Status of RADVOP – Efficient radar forward operator for data assimilation and model verification

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Motivation

- Possible enhancement of the short-range precipitation forecast by
  a) assimilation of radar data
  b) microphysics enhancements derived from comparisons of model results with radar data

- DWD:
  
  Up to now: latent heat nudging and simple nudging of radial winds. Future: ETKF assimilation system based on COSMO-DE-EPS. For assimilation of radar data so-called „forward operator“ necessary (simulation of radar quantities on the basis of model results on the native radar grid: radial winds, reflectivity, polarisation parameters).

- Such a forward operator also helpful for comparisons of model results and radar directly in terms of radar measurables - much easier than instead trying to derive model quantities like 3D wind, precipitation and hydrometeor contents from radar data.

- Project within the Extramural Research Program.

- Requirement: parallel / vectorized operator, integrated in COSMO code
Principle of radar measurement

Intensity $I \sim \frac{1}{r^2 \ell}$
Principle of radar measurement

$N(D, \vec{r}, t)$, concept of "spectral number density" of hydrometeors (number per size interval per volume)

$\Rightarrow$ field functions:

$\eta = \int_0^{\infty} \sigma_b(D) N(D) dD$  
$\Lambda = \int_0^{\infty} \sigma_{ext}(D) N(D) dD$

$\sim Z_e$  
useful for atten.  (assumption: incoherent single scattering)
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Principle of radar measurement

Meßzeit $\approx 10$ min

$\approx 100$ km

$\sim Z_e$ useful for atten. (assumption: incoherent single scattering)

Intensity $I \sim \frac{1}{r^2 \ell}$
Atmospheric ray propagation

From Fermat's principle:

\[ t = \frac{1}{c} \int_{R}^{P} n'(h(s)) dl = \min \Rightarrow \delta t = 0 \]

calculate \( h(s) \) by solving the Euler-Lagrange equation:

\[
\frac{d^2 h}{ds^2} - \left( \frac{1}{n' dh} + \frac{2}{a_e + h} \right) \left( \frac{dh}{ds} \right)^2 - \left( \frac{a_e + h}{a_e} \right)^2 \left( \frac{1}{n' dh} + \frac{1}{a_e + h} \right) = 0
\]

or by making use of its conserved integral:

\[ n' (h + a_e) \cos(\epsilon_{loc}) = \text{const.} \]

where \( \epsilon_{loc} = \arctan \left( \frac{dh}{dx} \right) = \arctan \left( \frac{a_e}{a_e + h} \frac{dh}{ds} \right) \)
Atmospheric ray propagation

Ray of radiation

\[ n' = \text{refr. index} = fct(p, T, e) \]

From Fermat's principle:

\[ t = \frac{1}{c} \int_{R}^{P} n'(h(s)) \, dl = \min \Leftrightarrow \delta t = 0 \]

calculate \( h(s) \) by solving the Euler-Lagrange equation:

\[
\frac{d^2h}{ds^2} - \left( \frac{1}{n' \, dh} + \frac{2}{a_e + h} \right) \left( \frac{dh}{ds} \right)^2 - \left( \frac{a_e + h}{a_e} \right)^2 \left( \frac{1}{n' \, dh} + \frac{1}{a_e + h} \right) = 0
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Radar operator (reflectivity)

\[
f^2(\phi, \theta) = \exp\left(-4 \ln 2 \left(\frac{\phi^2}{\phi_3^2} + \frac{\theta^2}{\theta_3^2}\right)\right)
\]

\[
\eta(r, \phi, \theta) = \int_0^\infty \sigma_b(D) N(D, r, \phi, \theta) dD
\]

\[
Z_e = \eta \frac{\lambda_0^4}{\pi^5 |K_w|^2}
\]

\[
\langle Z_e^{(R)} \rangle = \frac{\int_{r_0-\Delta r/2}^{r_0+\Delta r/2} \int_{-\pi/2}^{\pi/2} \int_{-\pi}^{\pi} Z_e(r, \phi, \theta) \exp\left(-2 \int_0^r \int_0^\infty \sigma_{\text{ext}}(D) N(D, r', \phi, \theta) dD dD' \right) \frac{f(\phi, \theta)^4}{r^2} \cos \theta d\theta d\phi dr}{\int_{r_0-\Delta r/2}^{r_0+\Delta r/2} \int_{-\pi/2}^{\pi/2} \int_{-\pi}^{\pi} \frac{f(\phi, \theta)^4}{r^2} \cos \theta d\theta d\phi dr}
\]
Radar operator (reflectivity)

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\[
\eta(r, \phi, \theta) = \int_0^\infty \sigma_b(D) N(D, r, \phi, \theta) \, dD
\]

\[
Z_e = \eta \frac{\lambda_0^4}{\pi^5 |K_w|^2}
\]

