A Limited-Area 4-D Pseudo Random Pattern Generator

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Pattern Generator (PG) Setup

- Goal
  The PG is intended to be used in a *model-error generator* in the COSMO model — both in the additive model-error and multiplicative model-error models (or mixed).
- Requirements
  1. The PG should produce (on-line) 4-D univariate pseudo-random spatio-temporal fields.
  2. The pseudo-random realizations should be reproducible.
  3. The PG should be fast enough.
  4. The variance and spatial and temporal length scales are to be tunable.
  5. The spatio-temporal interactions should be 'meaningful'.

The proposed solution

Approach: Generate fields by solving a stochastic partial differential equation.

The basic equation

$$\frac{\partial \xi}{\partial t} + \sum_{j=0}^{q} c_j(-\Delta)^j \xi = \sigma\alpha \tag{1}$$

The simplified form (gives rise to *Matérn* class spatial correlations)

$$\frac{\partial \xi}{\partial t} + \mu(1 - \lambda^2 \Delta)^q \xi = \sigma\alpha \tag{2}$$

Solving the equation

The domain: the 3-D torus (a cube with periodic boundary conditions in all three dimensions).

The spectral solver:

$$\xi(t; x, y, z) = \sum_{mnl} \tilde{\xi}_{mnl}(t) \mathrm{e}^{\mathbf{i}(mx+ny+lz)}. \tag{3}$$

The equation decouples into a series of 1-D (in time) equations for different wavenumber triples:

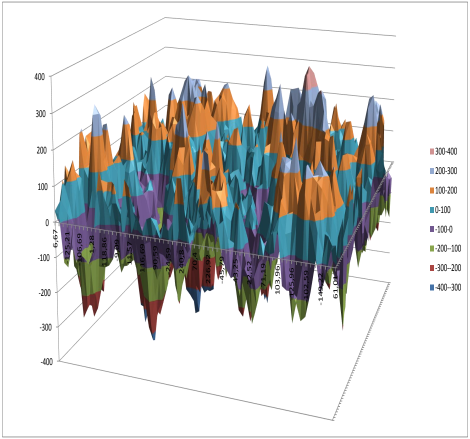
$$\frac{\mathrm{d}\tilde{\xi}_{mnl}}{\mathrm{d}t} + \mu[1 + \lambda^2(m^2 + n^2 + l^2)]^q \tilde{\xi}_{mnl}(t) = \sigma \tilde{\alpha}_{mnl}(t). \tag{4}$$

Selection of the structure parameters of the random field

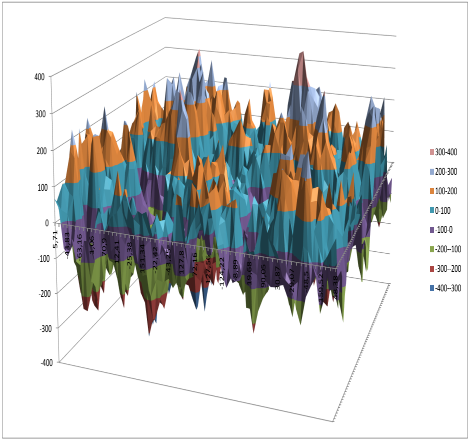
$$\frac{\partial \xi}{\partial t} + \mu(1 - \lambda^2 \Delta)^q \xi = \sigma \alpha \quad (5)$$

1. The spatial length scale L is controlled by the single internal parameter λ.
2. After λ is found, the temporal length scale T is controlled by the single parameter μ.
3. After λ and μ are found, the variance of ξ is controlled by the single parameter σ.
4. The degree of spatial smoothness of ξ is controlled by the single parameter q (the greater the smoother).

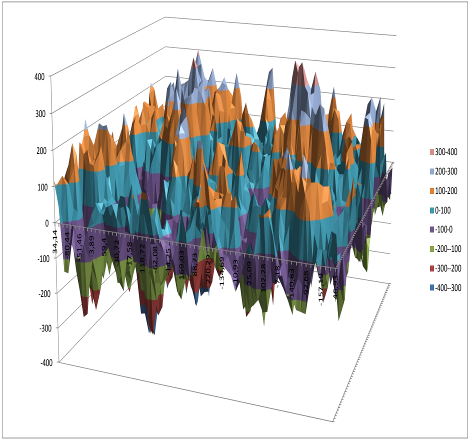
A horizontal realization of ξ at t=1 and lev=2



