# Influence of terrain-following coordinates on nearly hydrostatic flows in EULAG

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# OUTLINE

1. Motivation
2. Temperature profiles within anelastic framework
3. Setup of the experiment
4. Results
   - atmosphere at rest
   - shear flow
5. Conclusions

# 1. Motivation

Well known computational problem:

| coordinates | | hydrostatic flow |
|---|---|---|
| Cartesian | → | well balanced (vertical and horizontal equations of motion are separated) |
| curvilinear (e.g. terrain-following) | ↱ | transformation to model coordinates leads to residual buoyancy in model equations and may drive artifitial flow; multidimensional |

**OBJECT:** verify the influence of errors associated with terrain-following coordinates on the EULAG's solutions for nearly hydrostatic flows in a presence of orography

# 2. Temperature profiles within anelastic framework

- Generic form of inviscid anelastic ($\rho' \leqslant \bar{\rho}$) equations of motion (Lipps and Hemler, 1982, JAS):

$$\frac{D\vec{u}}{Dt} = -\nabla \frac{p-\bar{p}}{\bar{\rho}} + \vec{g}\frac{\theta-\bar{\theta}}{\bar{\theta}} - \vec{f}\times\vec{u} \quad , \qquad (1)$$

where $\{\bar{p}, \bar{\rho}, \bar{\theta}\}$ is hydrostatic basic state, horizontally homogeneous and of constatnt stability (Clark and Farley, 1984, JAS)

- Assuming underlaying particular geostrophically balanced solution of (1):

$$0 = -\nabla \frac{p_e-\bar{p}}{\bar{\rho}} + \vec{g}\frac{\theta_e-\bar{\theta}}{\bar{\theta}} - \vec{f}\times\vec{u}_e \qquad (2)$$

and subtracting (2) from (1) yields a common perturbational form:

$$\frac{D\vec{u}}{Dt} = -\nabla \frac{p-p_e}{\bar{\rho}} + \vec{g}\frac{\theta-\theta_e}{\bar{\theta}} - \vec{f}\times(\vec{u}-\vec{u}_e) \quad , \qquad (3)$$

where $\theta_e$ is thought to be reference (environmental) hydrostatic profile, no longer required to have constant stability.

# 2. Temperature profiles within anelastic framework

$$\frac{D\vec{u}}{Dt} = -\nabla\frac{\overbrace{p-p_e}^{p'}}{\bar{\rho}} + \vec{g}\,\frac{\overbrace{\theta-\theta_e}^{\theta'}}{\bar{\theta}} - \vec{f}\times\overbrace{(\vec{u}-\vec{u}_e)}^{u'} \qquad (3)$$

For a *mean* flow initial condition is said to be hydrostatically balanced, i.e.

$$\theta' = \theta - \theta_e = 0 \qquad (4)$$

around which perturbations are being enabled to develop.

… but imagine the situation (typical in weather forecasting) where reference state $\theta_e$ is some arbitrary (e.g. seasonal, annual, etc.) state and actual mean temperature is biased ($<\theta'> \neq 0$, e.g. >0 in Summer)?

# 2. Temperature profiles within anelastic framework

$$p' \quad \theta' \quad u'$$

$$\frac{D\vec{u}}{Dt} = -\nabla \frac{p - p_e}{\bar{\rho}} + \vec{g}\frac{\theta - \theta_e}{\bar{\theta}} - \vec{f} \times (\vec{u} - \vec{u}_e) \qquad (3)$$

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It is necessary to correct the form of pressure potential, in order to recall the environment to be initially well balanced.

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pressure potential $\Psi$

Correction to pressure potential comes from hydrostatic equation:

$$0 = -\nabla \Psi_{bias} + \vec{g}\frac{\theta_{bias}}{\bar{\theta}} \quad , \qquad (5)$$

and this modification ($\Psi_{bias}$) is put into initial environmental state $\Psi$. After this procedure pressure perturbation is being calculated around $\Psi + \Psi_{bias}$.

