# Stochastic Pattern Generator for use in ensemble assimilation and forecasting

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2 September 2013

# Pattern Generator (PG) Setup

- Goal

  The PG is intended to be used in a *model-error generator* in the COSMO model — both in the additive model-error and multiplicative model-error models (or mixed).

- Requirements

  1. The PG should produce (on-line) 4-D univariate pseudo-random spatio-temporal fields.
  2. The pseudo-random realizations should be reproducible.
  3. The PG should be fast enough.
  4. The variance and spatial and temporal length scales are to be tunable.
  5. The spatio-temporal interactions should be 'meaningful'.

# BPG: the proposed solution

Approach: Generate fields by solving a stochastic partial differential equation (vector auto-regression).

The basic equation

$$\frac{\partial \xi}{\partial t} + A\xi = \sigma\alpha \tag{1}$$

The simplified form (gives rise to *Matérn* class spatial correlations)

$$\frac{\partial \xi}{\partial t} + \mu(1 - \lambda^2\Delta)^q\xi = \sigma\alpha \tag{2}$$

# Solving the equation

The domain: the 3-D torus (a cube with periodic boundary conditions in all three dimensions).

The spectral solver:

$$\xi(t; x, y, z) = \sum_{mnl} \tilde{\xi}_{mnl}(t) \mathrm{e}^{\mathbf{i}(mx+ny+lz)}. \tag{3}$$

The equation decouples into a series of 1-D (in time) equations for different wavenumber triples:

$$\frac{\mathrm{d}\tilde{\xi}_{\mathbf{k}}}{\mathrm{d}t} + \mu(1 + \lambda^2 \cdot k^{2q}) \cdot \tilde{\xi}_{\mathbf{k}}(t) = \sigma\tilde{\alpha}_{\mathbf{k}}(t). \tag{4}$$

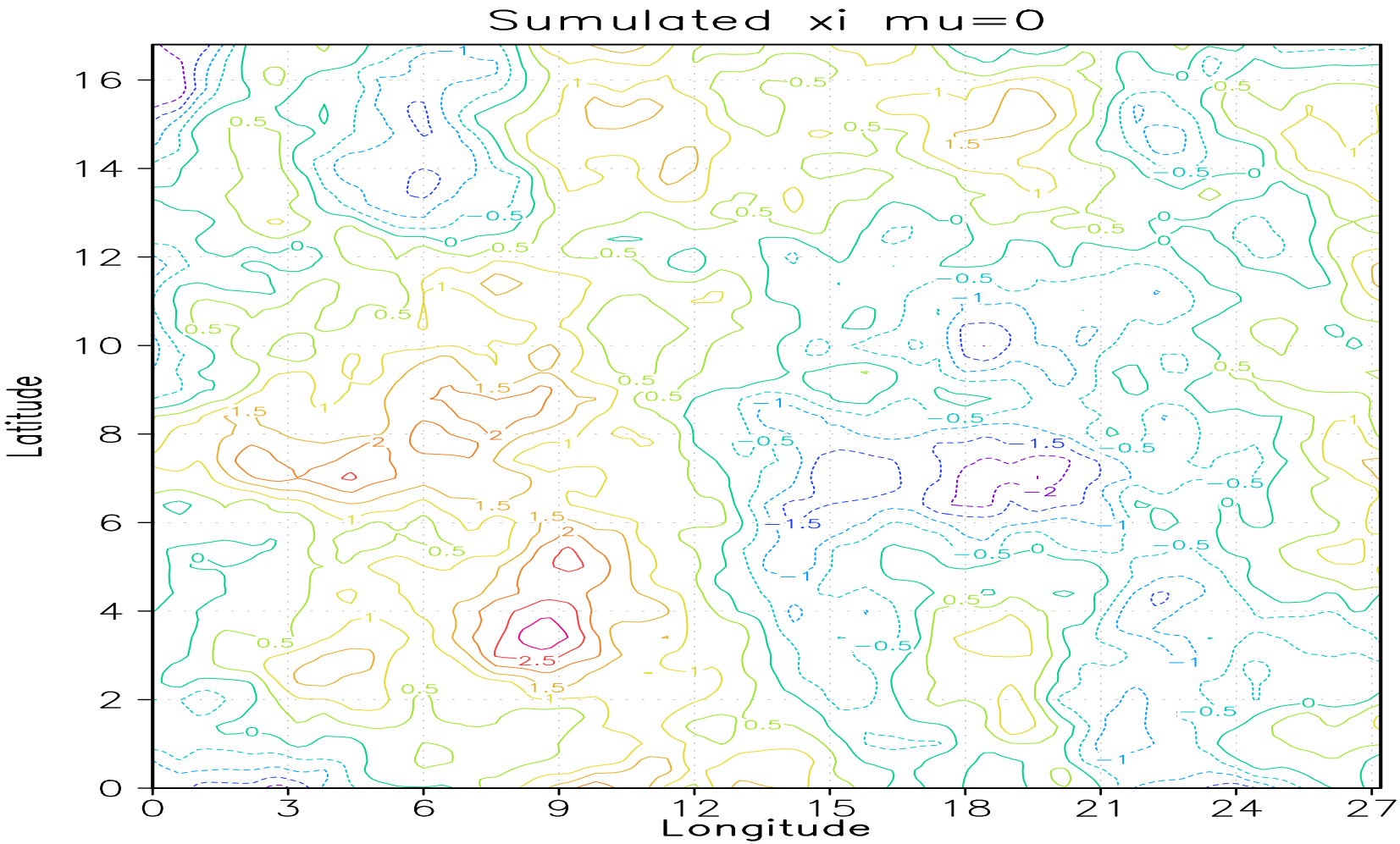
# User defined structure parameters of the random field (BPG)