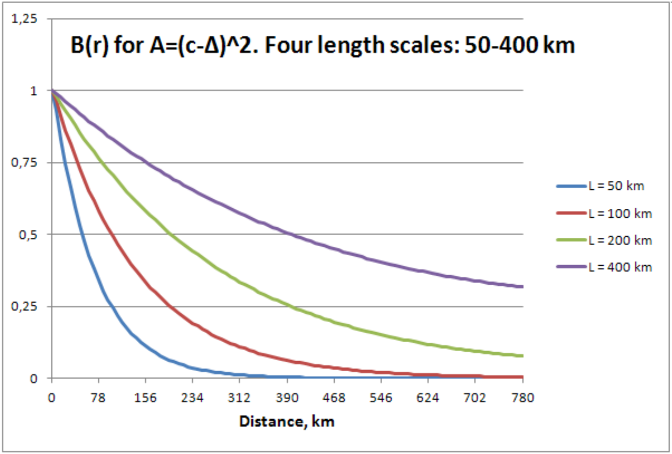
A horizontal realization of ξ at t=1 and lev=1



A horizontal realization of ξ at t=2 and lev=1

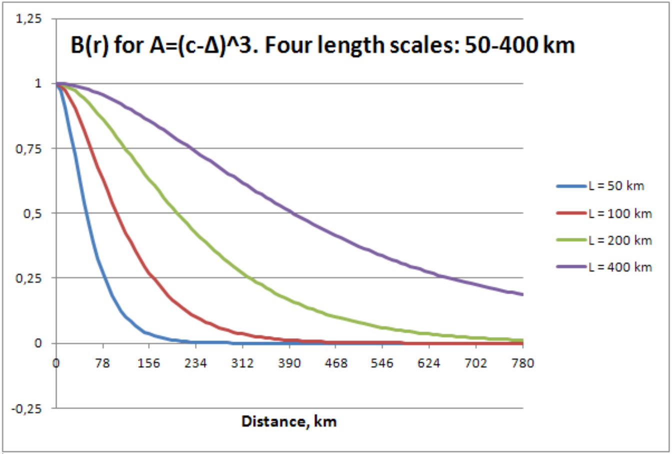


Theoretical spatial correlations for $q = 2$



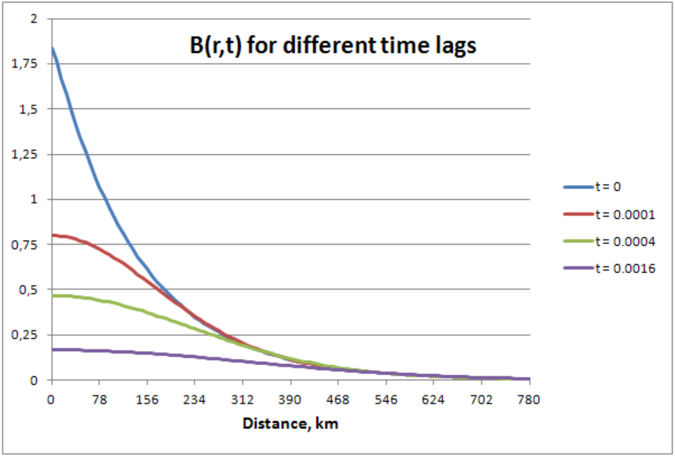
Different spatial length scales (50,100,200, and 400 km).

Theoretical spatial correlations for $q = 3$



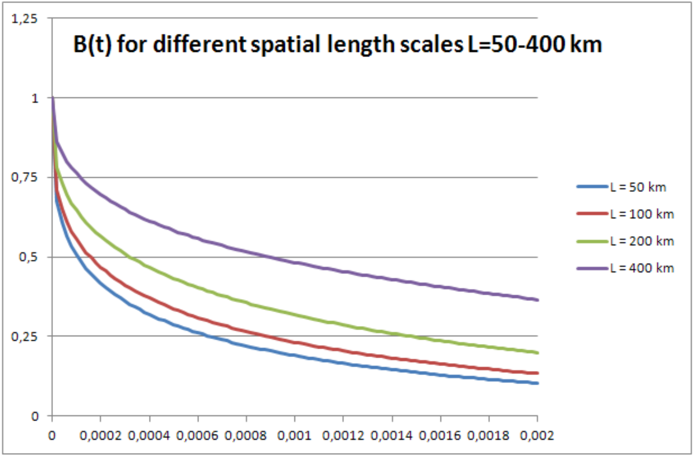
Different spatial length scales (50,100,200, and 400 km).