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Anelastic approximation in EULAG allows to work with three different temperature profiles $\{\bar{\theta}, \theta_e, \theta\}$.

# 3. Setup of the experiment

Background anelastic profile (Clark and Farley, 1984, JAS):

$$\bar{\theta}(z)=\theta_0 e^{\frac{N^2}{g}z}, \quad (6)$$

where N=0.01 1/s, ground values of temperature and pressure

$$\theta_0=T_0=288.15\mathrm{K}$$

$$p_0=10^5\,Pa$$

Reference (environmental) state was defined in log-pressure coordinates as:

$$C=\frac{dT}{dlogp}=0.42, \quad (7)$$

what means that temperature profile has a form:

$$T(z)=T_0\sqrt{1-az} \quad (8)$$

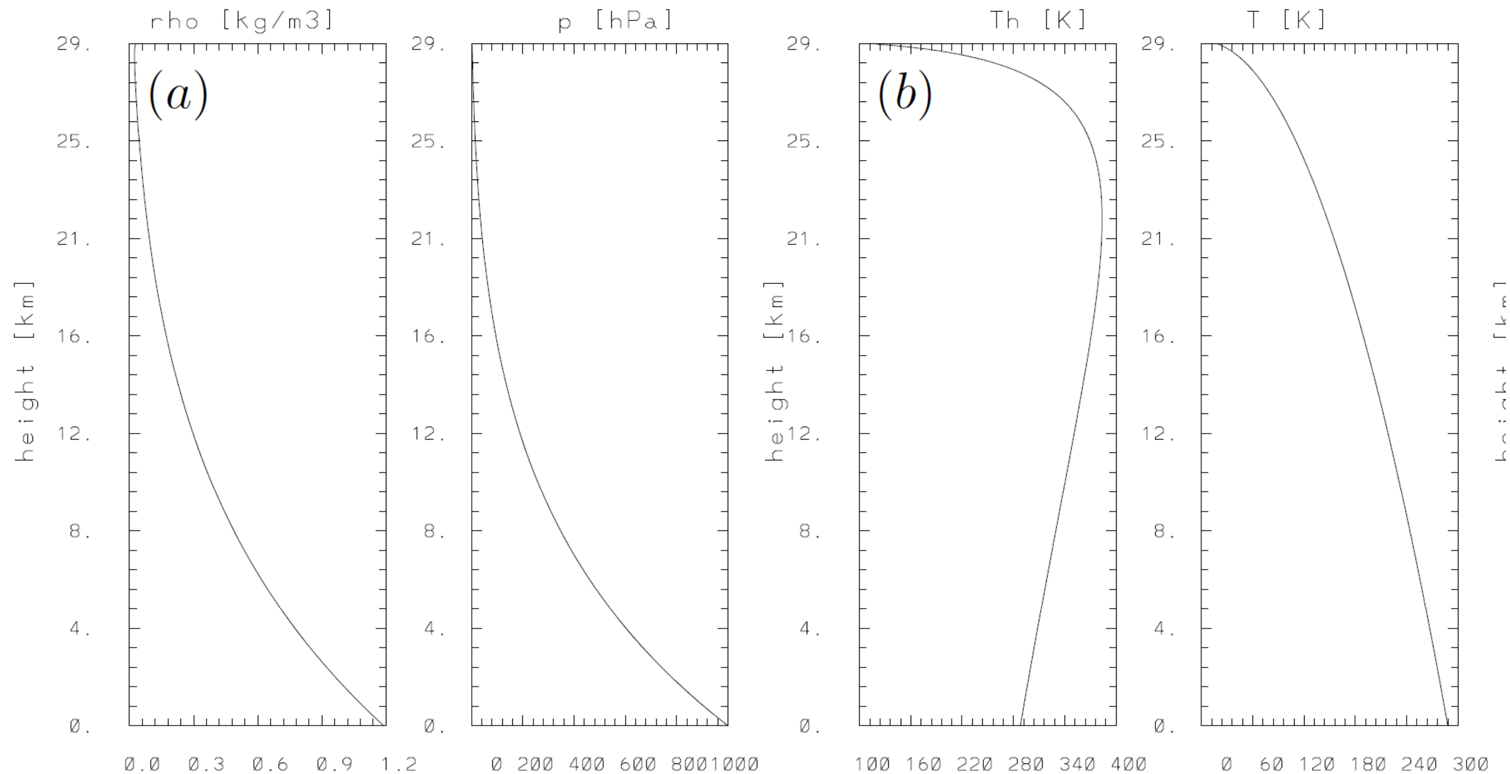
where:

