Objective model error estimation, modelling, and simulation. First results with COSMO

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Introduction. Model-error modelling approaches

1. Fully ad hoc
   - Covariance inflation
   - Multi-model
   - Multi-parametrization
   - Perturbed physical parameters
   - Stochastic physics (multiplicative ME perturbations)
   - Additive ME perturbations
2. Partly ad hoc
   - Stochastic kinetic energy backscatter
   - Stochastic physical parametrizations

General setup

1. Fix the model-error model (MEM): a mixed *multiplicative-additive* model
2. Develop an objective MEM **Estimator** using real forecast and observation data
3. Build a ME **Generator** and embed it into the COSMO code
4. Build a MEM **Validator** using the OSSE methodology: apply the Estimator to the simulated data generated with the *known* MEM
5. Using the Validator's results, adjust/calibrate the Estimator

Model error: definition

$$\frac{dX^m}{dt} = F(X^m)$$

$$\frac{dX^t}{dt} = F(X^t) - \varepsilon$$

$$\varepsilon = F(X^t) - \frac{dX^t}{dt}$$

Estimation methodology in general terms

$$\varepsilon = F(X^t) - \frac{dX^t}{dt}$$

1. The first term on the r.h.s. can be assessed using model forecast tendencies $F(X^m)$ — up to the difference $\delta F = F(X^m) - F(X^t)$.
2. The second term on the r.h.s. can be assessed using observations at isolated observation points — up to observation errors.

The model-error model

$$\varepsilon = b + \mu \cdot F(X^t) + \varepsilon_a$$

$$\varepsilon = b + \mu \cdot F_p(X^t) + \varepsilon_a$$

(The bias b proves to be nearly zero. F_p is the 'physical' tendency.)

Model-error-model parameters to be estimated:

$$\theta = (\mathbb{E}\mu,\ \mathbb{D}\mu,\ \mathbb{D}\varepsilon_a)$$

The estimator

The maximum likelihood technique:

$$\mathrm{lik}(\theta) := p(y;\theta)$$

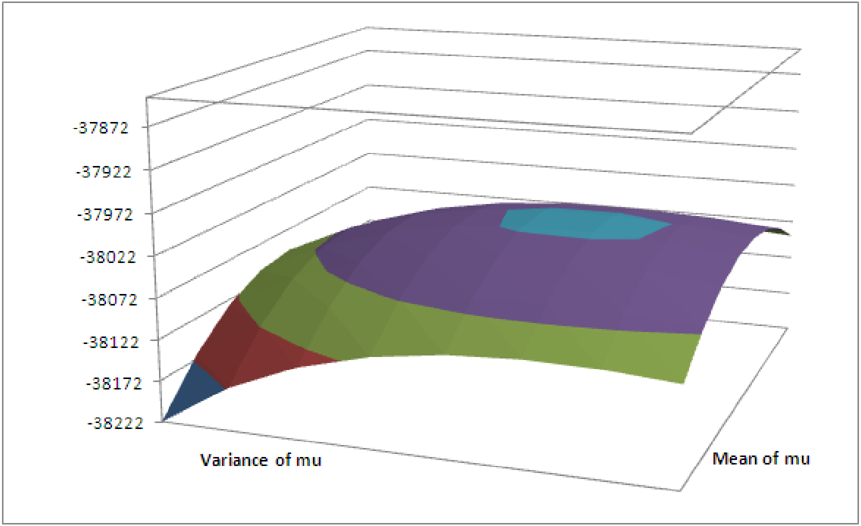
$$y := (d, f_p)$$

$$d := f - m$$

$$\hat{\theta} = \operatorname*{argmax}_{\theta} L(\theta)$$

$$\mathrm{lik}(\theta) \propto \frac{1}{\sqrt{\sigma_\mu^2 f_p^2 + \sigma_a^2 + \sigma_\Delta^2}} \exp\left[-\frac{1}{2}\cdot\frac{(d-\bar{\mu} f_p)^2}{\sigma_\mu^2 f_p^2 + \sigma_a^2 + \sigma_\Delta^2}\right].$$

A cross-section of the log-likelihood function (for the full tendency F)



Bootstrap tests (full tendency)

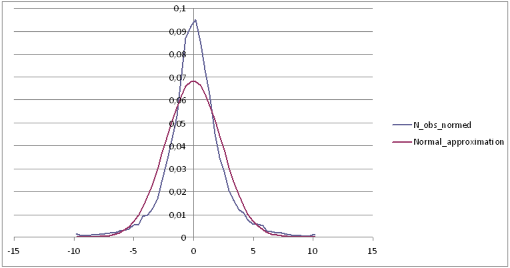
Sample size: corresponds to a 3-month archive of real data for COSMO-RU.

