Karlsruhe Institute of Technology

Deutscher Wetterdienst
Wetter und Klima aus einer Hand

Status of RADVOP – Efficient radar forward operator for data assimilation and model verification

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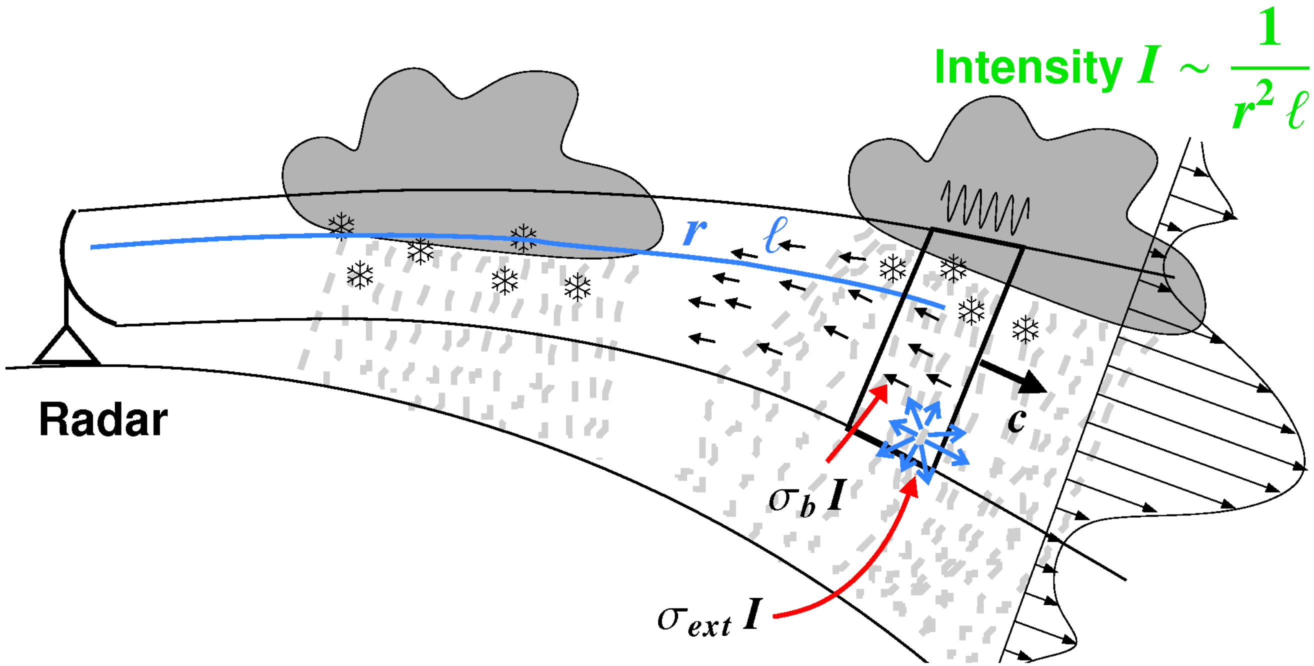
Motivation

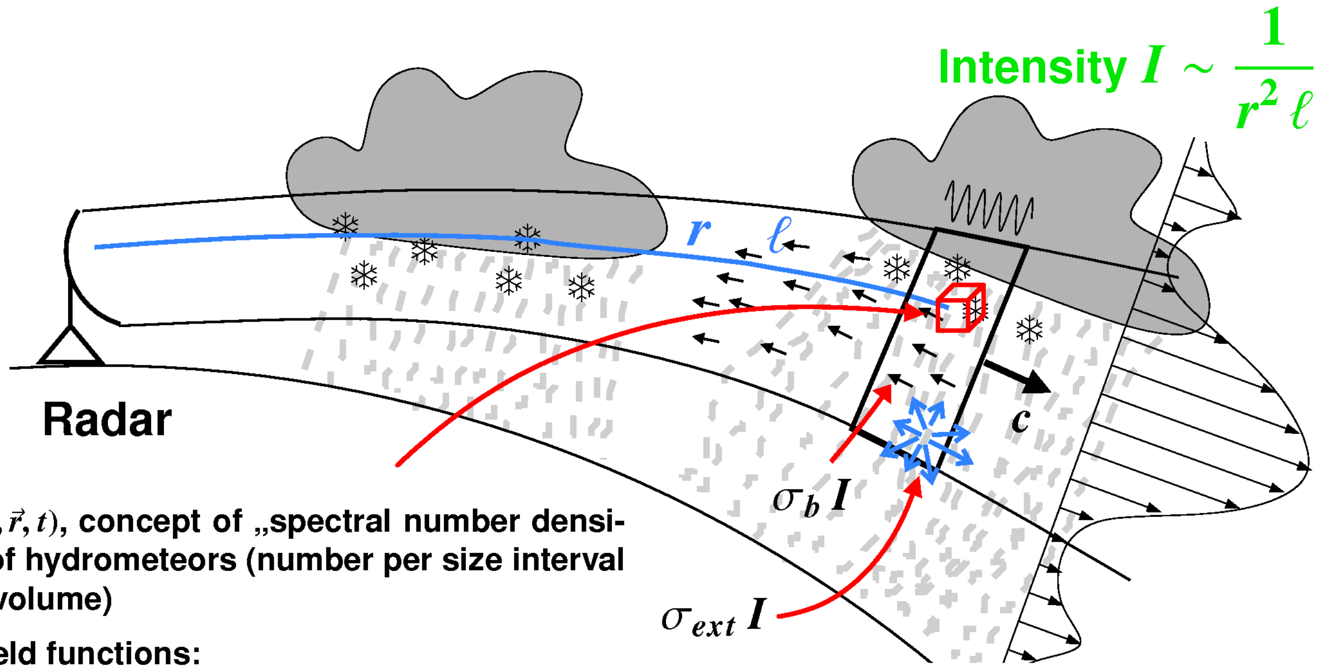
- Possible enhancement of the short-range precipitation forecast by
  a) assimilation of radar data
  b) microphysics enhance-ments derived from comparisons of
  model results with radar data
- DWD:

  Up to now: latent heat nudging and simple nudging of radial winds.
  Future: ENTKF assimilation system based on COSMO-DE-EPS.
  For assimilation of radar data socalled „forward operator“ necessary (simulation of radar quantities on the basis of model results on the native radar grid: radial winds, reflektivity, polarisation parameters).
- Such a forward operator also helpful for comparisons of model results and radar directly in terms of radar measurables - much easier than instead trying to derive model quantities like 3D wind, precipitation and hydrometeor contents from radar data.
- Project within the Extramural Research Program.
- Requirement: parallel / vectorized operator, integrated in COSMO code

Principle of radar measurement



Principle of radar measurement



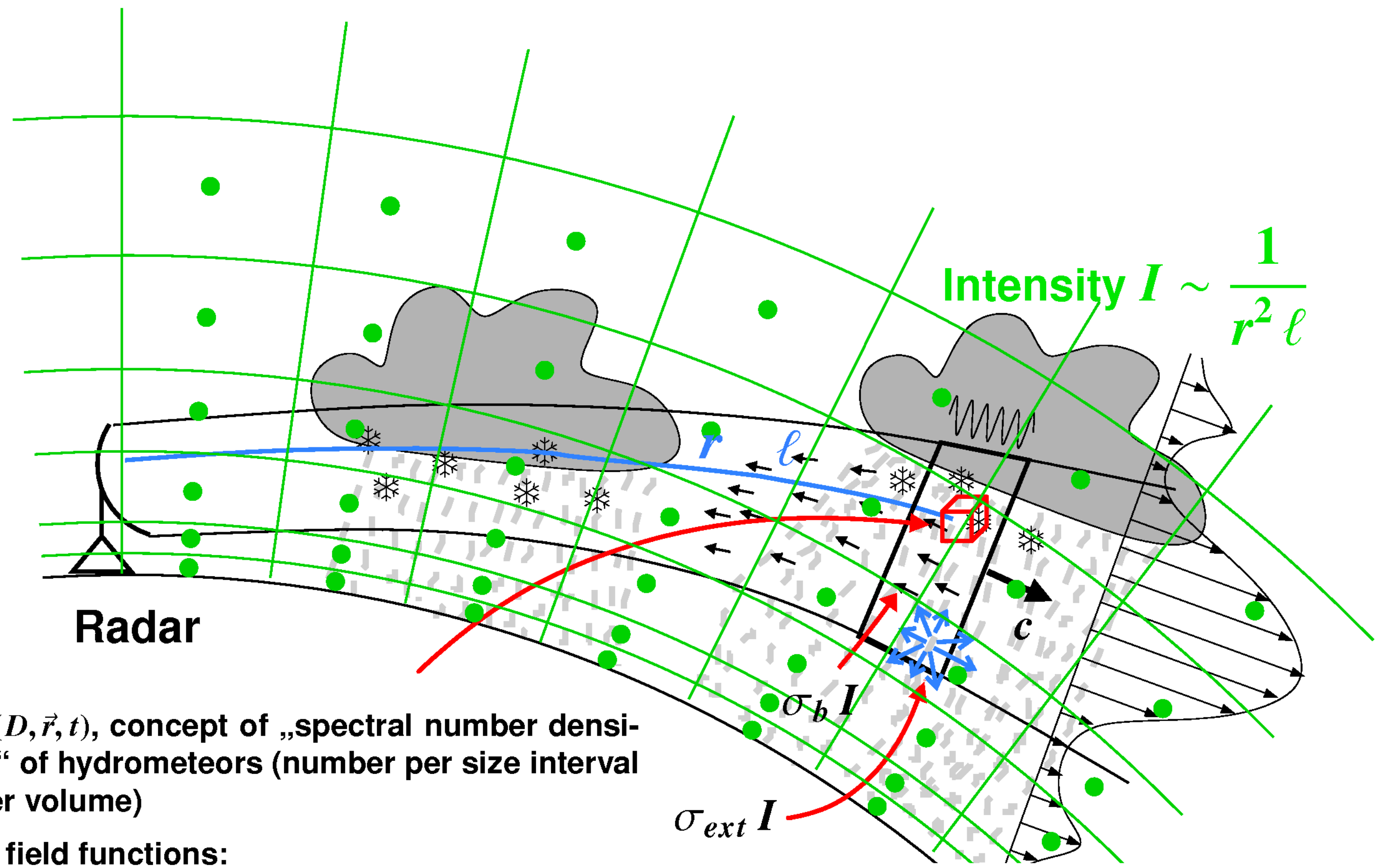
$N(D, \vec{r}, t)$, concept of „spectral number density" of hydrometeors (number per size interval per volume)