1. The spatial length scale $L$.
2. The temporal length scale $T$.
3. The variance of $\xi$.
4. The degree of spatial smoothness of $\xi$.

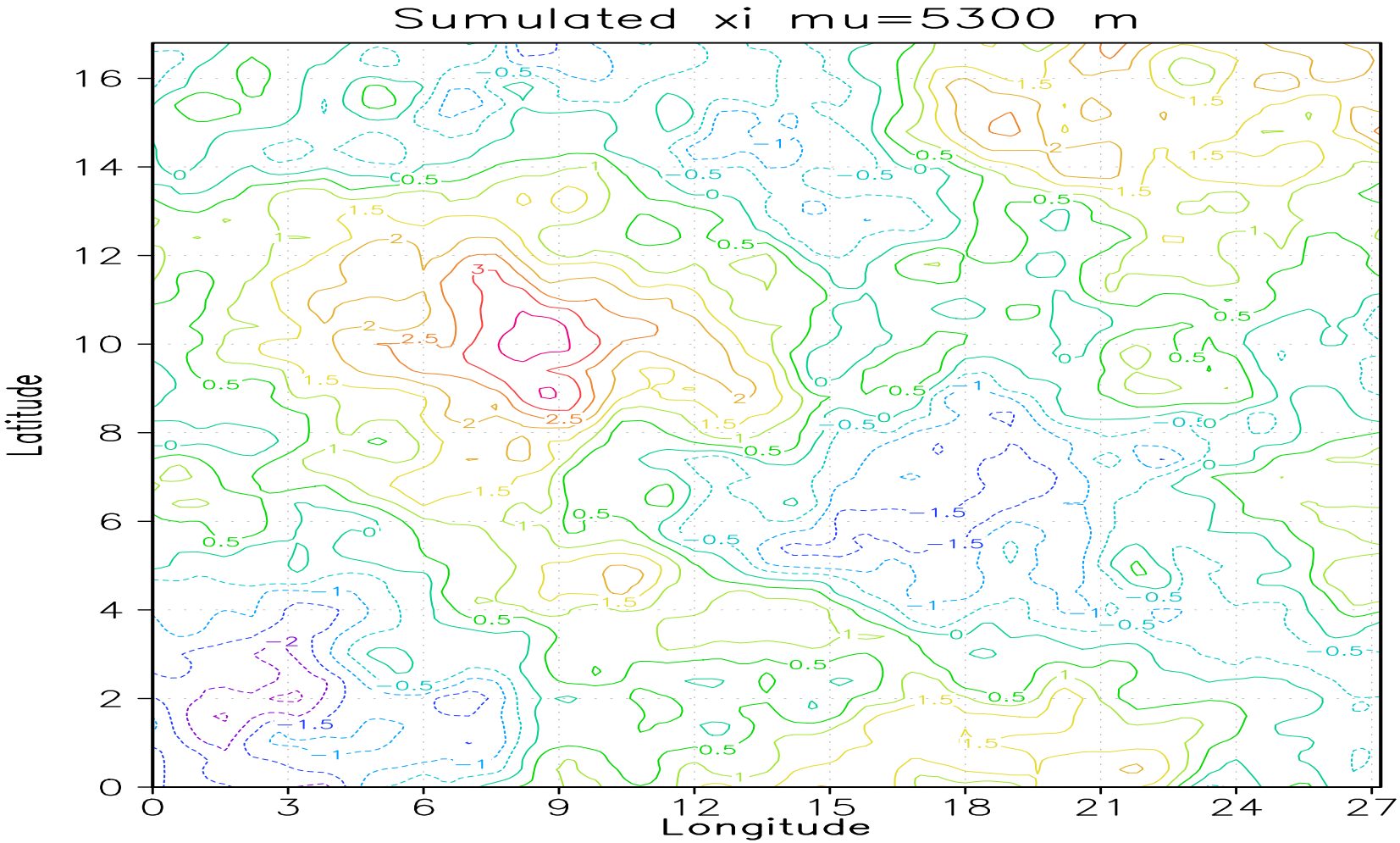
# From BPG to PG: mapping on the COSMO 3-D grid

- The mapper skips $2 \cdot L$-wide stripes in all 3D in order to eliminate spurious "periodic" correlations.
- Includes tunable inhomogeneity in the vertical.