θ :	$\sqrt{\mathbb{D}\mu}$	$\mathbb{E}\mu$	$\sqrt{\mathbb{D}\varepsilon_a}$
"Truth"	0.32	0.20	0.63
Experiment 1	0.33	0.19	0.61
Experiment 2	0.32	0.19	0.66
Experiment 3	0.31	0.20	0.63

Data

- COSMO-RU 14 km.
- Radiosonde temperature and wind observations.
- 12-h tendencies.
- A 3-month data archive is used.
- At each level, for each field, we have about 100 observed tendencies per day over the COSMO-RU area.

COSMO-RU results. Checking Guassianity of $F(X^m)$ (full tendency)



The estimator appeared to be *robust* to non-Gaussianity of this kind.
The true tendency was specified (using a Gaussian mixture) to have the same *leptokurtic* non-Gaussian behaviour (with the same excess kurtosis as observed for the forecast tendency).

Extrapolation to zero δF

As a proxy to

$$\delta F^m \equiv \delta = F(X^m) - F(X^t).$$

we use

$$\delta_a = F(X^m) - F(X^a) \equiv [F(X^m) - F(X^t)] - [F(X^a) - F(X^t)].$$

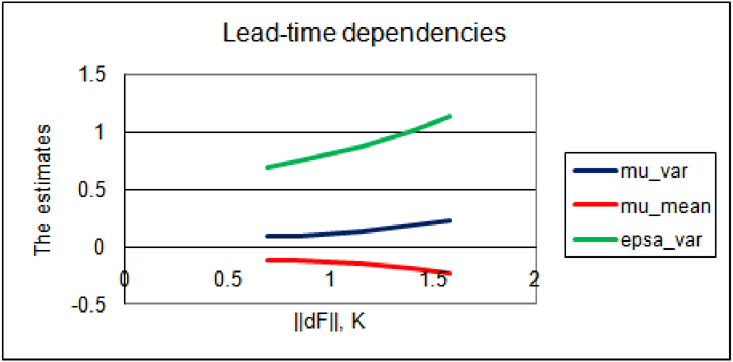
Both δ and δ_a represent *error growth*, which is significantly faster if the forecast is started from a forecast state than from an analysis (due to the "breeding errors" effect).

If, in addition, X^a is substantially more accurate than X^m, then we have the two reasons to neglect the $F(X^a) - F(X^t)$ term and, thus, use δ_a instead of unknown δ.

Then, we extrapolate the estimates to $\delta_a = 0$.

Dependence of the estimates on the st. dev. of $\delta F = F(X^m) - F(X^t)$ (full tendency)

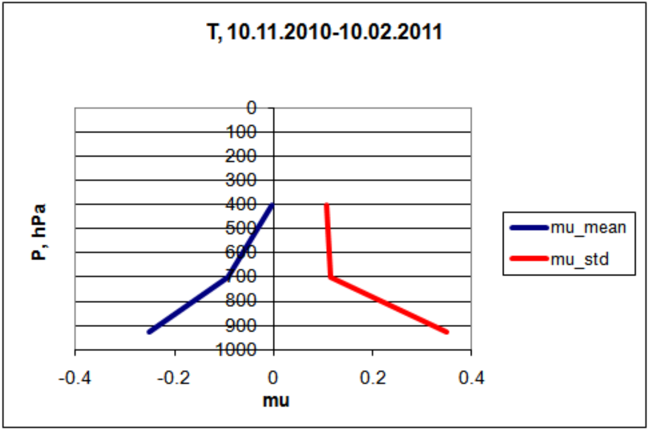
T-700



Mean and st. dev. of the multiplier **mu** (full tendency)

Temperature

$$\varepsilon = (\mathbb{E}\mu + \overset{\circ}{\mu}) \cdot F + \varepsilon_a$$



The MEM Validator: methodology

1. Generate the ‘truth’ with the known MEM.
2. Simulate the ‘observations’: the ‘truth’ plus obs err.
3. Develop a simplified analysis and run the ‘assimilation suite’ with the unperturbed COSMO and simulated observations.
4. Run 2-day forecasts from the ‘analyses’.
5. Apply the Estimator to the simulated both forecast and observational data.
6. Compare the estimated ME parameters with those specified by the generation of the ‘truth’.
   The systematic differences, if not too large, can be used to correct the real-world estimates.