⇒ field functions:

$$\eta = \int_0^\infty \sigma_b(D)\, N(D)\, dD \qquad \Lambda = \int_0^\infty \sigma_{ext}(D)\, N(D)\, dD$$

~ Z_e useful for atten. (assumption: incoherent single scattering)

Principle of radar measurement



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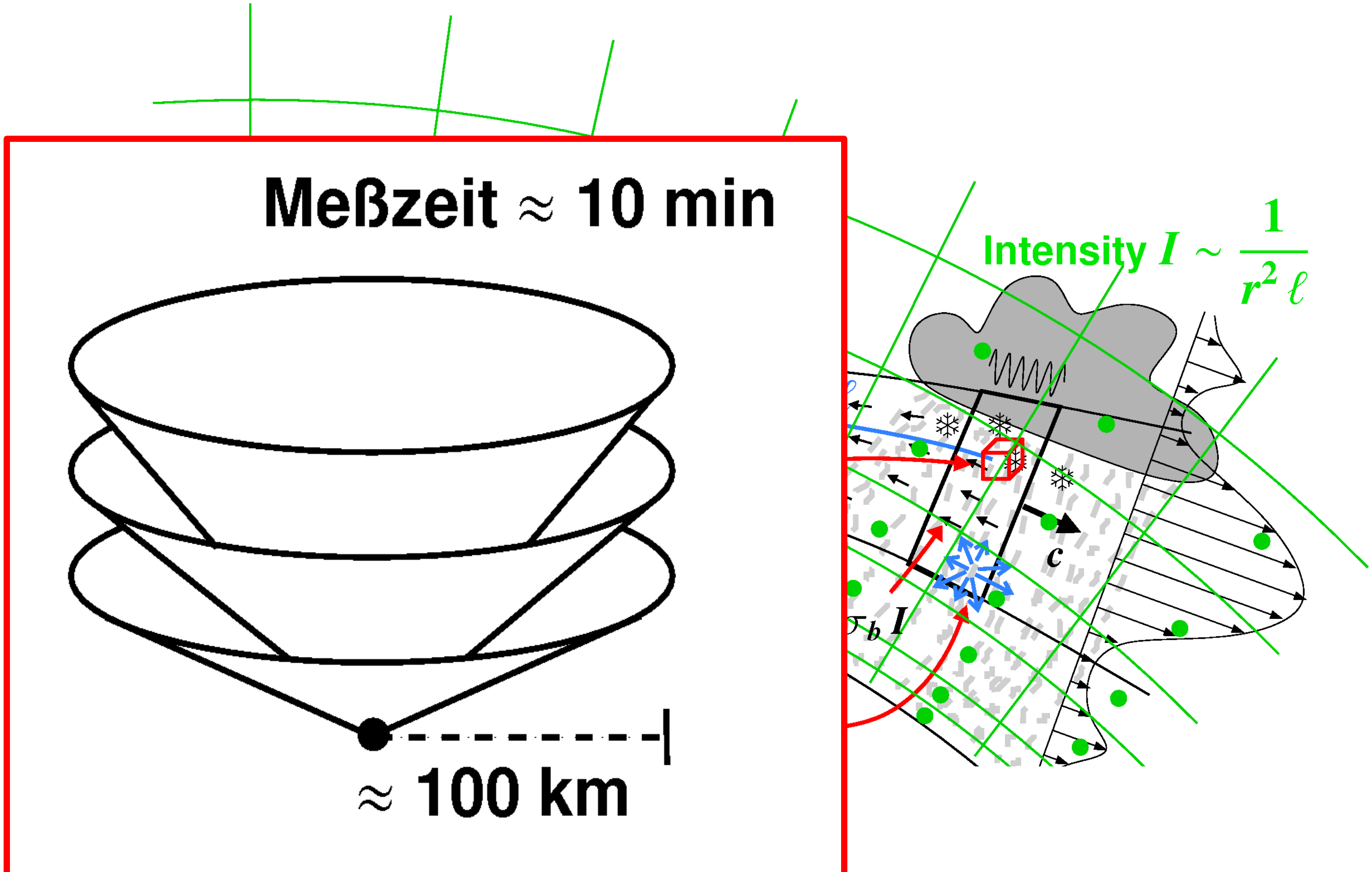

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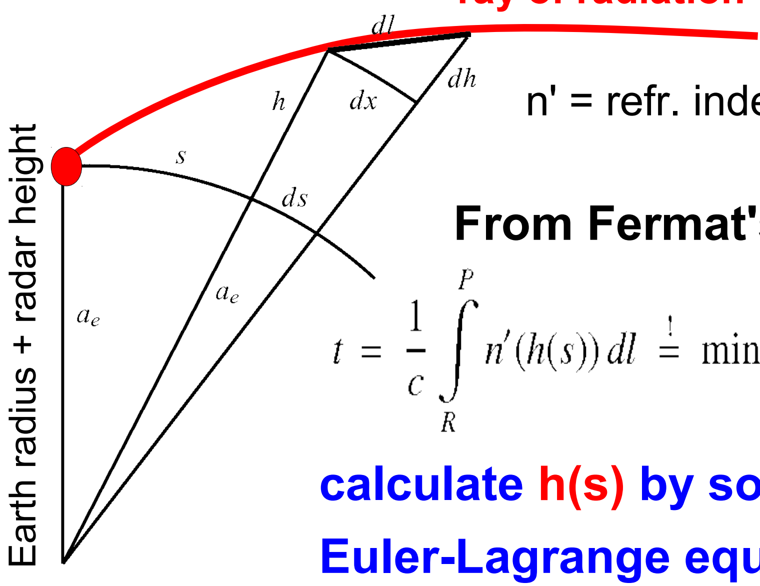
Atmospheric ray propagation



n' = refr. index = fct(p,T,e)

From Fermat's principle:

$$t = \frac{1}{c}\int_R^P n'(h(s))\,dl \stackrel{!}{=} \min \quad \Leftrightarrow \quad \delta t = 0$$

calculate h(s) by solving the Euler-Lagrange equation:

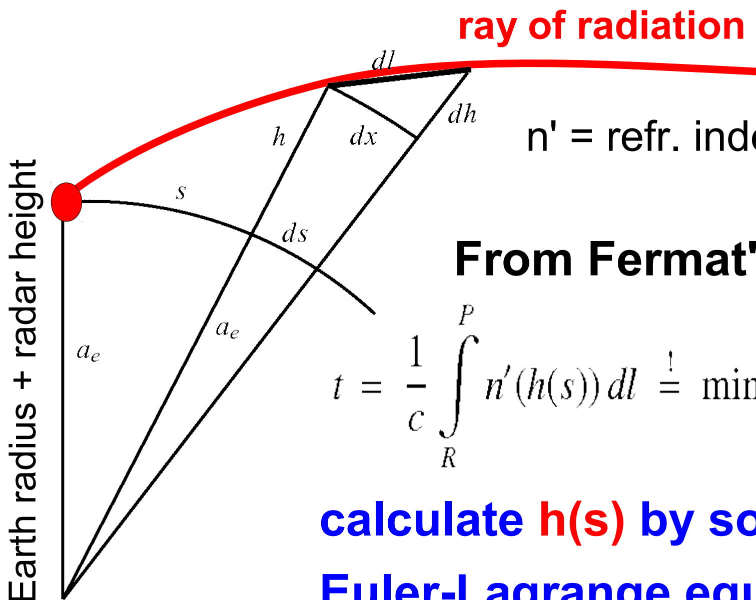
$$\frac{d^2h}{ds^2} - \left(\frac{1}{n'}\frac{dn'}{dh} + \frac{2}{a_e+h}\right)\left(\frac{dh}{ds}\right)^2 - \left(\frac{a_e+h}{a_e}\right)^2\left(\frac{1}{n'}\frac{dn'}{dh} + \frac{1}{a_e+h}\right) = 0$$

or by making use of its conserved integral:

$$n'\,(h+a_e)\cos(\epsilon_{loc}) = const.$$

$$\text{where} \quad \epsilon_{loc} = \arctan\left(\frac{dh}{dx}\right) = \arctan\left(\frac{a_e}{a_e+h}\frac{dh}{ds}\right)$$

Atmospheric ray propagation



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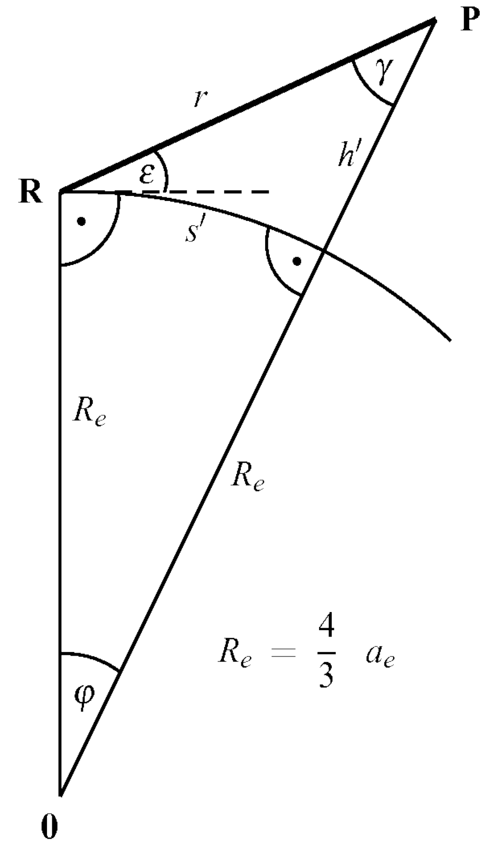
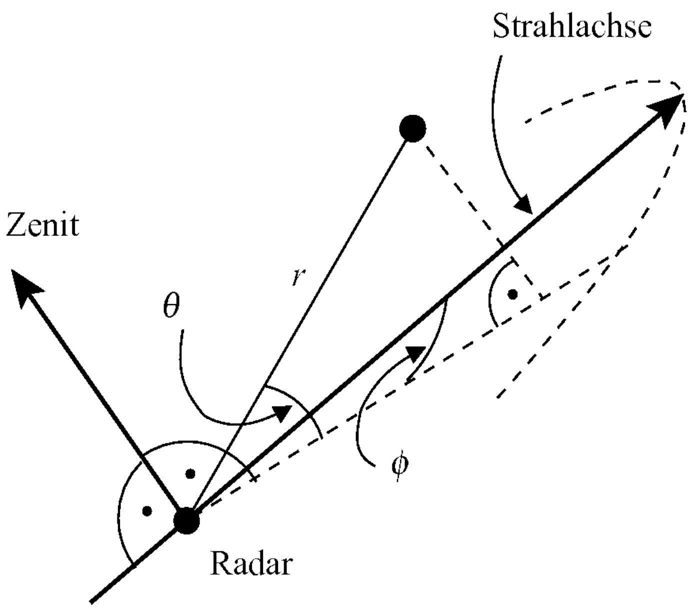
Efficient approximation for „standard“ conditions: 4/3 – earth - model



Radar operator (reflectivity)



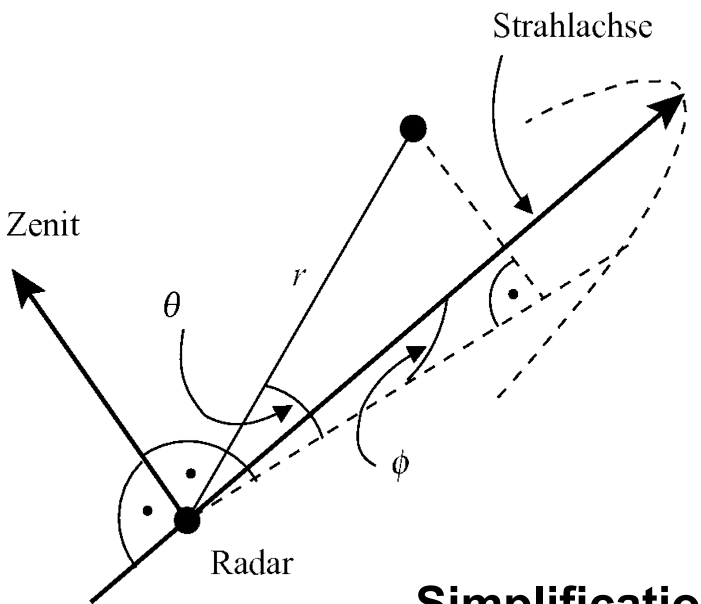
$$f^2(\phi,\theta) = \exp\left(-4\ln 2\left(\frac{\phi^2}{\phi_3^2} + \frac{\theta^2}{\theta_3^2}\right)\right)$$

$$\eta(r,\phi,\theta) = \int_0^\infty \sigma_b(D)\, N(D, r, \phi, \theta)\, dD$$

$$Z_e = \eta \frac{\lambda_0^4}{\pi^5 |K_w|^2}$$

$$\left\langle Z_e^{(R)} \right\rangle = \frac{\displaystyle\int_{r_0-\Delta r/2}^{r_0+\Delta r/2}\int_{-\pi}^{\pi}\int_{-\pi/2}^{\pi/2} Z_e(r,\phi,\theta)\, \overbrace{\exp\left[-2\int_0^r \overbrace{\int_0^\infty \sigma_{ext}(D)\, N(D,r',\phi,\theta)\, dD}^{\Lambda(r',\phi,\theta)}\, dr'\right]}^{l_N^{-2}(r,\phi,\theta)} \frac{f(\phi,\theta)^4}{r^2} \cos\theta\, d\theta\, d\phi\, dr}{\displaystyle\int_{r_0-\Delta r/2}^{r_0+\Delta r/2}\int_{-\pi}^{\pi}\int_{-\pi/2}^{\pi/2} \frac{f(\phi,\theta)^4}{r^2} \cos\theta\, d\theta\, d\phi\, dr}$$

Radar operator (reflectivity)



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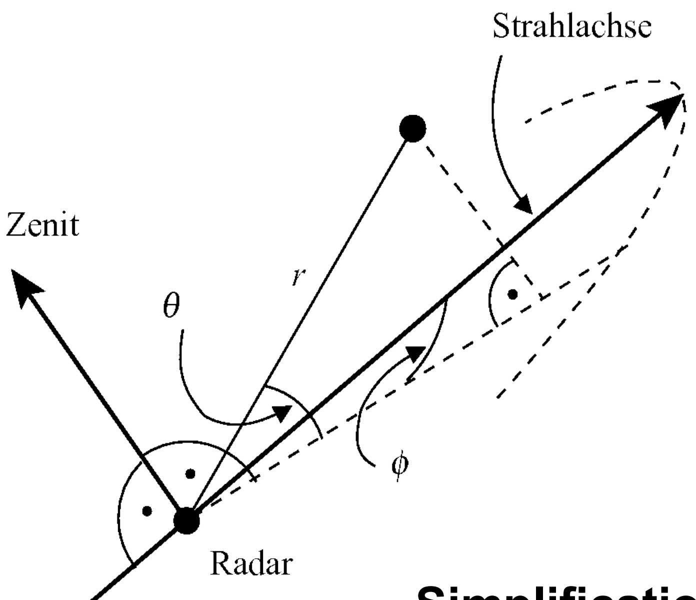
Simplification 1:

$$\langle Z_e^{(R)} \rangle = \frac{\int_{\pi/2}^{\pi/2} Z_e(r_0,0,\theta)\, \overbrace{\exp\left(-2\int_0^{r_0}\int_0^\infty \sigma_{ext}(D)\, N(D,r',\theta)\, dD\, dr'\right)}^{l_N^{-2}(r_0,0,\theta)} f(0,\theta)^4\, d\theta}{\int_{-\pi/2}^{\pi/2} f(0,\theta)^4\, d\theta}$$