$$a=\frac{2\mathrm{gC}}{R}T_0^2 \quad (9)$$

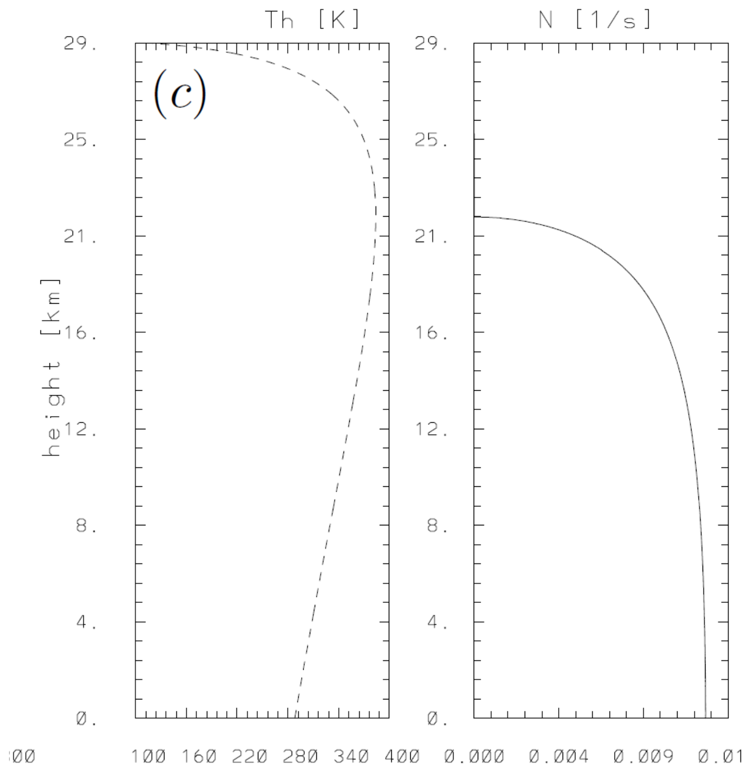
# 3. Setup of the experiment

$$\theta_e(z) = T_0\sqrt{1-az}\, e^{\frac{T_0 R}{Cc_p}\left(1-\sqrt{1-az}\right)} \qquad (10)$$

$$z_t = \frac{1}{a}\left(1-\frac{Cc_p}{RT_o}\right) \approx 21.3\, km$$ where stability drops to 0



# 3. Setup of the experiment



# 3. Setup of the experiment

Basic domain size 320x20km (1x0.4km)

2 types of flow:

- atmosphere initially at rest
- shear flow (Schar et al. 2002)

2 types of mountain:

- gaussian

$$h(x,z)=h_0 e^{(-((x-x_0)/\sigma)^2)}$$

- Schar mountain

$$\sigma=8\text{km}$$

$$h(x,z)=h_0 \cos(\pi(x-x_0)/x_c)^2 \cos((\pi(x-x_0))/\sigma)^2$$

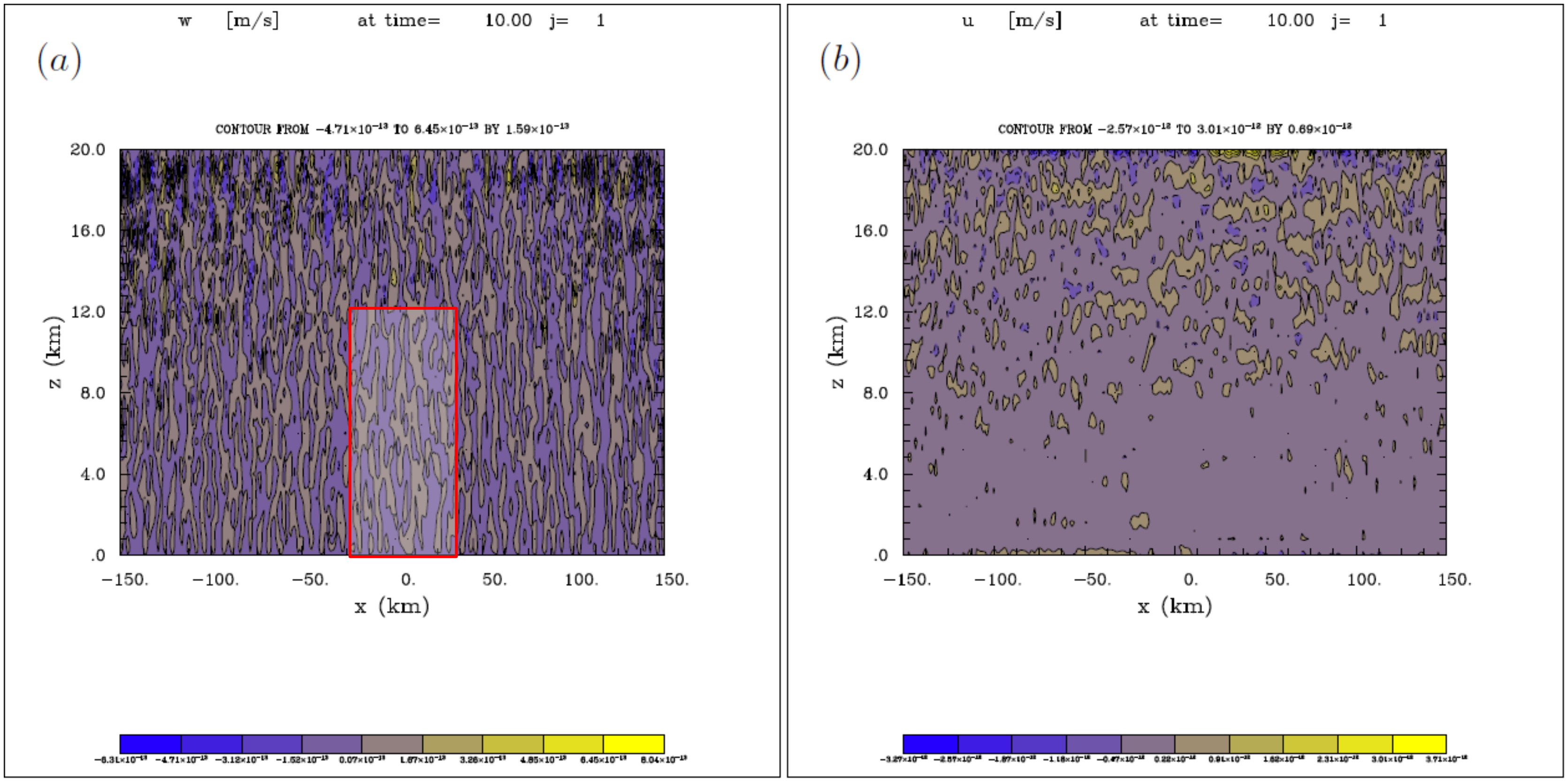
2 strategies:

- no initial perturbation
- initial perturbation introduced to velocity field (10e-3m/s)