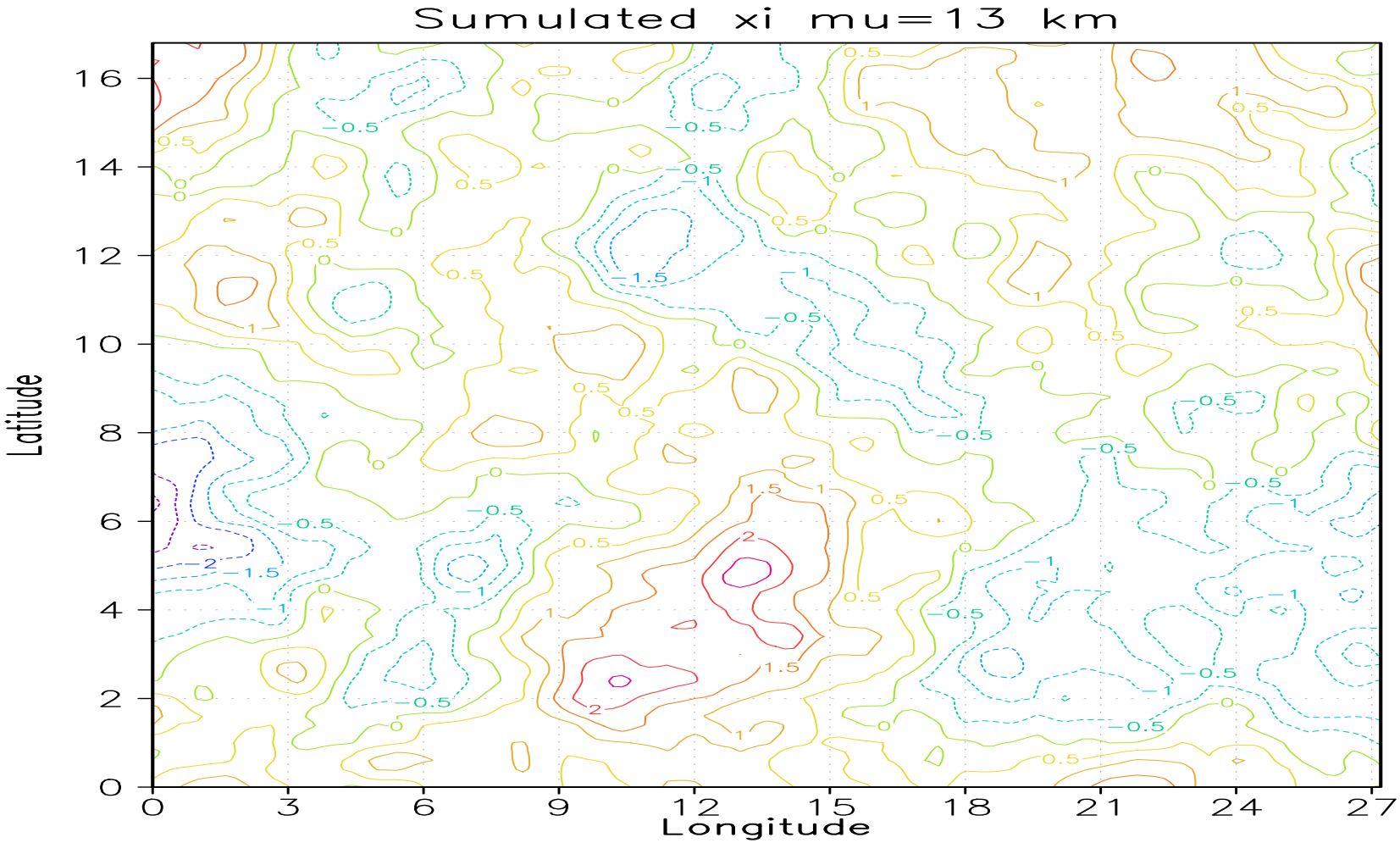
# A realization of $\xi$ on the COSMO grid: horizontal, $\mu = 0$