Radar operator (reflectivity)



$$f^2(\phi,\theta) = \exp\left(-4\ln 2\left(\frac{\phi^2}{\phi_3^2} + \frac{\theta^2}{\theta_3^2}\right)\right)$$

$$\eta(r,\phi,\theta) = \int_0^\infty \sigma_b(D)\, N(D, r, \phi, \theta)\, dD$$

$$Z_e = \eta \frac{\lambda_0^4}{\pi^5 |K_w|^2} \qquad k_2 = \frac{20}{\ln 10} \Lambda$$

Simplification 2:

$$\langle Z_e^{(R)} \rangle = Z_e(r_0,0,0) \overbrace{\exp\left(-2 \int_0^{r_0} \int_0^\infty \sigma_{ext}(D)\, N(D, r')\, dD\, dr'\right)}^{l_N^{-2}(r_0,0,0)}$$

Approximations for Z_e

In general: Mie-scattering (one- or two-layered spheres) :

$\sigma_{back} = f(D,m)$, m = refract. index hydrometeors (ice/water/air-mixtures)

$\sigma_{ext} = f(D,m)$

Approximations:

Rayleigh: $\sigma_{back} \sim D^6$

→ **water drops:** $Z_e \sim M_6$ (analytic for gamma size distr.)

→ **ice hydrom.:** m variable, still need to integrate over N(D)

→ **small dry ice hydrom.:** Rayleigh + Debye-approx. for m:

$$Z_e \sim \rho^2_{snow}(D)\, M_6$$

Radial wind operator and simplif.

$$\left\langle v_r^{(R)} \right\rangle = \frac{\int\limits_{r_0-\Delta r/2}^{r_0+\Delta r/2} \int\limits_{-\pi}^{\pi} \int\limits_{-\pi/2}^{\pi/2} (\vec{v} \cdot \vec{e}_r) \frac{\eta}{l_n^2} \frac{f^4}{r^2} \cos(\theta)\, d\theta\, d\phi\, dr}{\int\limits_{r_0-\Delta r/2}^{r_0+\Delta r/2} \int\limits_{-\pi}^{\pi} \int\limits_{-\pi/2}^{\pi/2} \frac{\eta}{l_n^2} \frac{f^4}{r^2} \cos(\theta)\, d\theta\, d\phi\, dr} - \frac{\int\limits_{r_0-\Delta r/2}^{r_0+\Delta r/2} \int\limits_{-\pi}^{\pi} \int\limits_{-\pi/2}^{\pi/2} (\vec{e}_3 \cdot \vec{e}_r)\, \bar{v}_T \frac{\eta}{l_n^2} \frac{f^4}{r^2} \cos(\theta)\, d\theta\, d\phi\, dr}{\int\limits_{r_0-\Delta r/2}^{r_0+\Delta r/2} \int\limits_{-\pi}^{\pi} \int\limits_{-\pi/2}^{\pi/2} \frac{\eta}{l_n^2} \frac{f^4}{r^2} \cos(\theta)\, d\theta\, d\phi\, dr}$$

With:

$$\bar{v}_T = \frac{\int\limits_0^\infty \sigma_b(D)\, N(D)\, v_T(D)\, dD}{\eta}$$

Refl.-weighted hydrom. fall speed (computed from model variables or somehow approximated)

$$l_n^{-2} = \exp\left(-2 \int\limits_0^r \Lambda(r')\, dr'\right)$$

Attenuation factor

Radial wind operator and simplif.

$$\langle v_r^{(R)} \rangle = \frac{\int_{r_0-\Delta r/2}^{r_0+\Delta r/2} \int_{-\pi}^{\pi} \int_{-\pi/2}^{\pi/2} (\vec{v} \cdot \vec{e}_r) \frac{\eta}{l_n^2} \frac{f^4}{r^2} \cos(\theta)\, d\theta\, d\phi\, dr}{\int_{r_0-\Delta r/2}^{r_0+\Delta r/2} \int_{-\pi}^{\pi} \int_{-\pi/2}^{\pi/2} \frac{\eta}{l_n^2} \frac{f^4}{r^2} \cos(\theta)\, d\theta\, d\phi\, dr} - \frac{\int_{r_0-\Delta r/2}^{r_0+\Delta r/2} \int_{-\pi}^{\pi} \int_{-\pi/2}^{\pi/2} (\vec{e}_3 \cdot \vec{e}_r)\, \overline{v}_T \frac{\eta}{l_n^2} \frac{f^4}{r^2} \cos(\theta)\, d\theta\, d\phi\, dr}{\int_{r_0-\Delta r/2}^{r_0+\Delta r/2} \int_{-\pi}^{\pi} \int_{-\pi/2}^{\pi/2} \frac{\eta}{l_n^2} \frac{f^4}{r^2} \cos(\theta)\, d\theta\, d\phi\, dr}$$

Simplifications (examples):

Only vertical smoothing:

$$\langle v_r^{(R)} \rangle = \frac{\frac{\Delta r}{r_0^2} \int_{-\pi/2}^{\pi/2} (\vec{v} \cdot \vec{e}_r) \frac{\eta}{l_n^2} f^4(0,\theta) \cos(\theta)\, d\theta}{\frac{\Delta r}{r_0^2} \int_{-\pi/2}^{\pi/2} \frac{\eta}{l_n^2} f^4(0,\theta) \cos(\theta)\, d\theta} - \frac{\frac{\Delta r}{r_0^2} \int_{-\pi/2}^{\pi/2} (\vec{e}_3 \cdot \vec{e}_r)\, \overline{v}_T \frac{\eta}{l_n^2} f^4(0,\theta) \cos(\theta)\, d\theta}{\frac{\Delta r}{r_0^2} \int_{-\pi/2}^{\pi/2} \frac{\eta}{l_n^2} f^4(0,\theta) \cos(\theta)\, d\theta}$$

\+ No reflectivity weighting.:

$$\langle v_r^{(R)} \rangle = \frac{\frac{\Delta r}{r_0^2} \int_{-\pi/2}^{\pi/2} (\vec{v} \cdot \vec{e}_r) f^4(0,\theta) \cos(\theta)\, d\theta}{\frac{\Delta r}{r_0^2} \int_{-\pi/2}^{\pi/2} f^4(0,\theta) \cos(\theta)\, d\theta} - \frac{\frac{\Delta r}{r_0^2} \int_{-\pi/2}^{\pi/2} (\vec{e}_3 \cdot \vec{e}_r)\, \overline{v}_T f^4(0,\theta) \cos(\theta)\, d\theta}{\frac{\Delta r}{r_0^2} \int_{-\pi/2}^{\pi/2} f^4(0,\theta) \cos(\theta)\, d\theta}$$

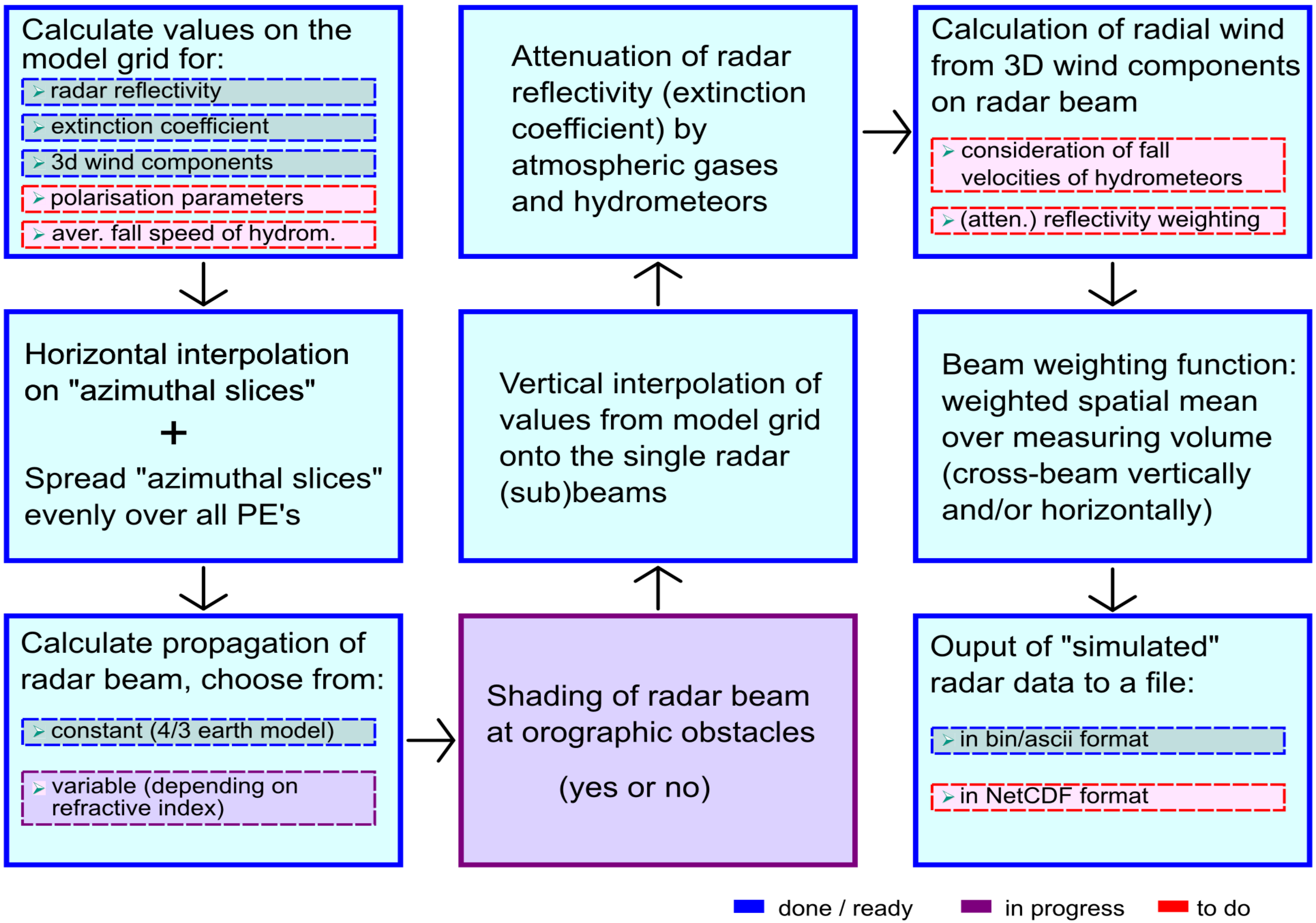
\+ No hydrometeor fall speed:

$$\langle v_r^{(R)} \rangle = \frac{\frac{\Delta r}{r_0^2} \int_{-\pi/2}^{\pi/2} (\vec{v} \cdot \vec{e}_r) f^4(0,\theta) \cos(\theta)\, d\theta}{\frac{\Delta r}{r_0^2} \int_{-\pi/2}^{\pi/2} f^4(0,\theta) \cos(\theta)\, d\theta}$$

\+ No smoothing, but with fall speed:

$$\langle v_r^{(R)} \rangle = \vec{v} \cdot \vec{e}_r + \overline{v}_T\, \vec{e}_3 \cdot \vec{e}_r$$

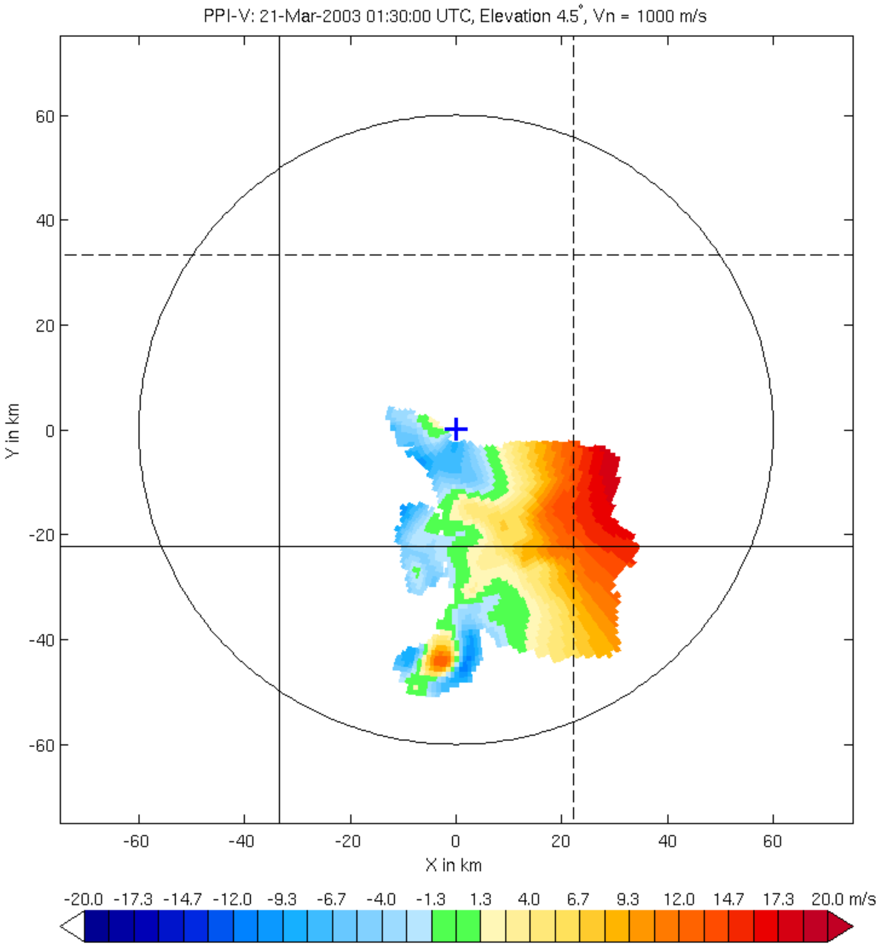
Block diagram and status



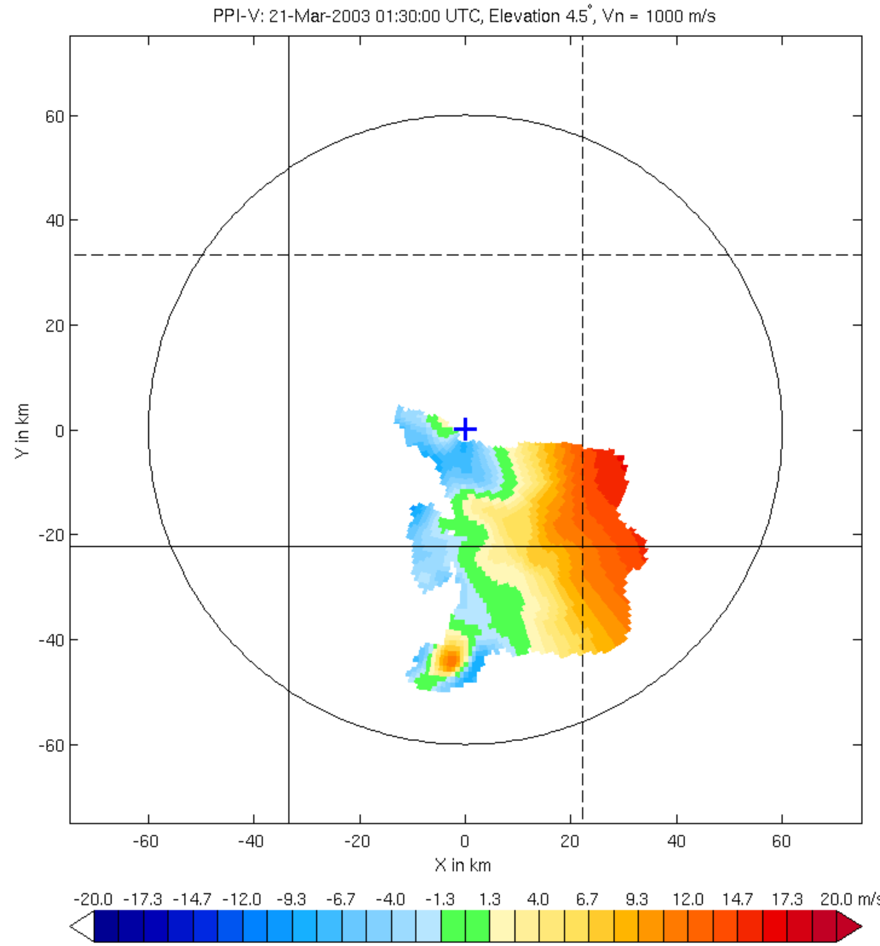
Results idealized convection 1 km

Radial wind

4/3 earth, no smoothing



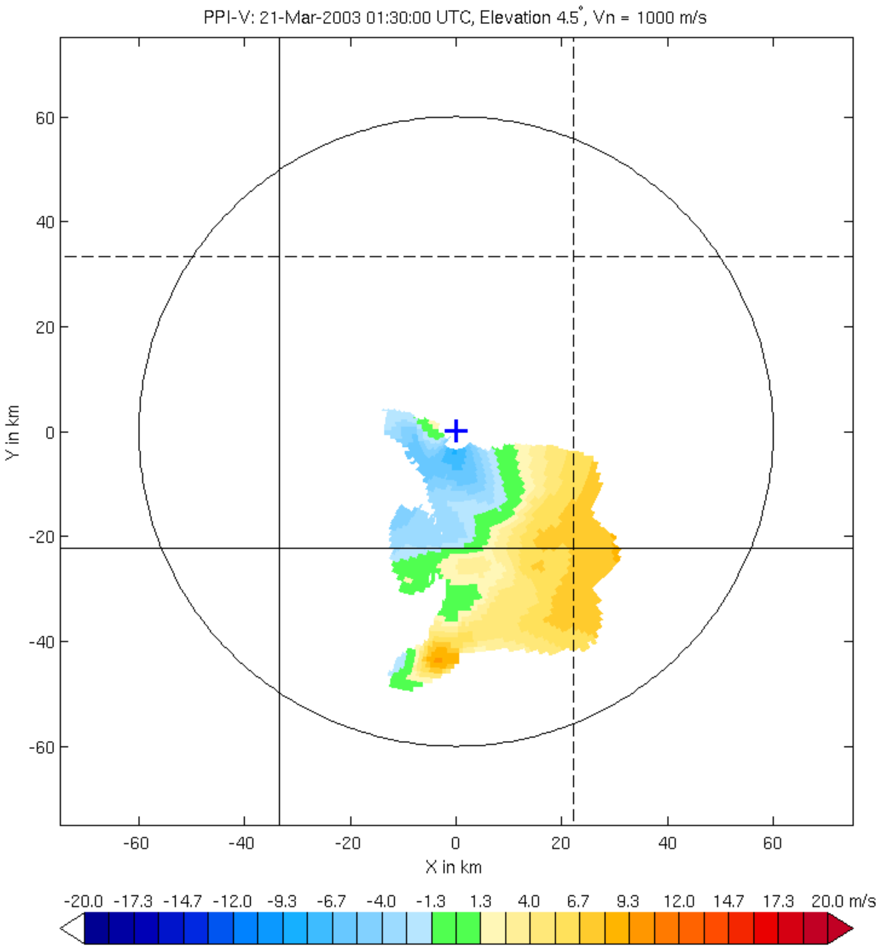