**Simplification 1:**

\[
\left\langle Z_e^{(R)} \right\rangle = \frac{\int_{\pi/2}^{\pi/2} Z_e(r_0, 0, \theta) \exp \left( -2 \int_0^{r_0} \int_0^\infty \sigma_{ext}(D) N(D, r', \theta) \, dD \, dr' \right) f(0, \theta)^4 \, d\theta}{\int_{-\pi/2}^{\pi/2} f(0, \theta)^4 \, d\theta}
\]
Radar operator (reflectivity)

Simplification 2:

\[
\langle Z_e^{(R)} \rangle = Z_e(r_0, 0, 0) \exp \left( -2 \int_0^{r_0} \int_0^\infty \sigma_{ext}(D) N(D, r') dD dr' \right)
\]

\[
f^2(\phi, \theta) = \exp \left( -4 \ln 2 \left( \frac{\phi^2}{\phi_3^2} + \frac{\theta^2}{\theta_3^2} \right) \right)
\]

\[
\eta(r, \phi, \theta) = \int_0^\infty \sigma_b(D) N(D, r, \phi, \theta) dD
\]

\[
Z_e = \eta \frac{\lambda_0^4}{\pi^5 |K_w|^2}
\]

\[
k_2 = \frac{20}{\ln 10} \Lambda
\]
Approximations for $Z_e$

**In general:** Mie-scattering (one- or two-layered spheres):

$$\sigma_{\text{back}} = f(D,m) \ , \ m = \text{refract. index hydrometeors}$$

(ice/water/air-mixtures)

$$\sigma_{\text{ext}} = f(D,m)$$

**Approximations:**

**Rayleigh:**

$$\sigma_{\text{back}} \sim D^6$$

→ **water drops:** $Z_e \sim M_6$ (analytic for gamma size distr.)

→ **ice hydrom.:** $m$ variable, still need to integrate over $N(D)$

→ **small dry ice hydrom.:** Rayleigh + Debye-approx. for $m$:

$$Z_e \sim \rho^2_{\text{snow}}(D) \ M_6$$
Radial wind operator and simplif.

\[ \langle v_r^{(R)} \rangle = \frac{\int_{r_0 - \Delta r/2}^{r_0 + \Delta r/2} \int_{-\pi}^{\pi/2} \int_{-\pi/2}^{\pi/2} (\hat{v} \cdot \hat{e}_r) \frac{n f^4}{r^2} \cos(\theta) \ d\theta \ d\phi \ dr}{\int_{r_0 - \Delta r/2}^{r_0 + \Delta r/2} \int_{-\pi}^{\pi/2} \int_{-\pi/2}^{\pi/2} \frac{n f^4}{r^2} \cos(\theta) \ d\theta \ d\phi \ dr} - \frac{\int_{r_0 - \Delta r/2}^{r_0 + \Delta r/2} \int_{-\pi}^{\pi/2} \int_{-\pi/2}^{\pi/2} (\hat{e}_3 \cdot \hat{e}_r) \bar{v}_T \frac{n f^4}{r^2} \cos(\theta) \ d\theta \ d\phi \ dr}{\int_{r_0 - \Delta r/2}^{r_0 + \Delta r/2} \int_{-\pi}^{\pi/2} \int_{-\pi/2}^{\pi/2} \frac{n f^4}{r^2} \cos(\theta) \ d\theta \ d\phi \ dr} \]

With:

\[ \bar{v}_T = \frac{\int_{0}^{\infty} \sigma_b(D) N(D) v_T(D) dD}{\eta} \]

Refl.-weighted hydrom. fall speed (computed from model variables or somehow approximated)

\[ l_n^{-2} = \exp \left( -2 \int_{0}^{r} \Lambda(r') \ dr' \right) \]

Attenuation factor
Radial wind operator and simplif.

\[ \langle V_r^{(R)} \rangle = \frac{\int_{r_0 - \Delta r/2}^{r_0 + \Delta r/2} \int_{-\pi}^{\pi} \int_{-\pi/2}^{\pi/2} (v_r \cdot \mathbf{e}_r) \frac{n}{l_n^2} \frac{f^4}{r^2} \cos(\theta) \, d\theta \, d\phi \, dr}{\int_{r_0 - \Delta r/2}^{r_0 + \Delta r/2} \int_{-\pi}^{\pi} \int_{-\pi/2}^{\pi/2} \frac{n}{l_n^2} \frac{f^4}{r^2} \cos(\theta) \, d\theta \, d\phi \, dr} - \frac{\int_{r_0 - \Delta r/2}^{r_0 + \Delta r/2} \int_{-\pi}^{\pi} \int_{-\pi/2}^{\pi/2} (e_3 \cdot \mathbf{e}_r) \overline{v}_T \frac{n}{l_n^2} \frac{f^4}{r^2} \cos(\theta) \, d\theta \, d\phi \, dr}{\int_{r_0 - \Delta r/2}^{r_0 + \Delta r/2} \int_{-\pi}^{\pi} \int_{-\pi/2}^{\pi/2} \frac{n}{l_n^2} \frac{f^4}{r^2} \cos(\theta) \, d\theta \, d\phi \, dr} \]

