decomposition of ξ

$$\xi(x, y, z, t) = \sum_{m,n,l} \xi_{nml}(t) e^{i(xm+yn+zm)} \tag{3}$$

We note that complex exponents are the eigenfunctions of the Laplacian operator.

Integration in spectral space

So, the big equation (1) is splited into many ordinary differential equations: one for each wavevector (m, n, l). They have the form

$$\left(\frac{d}{dt} + \mu(1 + \lambda^2 \left(m^2 + n^2 + l^2 \right) \right)^q \Bigg)^p \xi_{nml} = \sigma\alpha, \tag{4}$$

Choice of p and q

Choice of q

Consider the elementary random process ξ_{nml}. The characteristic time scale of ξ_{nml} is

$$T = \frac{1}{\mu(1+\lambda\overline{k}^2)^q} \tag{5}$$

Physically, we should ensure that $T \sim \frac{1}{k}$. Thus, q should be $0.4 - 0.6$. Also, the step of integration should be considerably less than T.

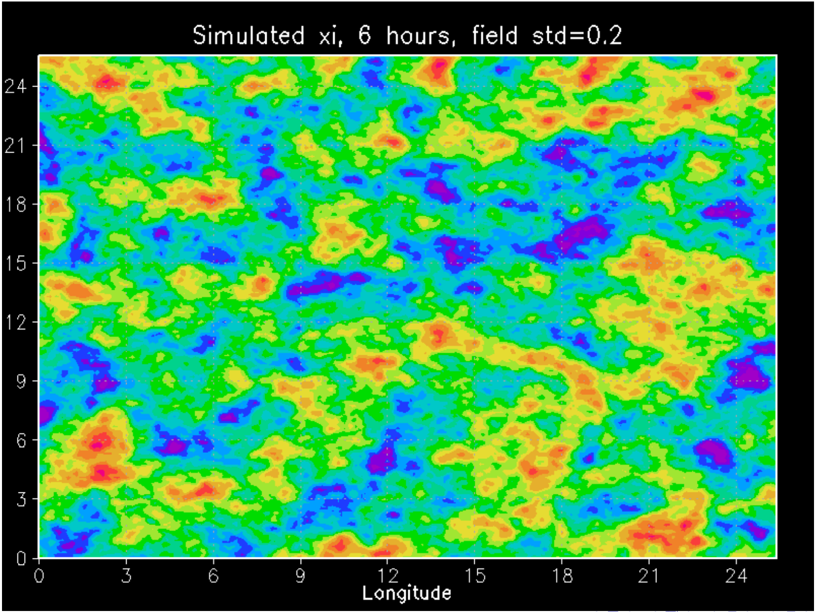
Choice of p

The variance b(k) of the solution of the equation (4) in spectral space is

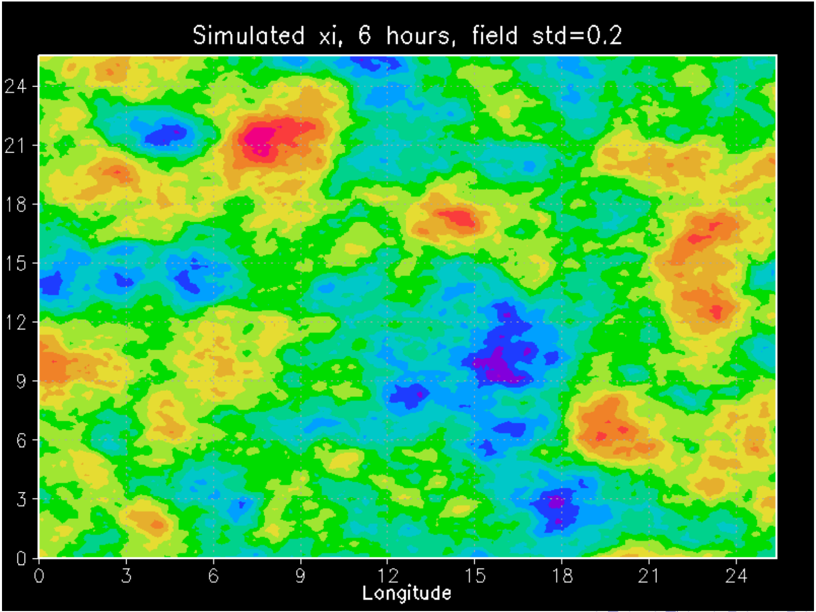
$$b(k) \sim \frac{1}{\mu(1+\lambda\overline{k}^2)^{q(2p-1)}} \tag{6}$$

The variance of ξ is the sum of $b(k)$ over all wavevectors. Because $Var(\xi)$ cannot be infinite, this sum should converge. That's why $b(k)$ should decay fast enough and p cannot be equal to 1. Our calculations show that $p = 3$ is an optimal value of this parameter.