4/3 earth, vertical smoothing



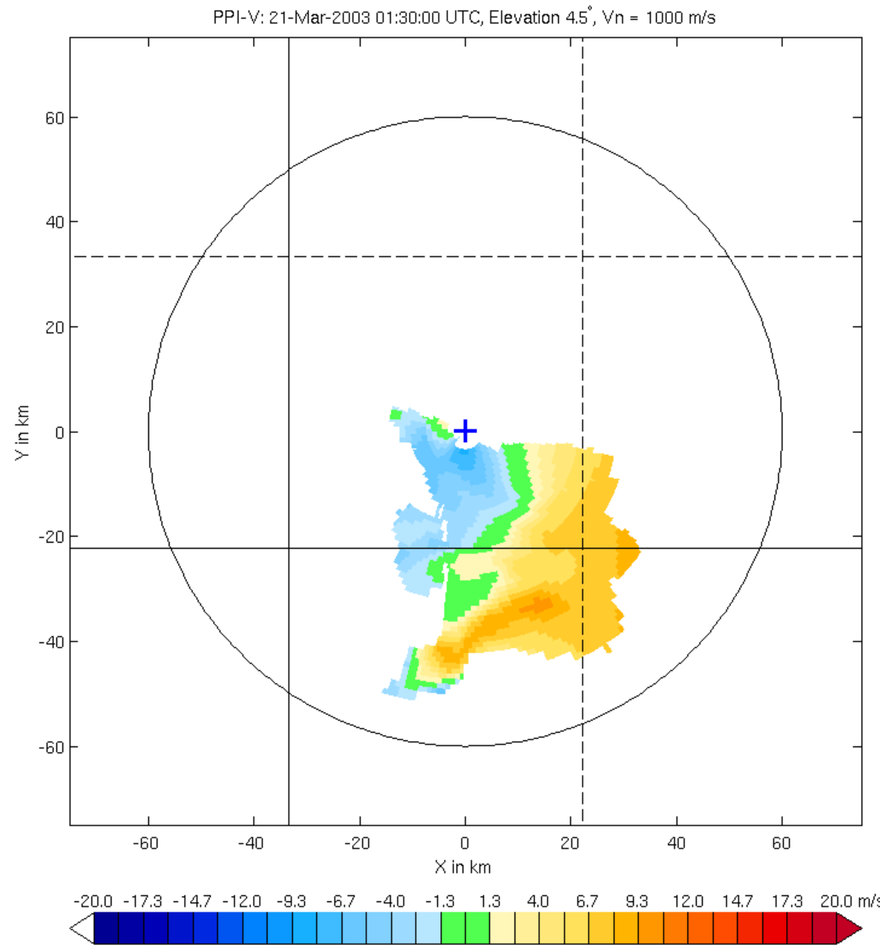
Results idealized convection 1 km

Radial wind

Online propag., no smoothing



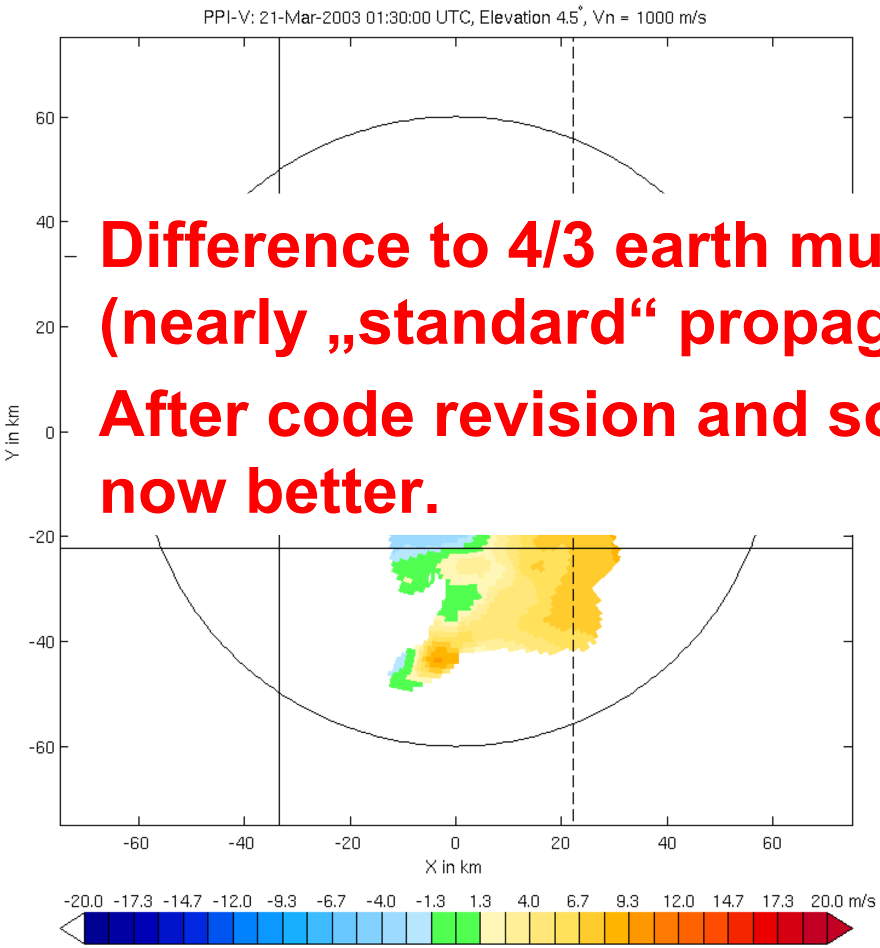
Online propag., vertical smoothing



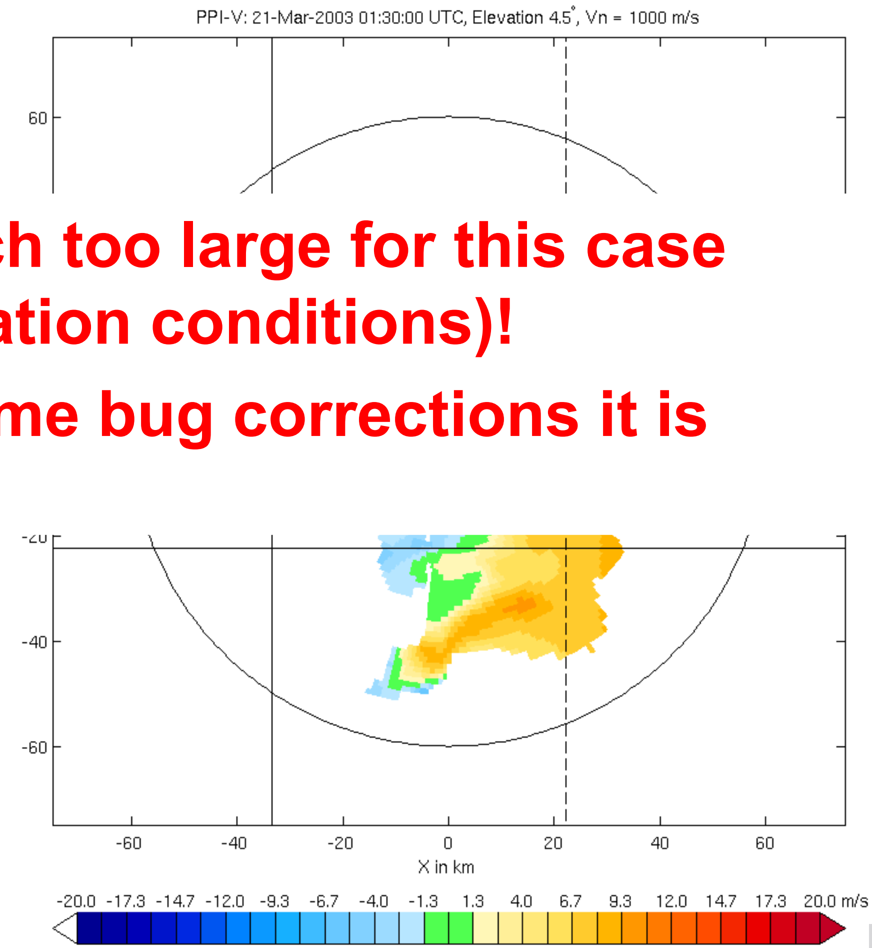
Results idealized convection 1 km

Radial wind

Online propag., no smoothing



Online propag., vertical smoothing



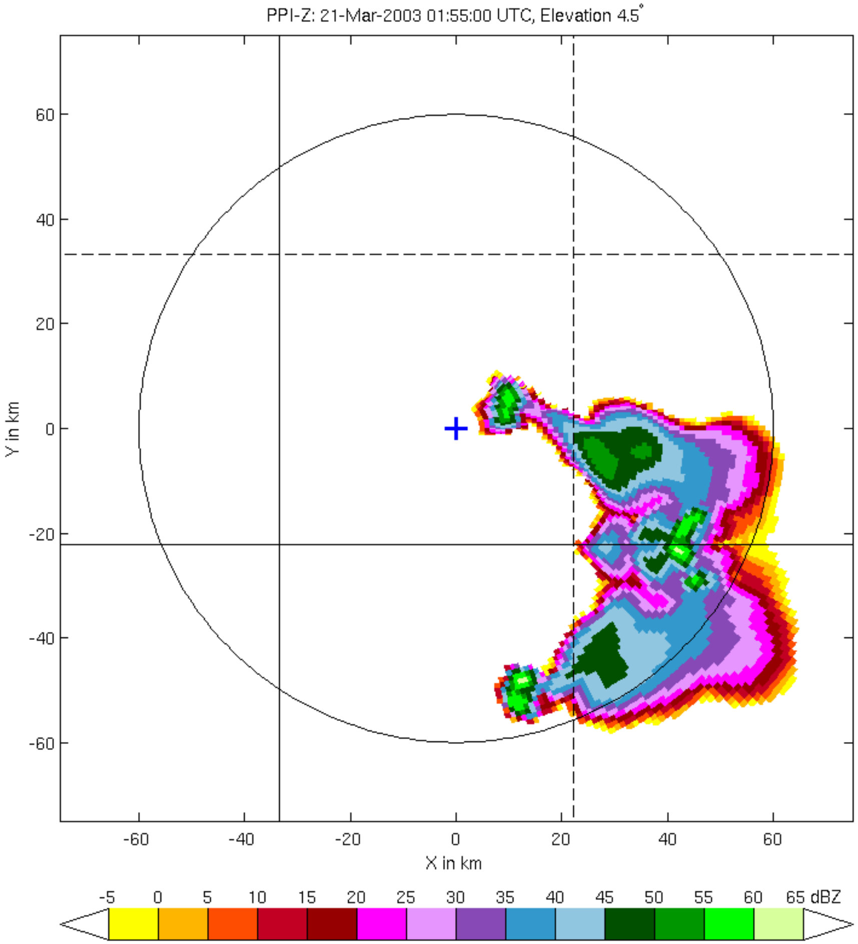
Difference to 4/3 earth much too large for this case (nearly „standard“ propagation conditions)!

After code revision and some bug corrections it is now better.