Simplifications (examples):

- Only vertical smoothing:

\[ \langle V_r^{(R)} \rangle = \frac{\int_{-\pi/2}^{\pi/2} (v_r \cdot \mathbf{e}_r) \frac{n}{l_n} f^4(0, \theta) \cos(\theta) \, d\theta}{\int_{-\pi/2}^{\pi/2} f^4(0, \theta) \cos(\theta) \, d\theta} - \frac{\int_{-\pi/2}^{\pi/2} (e_3 \cdot \mathbf{e}_r) \overline{v}_T \frac{n}{l_n} f^4(0, \theta) \cos(\theta) \, d\theta}{\int_{-\pi/2}^{\pi/2} f^4(0, \theta) \cos(\theta) \, d\theta} \]

- + No reflectivity weighting:

\[ \langle V_r^{(R)} \rangle = \frac{\int_{-\pi/2}^{\pi/2} (v_r \cdot \mathbf{e}_r) f^4(0, \theta) \cos(\theta) \, d\theta}{\int_{-\pi/2}^{\pi/2} f^4(0, \theta) \cos(\theta) \, d\theta} - \frac{\int_{-\pi/2}^{\pi/2} (e_3 \cdot \mathbf{e}_r) \overline{v}_T f^4(0, \theta) \cos(\theta) \, d\theta}{\int_{-\pi/2}^{\pi/2} f^4(0, \theta) \cos(\theta) \, d\theta} \]

- + No hydrometeor fall speed:

\[ \langle V_r^{(R)} \rangle = \frac{\int_{-\pi/2}^{\pi/2} (v_r \cdot \mathbf{e}_r) f^4(0, \theta) \cos(\theta) \, d\theta}{\int_{-\pi/2}^{\pi/2} f^4(0, \theta) \cos(\theta) \, d\theta} - \frac{\int_{-\pi/2}^{\pi/2} (e_3 \cdot \mathbf{e}_r) \overline{v}_T f^4(0, \theta) \cos(\theta) \, d\theta}{\int_{-\pi/2}^{\pi/2} f^4(0, \theta) \cos(\theta) \, d\theta} \]

- + No smoothing, but with fall speed:

\[ \langle V_r^{(R)} \rangle = \mathbf{v} \cdot \mathbf{e}_r + \overline{v}_T \mathbf{e}_3 \cdot \mathbf{e}_r \]
Block diagram and status

Calculate values on the model grid for:
- radar reflectivity
- extinction coefficient
- 3d wind components
- polarisation parameters
- aver. fall speed of hydrometeors

- Horizontal interpolation on "azimuthal slices"
  + Spread "azimuthal slices" evenly over all PE's

- Calculate propagation of radar beam, choose from:
  - constant (4/3 earth model)
  - variable (depending on refractive index)

- Attenuation of radar reflectivity (extinction coefficient) by atmospheric gases and hydrometeors

- Vertical interpolation of values from model grid onto the single radar (sub)beams

- Shading of radar beam at orographic obstacles (yes or no)

- Calculation of radial wind from 3D wind components on radar beam
  - consideration of fall velocities of hydrometeors
  - (atten.) reflectivity weighting

- Beam weighting function: weighted spatial mean over measuring spatial volume (cross-beam vertically and/or horizontally)

- Output of "simulated" radar data to a file:
  - in bin/ascii format
  - in NetCDF format
Results idealized convection 1 km

Radial wind

4/3 earth, no smoothing

4/3 earth, vertical smoothing
Results idealized convection 1 km

Radial wind

Online propag., no smoothing

Online propag., vertical smoothing
Results idealized convection 1 km

Radial wind

Online propag., no smoothing

Online propag., vertical smoothing

Difference to 4/3 earth much too large for this case (nearly „standard“ propagation conditions)!
After code revision and some bug corrections it is now better.
Results idealized convection 1 km

Reflectivity @ wavelength 5.5 cm

Simple Rayleigh approx. no attenuation

Mie scattering + attenuation
Results idealized convection 1 km

Reflectivity @ wavelength 3.0 cm

Simple Rayleigh approx. no attenuation

Mie scattering + attenuation
Efficiency @ 1 km resol.