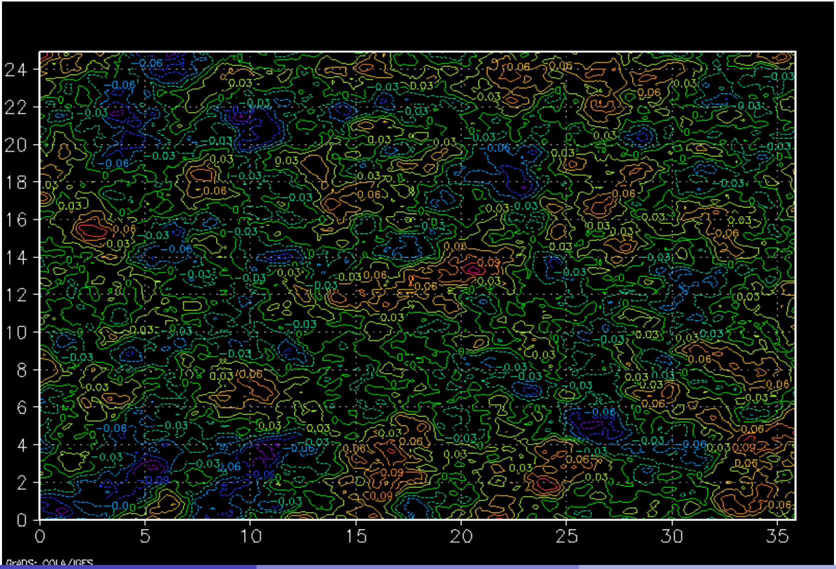
Random field, small scale



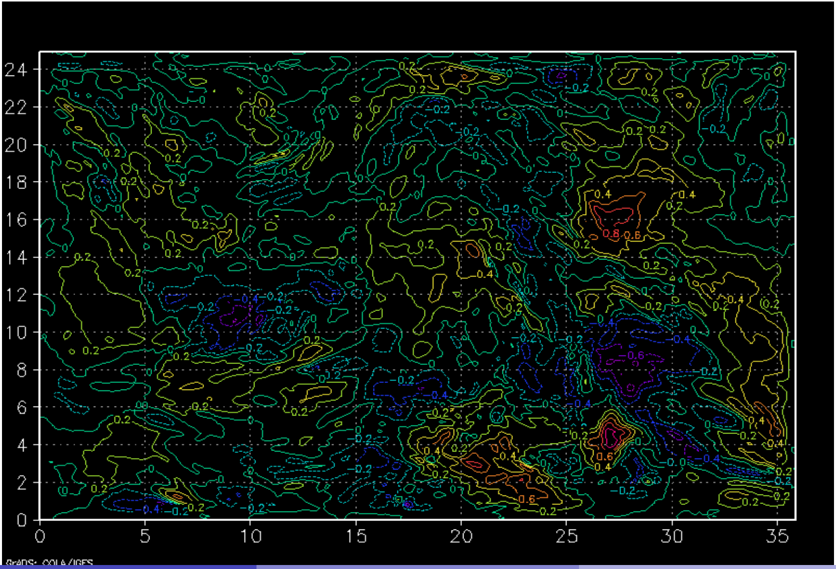
Random field, large scale



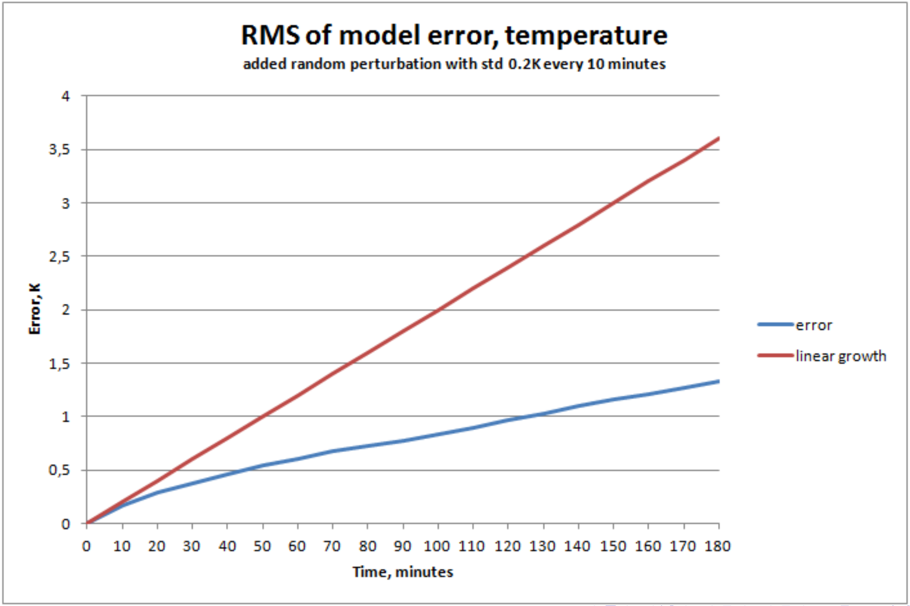
Forecast perturbation (the very start of the 6-h fcst), V-wind model error



Forecast perturbation (the end of the 6-h fcst), V-wind model error



Forecast error growth (T perturbed)



Conclusions

- The Pattern Generator (PG) based on stochastic partial dirreferential equation approach is developed and implemented into COSMO code.
- The PG can produce 2-D and 3-D random fields.
- PG is easily tunable: variance, spatial length scale, temporal length scale, degree of spatial smoothness, and degree of vertical inhomogeneity can be selected by the user.
- Computational time of the generator is about 3% of that of the COSMO model.

Thank you!