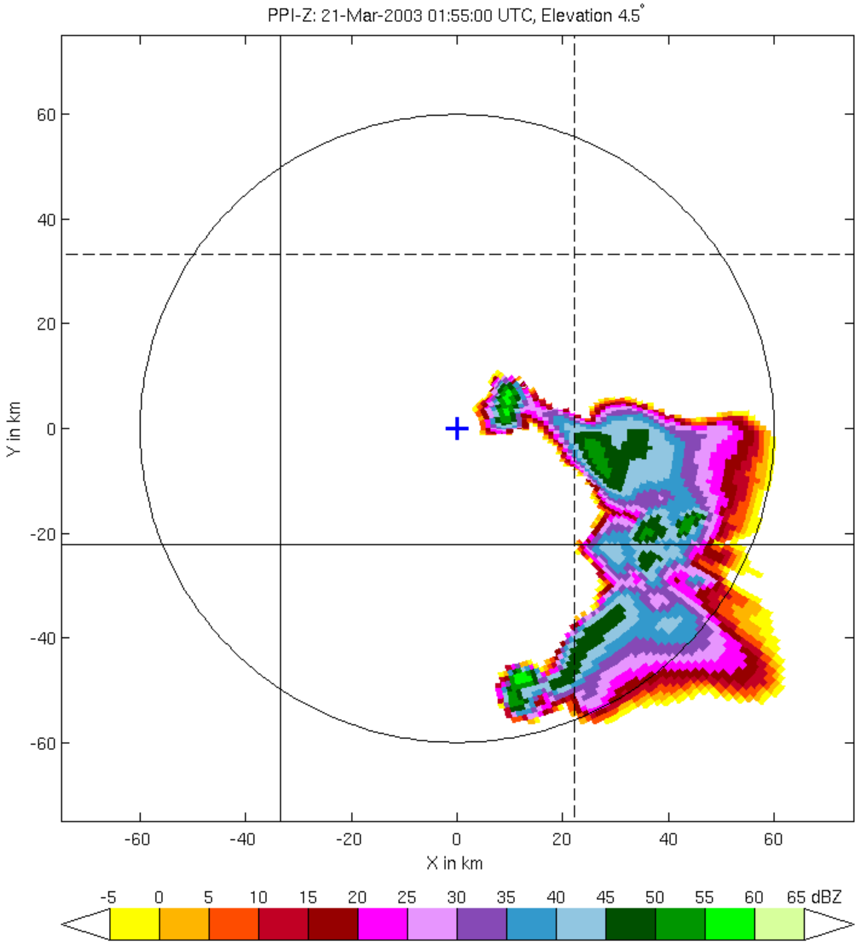
Results idealized convection 1 km

Reflectivity @ wavelength **5.5 cm**

Simple Rayleigh approx. no attenuation



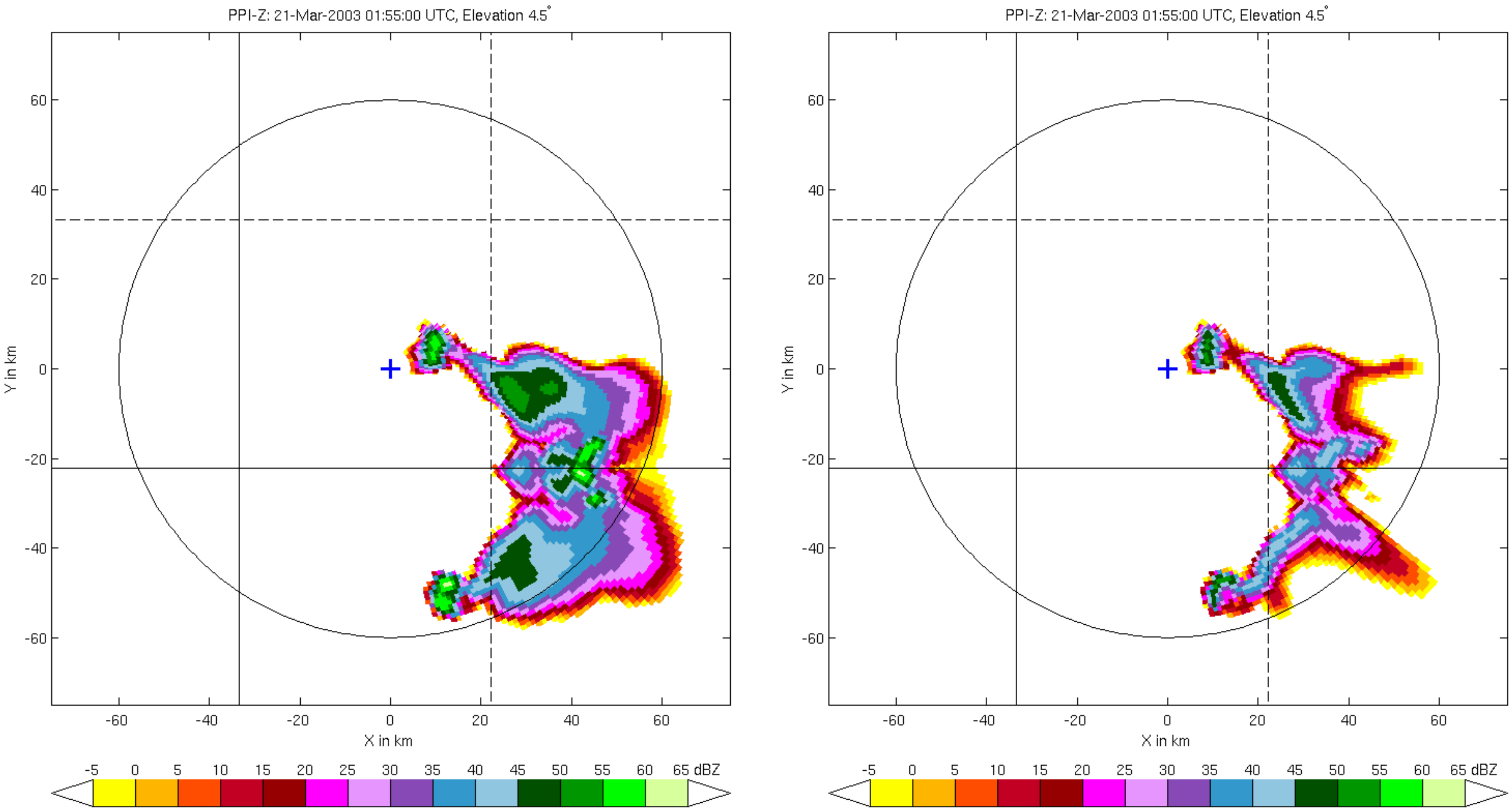
Mie scattering + attenuation



Results idealized convection 1 km

Reflectivity @ wavelength **3.0 cm**

Simple Rayleigh approx. no attenuation | Mie scattering + attenuation



Efficiency @ 1 km resol.

(version with online propag. calculations)

Vectorization degree [%] and CPU time [s]				
Subroutine	Without space averaging		With space averaging	
calc_geometry_grid	97.63%	0.019s($\cong$ 0.0%)	97.63%	0.019s($\cong$ 0.0%)
calc_grd_rfridx	99.91%	0.229s($\cong$ 0.0%)	99.91%	0.219s($\cong$ 0.0%)
calc_grd_winduvw	99.78%	0.374s($\cong$ 0.0%)	99.78%	0.376s($\cong$ 0.0%)
calc_grd_reflectivity	99.34%	0.330s($\cong$ 0.0%)	99.34%	0.326s($\cong$ 0.0%)
calc_geometry_online	85.39%	8.539s($\cong$ 0.2%)	84.30%	48.89s($\cong$ 1.1%)
calc_mod_radialwind_online	99.58%	0.461s($\cong$ 0.0%)	99.81%	1.771s($\cong$ 0.1%)
calc_mod_reflectivity_online	99.60%	0.221s($\cong$ 0.0%)	99.81%	0.801s($\cong$ 0.0%)
output_radar	91.29%	3.512s($\cong$ 0.1%)	97.58%	6.063s($\cong$ 0.1%)
communication expenses	$>$ 1319s($\cong$ 30.6%)		$>$ 1281s($\cong$ 30.2%)	
total CPU time	4217.903s		4243.724s	

actual Ze-calc. (Mie-scattering) is done elsewhere and hides in the „communication expenses“ (load imbalance)!

Efficiency @ 1 km resol.

For both online propag. and 4/3 earth model:

- Communication amount for online propag. higher, but not so critical (might be different for 2.8 km !)
- „Bottleneck“: Mie-scattering in combination with load imbalance causes long „waiting times“ for idle processors

In consequence:

- Optimization of the scattering parameter calculations on the model grid necessary.

  Easiest way: (regular) lookup tables, computed only once at model start, (multi-)linear table interpolation.

  First: reflectivity lookup tables, later similar concept for polarisation parameters.

Weitergehende Fragestellungen

- **Wie kann man Modellverifikation mittels 3D Radardaten betreiben? Was kann man gewinnen?**
- **Assimilation von Radardaten: wie reagiert das Modell auf die Assimilation solcher Massendaten? Wo liegen die Probleme? Was kann man gewinnen?**
- **Abschätzung von Effekten der nichtgleichmäßigen Strahlfüllung auf Dämpfung (k2-Ze-Beziehung).**