The MEM Validator: the 'truth'

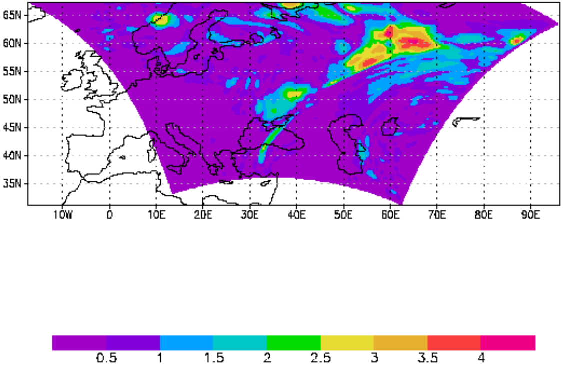
Fix the boundary conditions and run COSMO with the MEG switched on (for, say, one month). The additive ME is dropped here:

$$\frac{dX^t}{dt} = F(X^t) - (\mathbb{E}\mu - \overset{\circ}{\mu}) \cdot F_p(X^t)$$

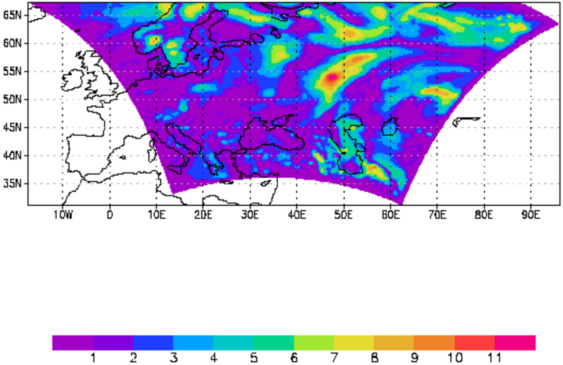
The resulting 'long-range' forecast will serve as (a realization of) the simulated truth.

The resulting 'truth' can be compared with:
(1) the control long-range forecast (downscaled LBC)
(2) the real-world COSMO (GME) analyses

The simulated ‘truth’ minus the control fcst (T700,day 5)



The control fcst minus the real-world analysis (T700,day 5)



The MEM Validator: the 'analysis'

The goal: build a 'toy analysis'.

The solution: specify a diagonal gain matrix K.

This can be justified if
(i) All degrees of freedom are observed: $H = I$
(ii) $R \propto B$.

So, we try to specify R with spatial obs-err correlations similar to those for B.
Then, the analysis reduces to point-wise corrections:

$$K = B(B+R)^{-1} = \frac{B}{B+R} = \frac{1}{1+r/b} \cdot I \equiv w \cdot I$$

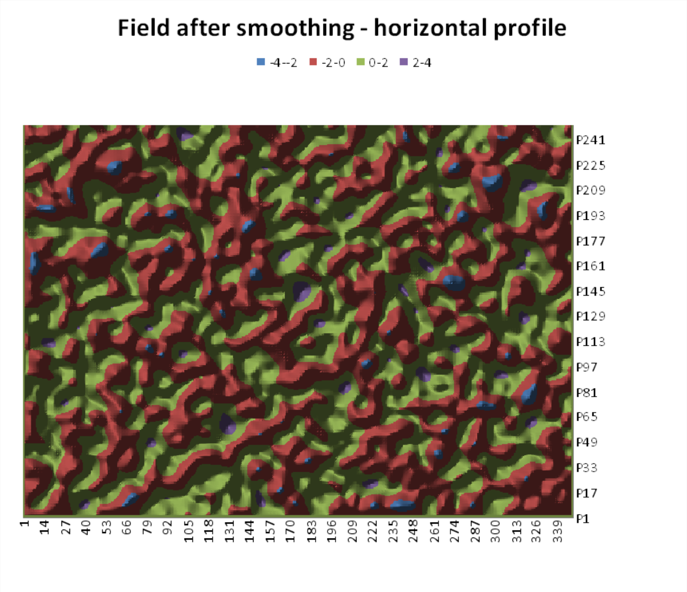
$$X^a_{ijke} = X^f_{ijke} + w_{(ke)} \cdot (X^o_{ijke} - X^f_{ijke})$$