Theoretical spatio-temporal correlations



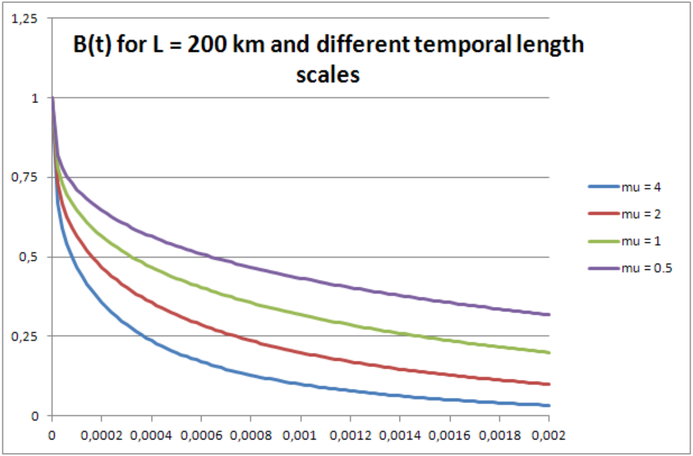
Different time lags (0,1,4, and 16 h).

Theoretical temporal correlations for different imposed spatial length scales



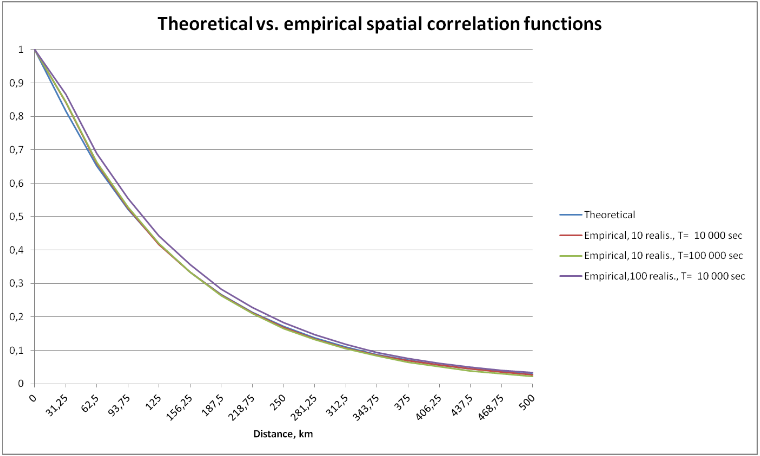
Different length scales (50,100,200, and 400 km).

Theoretical temporal correlations for different imposed temporal length scales



Different μ.

Comparison of theoretical spatial correlations with those estimated using simulated pseudo-random-field realizations



The correlations almost coincide.

CPU time and discretization errors

Spatial scale = 100 km, Time scale = 6 h., 1-D grid size = 64

Forecast time= 15 min., 10 realizations

1 CPU

Δt multiplier	σ	Spectral solver cpu time, sec.	FFT cpu time, sec.
0.0001	164.8	2614	5
0.001	168.5	254	5
0.003	167.9	85	5
0.01	165.2	26	5
0.03	167.9	9	5
0.1	165.3	3	5
0.3	167.9	1.5	5
1.0	167.9	1	5
3.0	165.7	1	5
10.0	168.1	1	6
100.0	168.1	1	5
10000.0	168.1	1	5

Extensions

- Inhomogeneity in the vertical.
- Second-order in time auto-regression (smoother in time fields).
- A physical space solver (?)
- Introduction of flow dependence (e.g. advection with wind).
- Non-Gaussianity. Boundedness.
- Etc.

Conclusions

- A Pattern Generator (PG) based on stochastic partial dirreferential equation approach is proposed and developed.
- A spectral-space solver is currently used.
- PG is easily tunable: variance, spatial length scale, temporal length scale, and degree of spatial smoothness can be selected by the user.
- Theoretical correlations are confirmed in pseudo-random simulations.
- CPU time seems to be small enough.
- The PG can be used to generate perturbations of boundary conditions (lateral, upper, and lower).

Thank you!