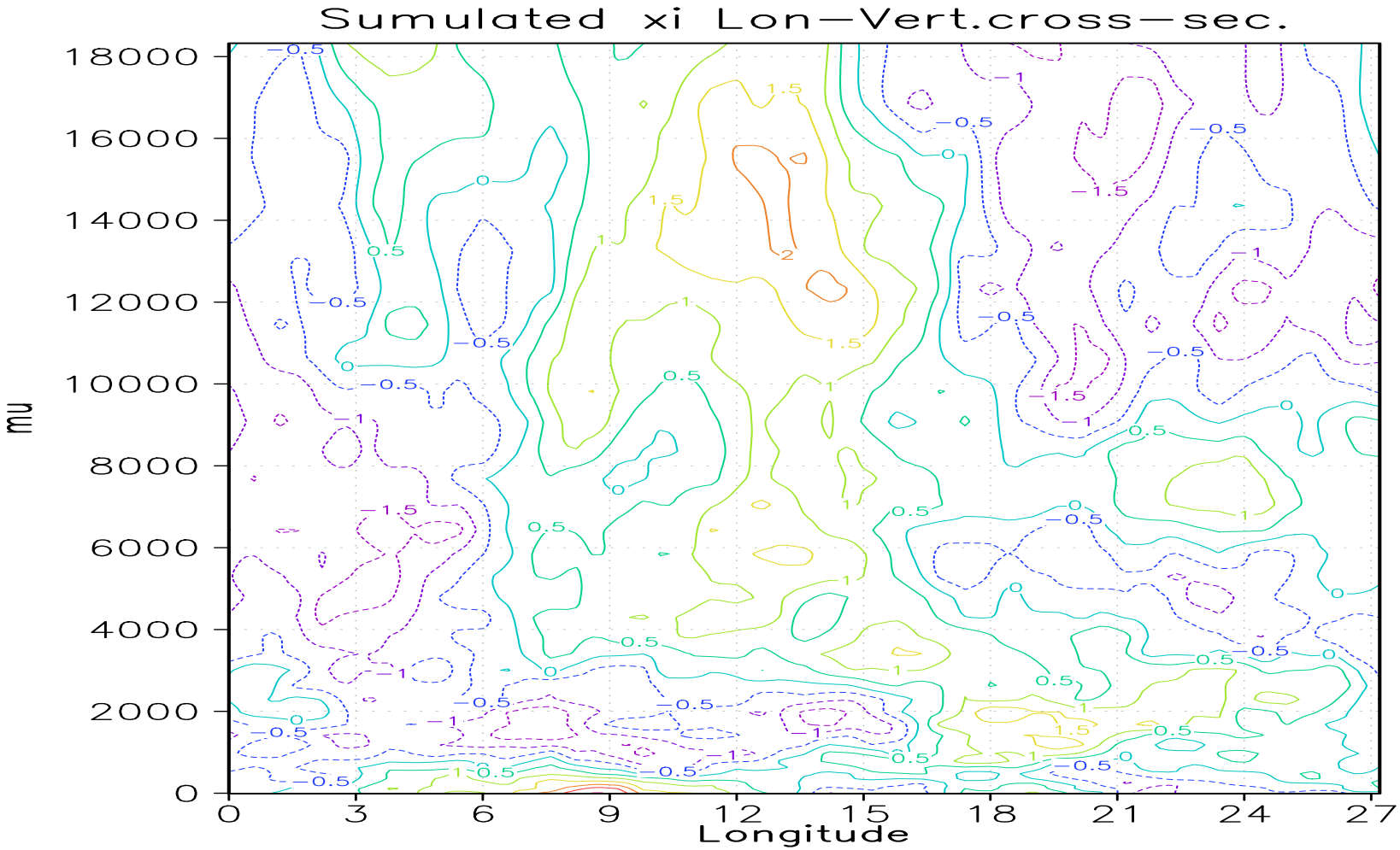
# $\xi$ on the COSMO grid: horizontal, $\mu = 5.3$ km