The MEM Validator: the 'observations'

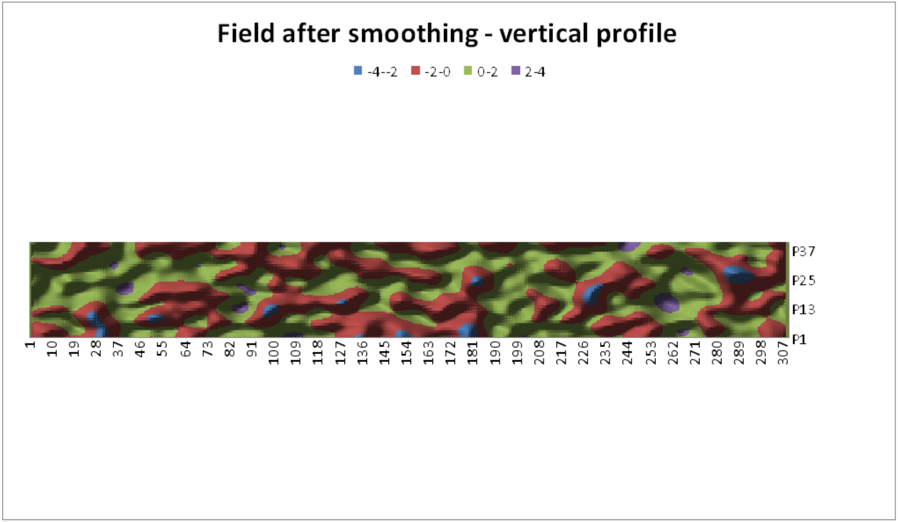
The 'observations' are the 'truth' plus the noise.
The noise is specified as follows.

- Independent Gaussian pseudo-random variables are assigned to points of a *sparse* ($5 \times 5 \times 3$ thinned) 3-D grid. So, on the sparse grid we have the white noise.
- The tri-linear interpolation is performed from the sparse to the real COSMO grid. The spatial correlations emerge on the length scales of the sparse-grid mesh sizes, but some inhomogeneity arises.
- Several sweeps of a $5 \times 5 \times 3$ running smoother are performed to homogenize and further increase the spatial scales of the noise – so that R become closer to B.

The observational noise (hor)



The observational noise (vert)



The MEM Validator: the 'assimilation suite'

The classical 'analysis-forecast' suite is built with the cycling frequency of 6 h.

FG-error and analysis-error rms statistics (level 20, asml day 30):

	T (K)	U (m/s)	V (m/s)
FG minus 'truth'	0.8	3.2	2.8
Anls minus 'truth'	0.6	1.9	1.6
Free fcst minus 'truth'	3	8	9.5

Reproducibility of the MEM parameters

Is the 'output' $\hat{\theta}$ close to the 'input' (specified or postulated) θ?

Not yet...

Conclusions

- A new maximum likelihood based approach to the estimation of the mixed multiplicative-additive ME model is suggested and tested (using bootstrap).
- The developed Estimator is applied to COSMO-RU-14 forecast data.
- The ME Generator is developed and built into the COSMO code.
- The MEM Validator is developed: specify the 'true' ME, produce (by running COSMO with MEG) the 'truth', define noisy 'observations', mimic the assimilation suite, mimic the 6 to 48-h. free forecasts (the 12-h. tendencies from which are used as input to the Estimator), and try to restore the MEM parameter with the Estimator.

  The reproducibility of the MEM parameters is not yet established.

Issues and further steps

- There is an inconsistency in the setup: MEM is estimated using accumulated physical tendencies, whereas MEG acts on simultaneous (80-sec.) tendencies.

  Tentative remedy: multiply μ by a moving-accumulated physical tendency, with the accumulation time scale to be identified experimentally (1 h can be a starting point).
- Introduce spatial (not just temporal) averaging of physical tendencies: the model error need not be maximal at exactly the same place where the forecast tendency is maximal, it can be maximal somewhere nearby (normally, the forecast fails to exactly locate a feature like front or convective system).
- Turn to spatial, multivariate, and spatio-temporal estimation of the μ and ε_a random fields—and, correspondingly, upgrade the MEG.
- Another possible approach: make use of 'perfect parametrization' schemes (if available): on the forecast trajectory, compute 1-time-step 'perfect tendencies' and confront them with the COSMO ones.

  This approach can result in different μ for different parametrizations.