Radial wind operator

$$\left\langle v_r^{(R)} \right\rangle = \frac{\int\limits_{r_0-\Delta r/2}^{r_0+\Delta r/2} \int\limits_{-\pi}^{\pi} \int\limits_{-\pi/2}^{\pi/2} \left(\int\limits_0^\infty \sigma_b(D)\, N(D,r,\phi,\theta)\ \left[(\vec{v} - v_T(D)\vec{e}_3) \cdot \vec{e}_r \right] dD \right) \frac{f(\phi,\theta)^4}{r^2} \cos\theta\, d\theta\, d\phi\, dr}{\int\limits_{r_0-\Delta r/2}^{r_0+\Delta r/2} \int\limits_{-\pi}^{\pi} \int\limits_{-\pi/2}^{\pi/2} \eta(r,\phi,\theta) \frac{f(\phi,\theta)^4}{r^2} \cos\theta\, d\theta\, d\phi\, dr}$$

$$= \frac{\int\limits_{r_0-\Delta r/2}^{r_0+\Delta r/2} \int\limits_{-\pi}^{\pi} \int\limits_{-\pi/2}^{\pi/2} (\vec{v} \cdot \vec{e}_r) \frac{\eta}{l_n^2} \frac{f^4}{r^2} \cos(\theta)\, d\theta\, d\phi\, dr}{\int\limits_{r_0-\Delta r/2}^{r_0+\Delta r/2} \int\limits_{-\pi}^{\pi} \int\limits_{-\pi/2}^{\pi/2} \frac{\eta}{l_n^2} \frac{f^4}{r^2} \cos(\theta)\, d\theta\, d\phi\, dr} - \frac{\int\limits_{r_0-\Delta r/2}^{r_0+\Delta r/2} \int\limits_{-\pi}^{\pi} \int\limits_{-\pi/2}^{\pi/2} \left(\vec{e}_3 \cdot \vec{e}_r \int\limits_0^\infty \sigma_b(D)\, N(D)\, v_T(D)\, dD \right) \frac{1}{l_n^2} \frac{f^4}{r^2} \cos(\theta)\, d\theta\, d\phi\, dr}{\int\limits_{r_0-\Delta r/2}^{r_0+\Delta r/2} \int\limits_{-\pi}^{\pi} \int\limits_{-\pi/2}^{\pi/2} \frac{\eta}{l_n^2} \frac{f^4}{r^2} \cos(\theta)\, d\theta\, d\phi\, dr}$$

$$= \frac{\int\limits_{r_0-\Delta r/2}^{r_0+\Delta r/2} \int\limits_{-\pi}^{\pi} \int\limits_{-\pi/2}^{\pi/2} (\vec{v} \cdot \vec{e}_r) \frac{\eta}{l_n^2} \frac{f^4}{r^2} \cos(\theta)\, d\theta\, d\phi\, dr}{\int\limits_{r_0-\Delta r/2}^{r_0+\Delta r/2} \int\limits_{-\pi}^{\pi} \int\limits_{-\pi/2}^{\pi/2} \frac{\eta}{l_n^2} \frac{f^4}{r^2} \cos(\theta)\, d\theta\, d\phi\, dr} - \frac{\int\limits_{r_0-\Delta r/2}^{r_0+\Delta r/2} \int\limits_{-\pi}^{\pi} \int\limits_{-\pi/2}^{\pi/2} (\vec{e}_3 \cdot \vec{e}_r)\, \overline{v}_T \frac{\eta}{l_n^2} \frac{f^4}{r^2} \cos(\theta)\, d\theta\, d\phi\, dr}{\int\limits_{r_0-\Delta r/2}^{r_0+\Delta r/2} \int\limits_{-\pi}^{\pi} \int\limits_{-\pi/2}^{\pi/2} \frac{\eta}{l_n^2} \frac{f^4}{r^2} \cos(\theta)\, d\theta\, d\phi\, dr}$$

- **Aufzählung**
  - **asdfasdf**
  - **asfdasdf**
- **Zweiter Punkt**
  - **asdfasdf**
  - **asdasdf**