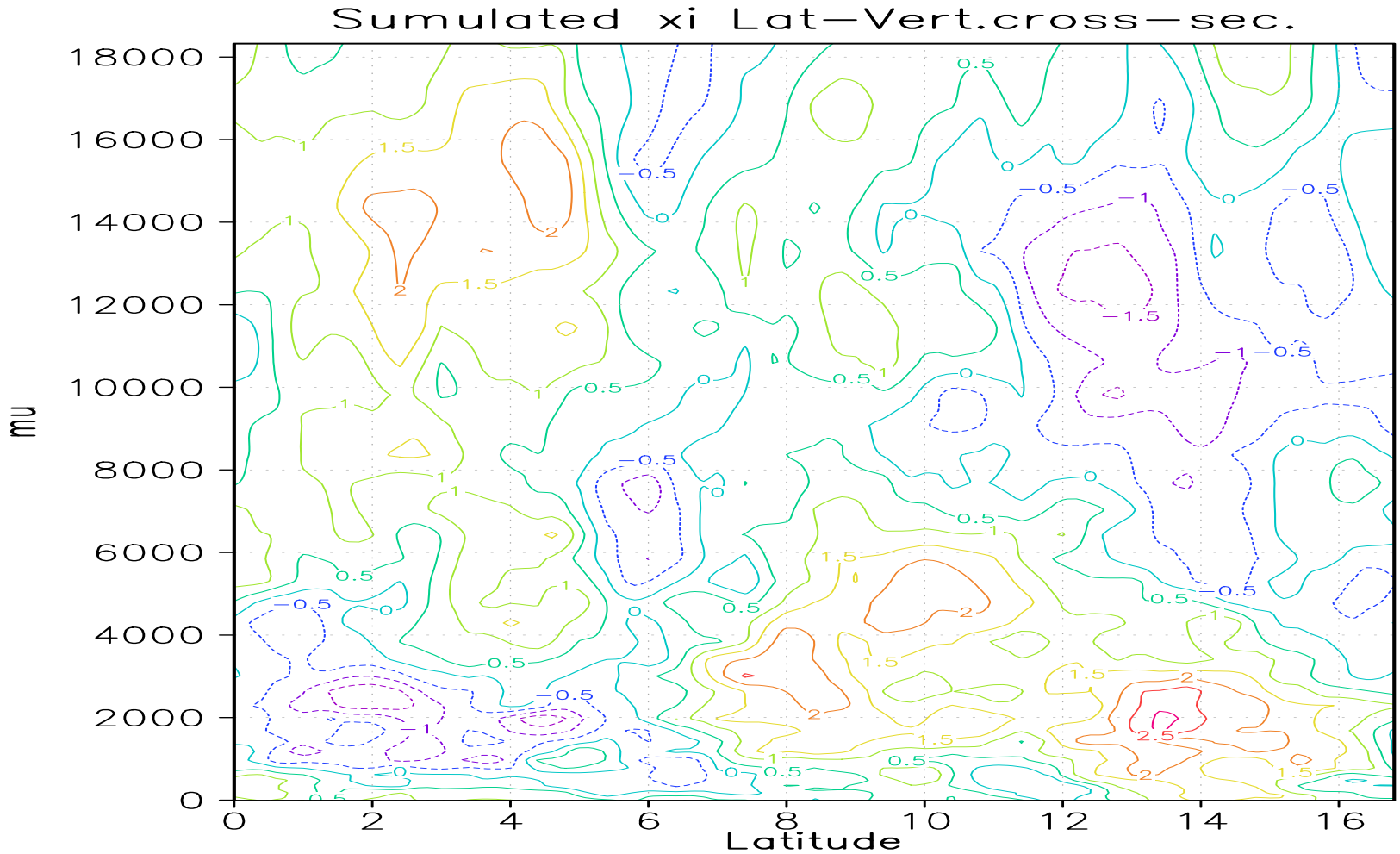
# $\xi$ on the COSMO grid: horizontal, $\mu = 13$ km



# $\xi$ on the COSMO grid: lon*vertical



# $\xi$ on the COSMO grid: lat*vertical



# Next steps

The spectral approach

0) Switch to the 4-th order in time (to balance *space* vs. *time* model orders and to prevent the time scales to be too small for small spatial scales). Should substantiallly improve the speed and enable higher resolutions.

$$\frac{\mathrm{d}\tilde{\xi}_{\mathbf{k}}}{\mathrm{d}t} + \mu(1 + \lambda^2 \cdot k^{2q}) \cdot \tilde{\xi}_{\mathbf{k}}(t) = \sigma\tilde{\alpha}_{\mathbf{k}}(t). \tag{5}$$

$$\tau_{\mathbf{k}} = \frac{1}{\mu(1 + \lambda^2 \cdot k^{2q})}. \tag{6}$$

# Next steps (cont.)

1. The spectral approach
   - Introduce vertical dependence into the *horizontal* lenght scale.
   - Prepare the 2-D version.
   - Finalize the spectral-space version and check it within the COSMO model.
2. The physical-space approach
   - Try a multi-scale physical space setup. If successful, the approach will be used for the global generator as well. (It is more flexible and, presumable, faster.)

# Conclusions

- The development of the Pattern Generator (PG) based on stochastic partial dirreferential equation approach is continued.
- The PG is embedded into the COSMO model (but not yet working). The fields are produced on the COSMO grids.
- PG is easily tunable: variance, spatial length scale, temporal length scale, degree of spatial smoothness, and degree of vertical inhomogeneity can be selected by the user.
- The *spectral-space* version will be finalized and a *physical-space* version extendable to the global domain will be developed next year.

Thank you!