(version with online propag. calculations)

<table>
<thead>
<tr>
<th>Subroutine</th>
<th>Without space averaging</th>
<th>With space averaging</th>
</tr>
</thead>
<tbody>
<tr>
<td>calc_geometry_grid</td>
<td>97.63% 0.019s (≈ 0.0%)</td>
<td>97.63% 0.019s (≈ 0.0%)</td>
</tr>
<tr>
<td>calc_grd_rfridx</td>
<td>99.91% 0.229s (≈ 0.0%)</td>
<td>99.91% 0.219s (≈ 0.0%)</td>
</tr>
<tr>
<td>calc_grd_winduvw</td>
<td>99.78% 0.374s (≈ 0.0%)</td>
<td>99.78% 0.376s (≈ 0.0%)</td>
</tr>
<tr>
<td>calc_grd_reflectivity</td>
<td>99.34% 0.330s (≈ 0.0%)</td>
<td>99.34% 0.326s (≈ 0.0%)</td>
</tr>
<tr>
<td>calc_geometry_online</td>
<td>85.39% 8.539s (≈ 0.2%)</td>
<td>84.30% 48.89s (≈ 1.1%)</td>
</tr>
<tr>
<td>calc_mod_radialwind_online</td>
<td>99.58% 0.461s (≈ 0.0%)</td>
<td>99.81% 1.771s (≈ 0.1%)</td>
</tr>
<tr>
<td>calc_mod_reflectivity_online</td>
<td>99.60% 0.221s (≈ 0.0%)</td>
<td>99.81% 0.801s (≈ 0.0%)</td>
</tr>
<tr>
<td>output_radar</td>
<td>91.29% 3.512s (≈ 0.1%)</td>
<td>97.58% 6.063s (≈ 0.1%)</td>
</tr>
<tr>
<td>communication expenses</td>
<td>&gt; 1319s (≈ 30.6%)</td>
<td>&gt; 1281s (≈ 30.2%)</td>
</tr>
<tr>
<td>total CPU time</td>
<td>4217.903s</td>
<td>4243.724s</td>
</tr>
</tbody>
</table>

actual Ze-calc. (Mie-scattering) is done elsewhere and hides in the „communication expenses“ (load imbalance)!
Efficiency @ 1 km resol.

For both online propag. and 4/3 earth model:

- Communication amount for online propag. higher, but not so critical (might be different for 2.8 km !)

- „Bottleneck“: Mie-scattering in combination with load imbalance causes long „waiting times“ for idle processors

In consequence:

- Optimization of the scattering parameter calculations on the model grid necessary.
  Easiest way: (regular) lookup tables, computed only once at model start, (multi-)linear table interpolation.
  First: reflectivity lookup tables, later similar concept for polarisation parameters.
Weitergehende Fragestellungen

- Wie kann man Modellverifikation mittels 3D Radardaten betreiben? Was kann man gewinnen?

- Assimilation von Radardaten: wie reagiert das Modell auf die Assimilation solcher Massendaten? Wo liegen die Probleme? Was kann man gewinnen?

- Abschätzung von Effekten der nichtgleichmäßigen Strahlfüllung auf Dämpfung (k2-Ze-Beziehung).
Radial wind operator

\[ \langle v_r^{(R)} \rangle = \left( \int_{r_0 - \Delta r/2}^{r_0 + \Delta r/2} \int_{-\pi}^{\pi} \int_{-\pi/2}^{\pi/2} \left( \int_0^{\infty} \sigma_b(D) N(D, r, \phi, \theta) \left[ (\vec{v} - v_T(D) \vec{e}_3) \cdot \vec{e}_r \right] dD \right) \frac{f(\phi, \theta)^4}{r^2} \cos \theta d\theta d\phi dr \right) \]

\[ \left( \int_{r_0 - \Delta r/2}^{r_0 + \Delta r/2} \int_{-\pi}^{\pi} \int_{-\pi/2}^{\pi/2} \eta(r, \phi, \theta) \frac{f(\phi, \theta)^4}{r^2} \cos \theta d\theta d\phi dr \right) \]

\[ = \left( \int_{r_0 - \Delta r/2}^{r_0 + \Delta r/2} \int_{-\pi}^{\pi} \int_{-\pi/2}^{\pi/2} (\vec{v} \cdot \vec{e}_r) \frac{\eta}{I_n^2} \frac{f^4}{r^2} \cos(\theta) d\theta d\phi dr \right) \]

\[ - \left( \int_{r_0 - \Delta r/2}^{r_0 + \Delta r/2} \int_{-\pi}^{\pi} \int_{-\pi/2}^{\pi/2} (\vec{v}_r \cdot \vec{e}_r) \int_0^{\infty} \sigma_b(D) N(D) v_T(D) dD \frac{1}{I_n^2} \frac{f^4}{r^2} \cos(\theta) d\theta d\phi dr \right) \]
Aufzählung
- asdfasdf
- asfdasdf

Zweiter Punkt
- asdfasdf
- asdasdf