Time of simulations = 10h

# 4. Results

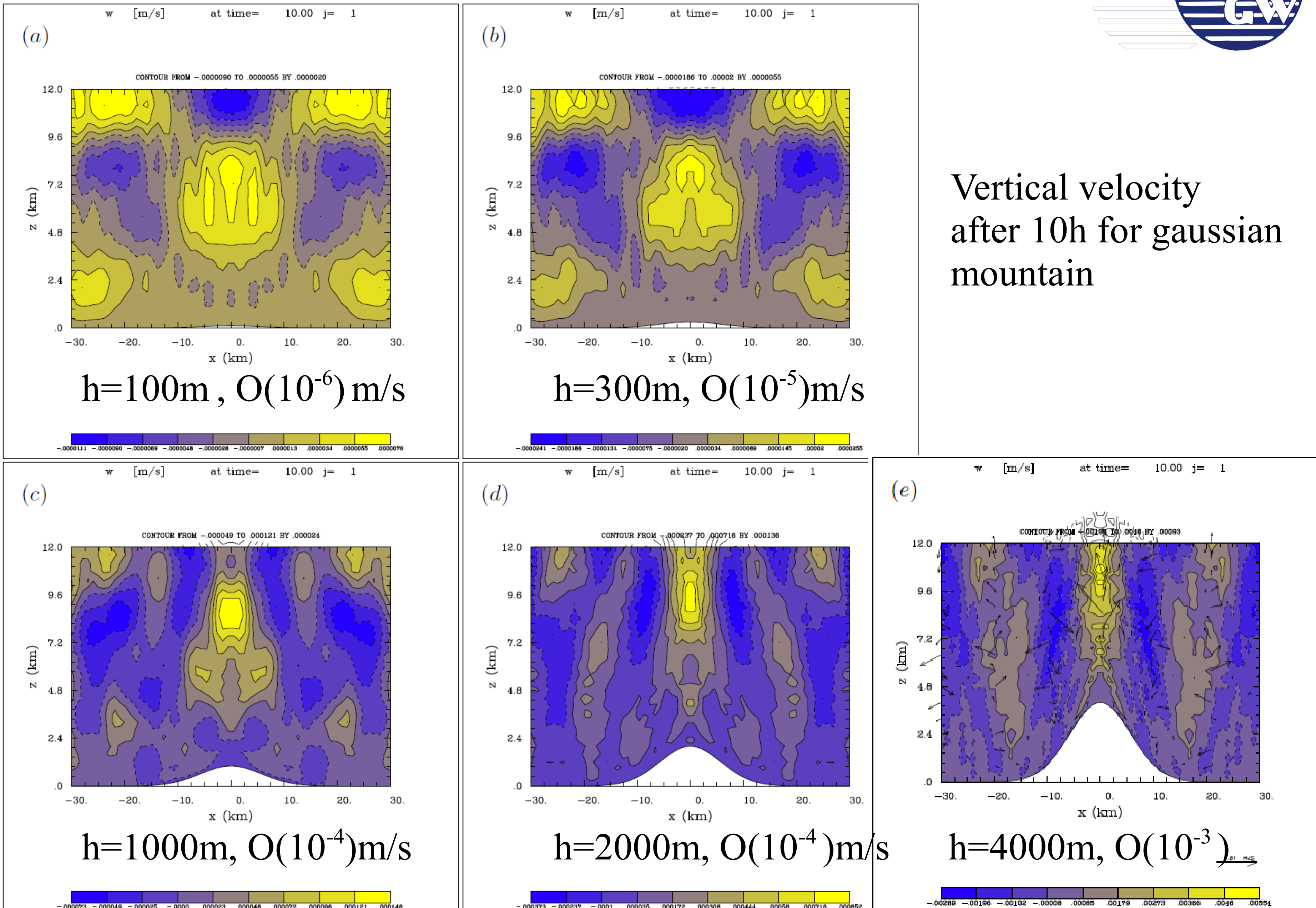
Atmosphere initially at rest /periodic and rigid b.c. tested/




Vertical (a) and horizontal velocity (b) after 10h of simulation – max. fluctuations $O(10^{-13})$m/s

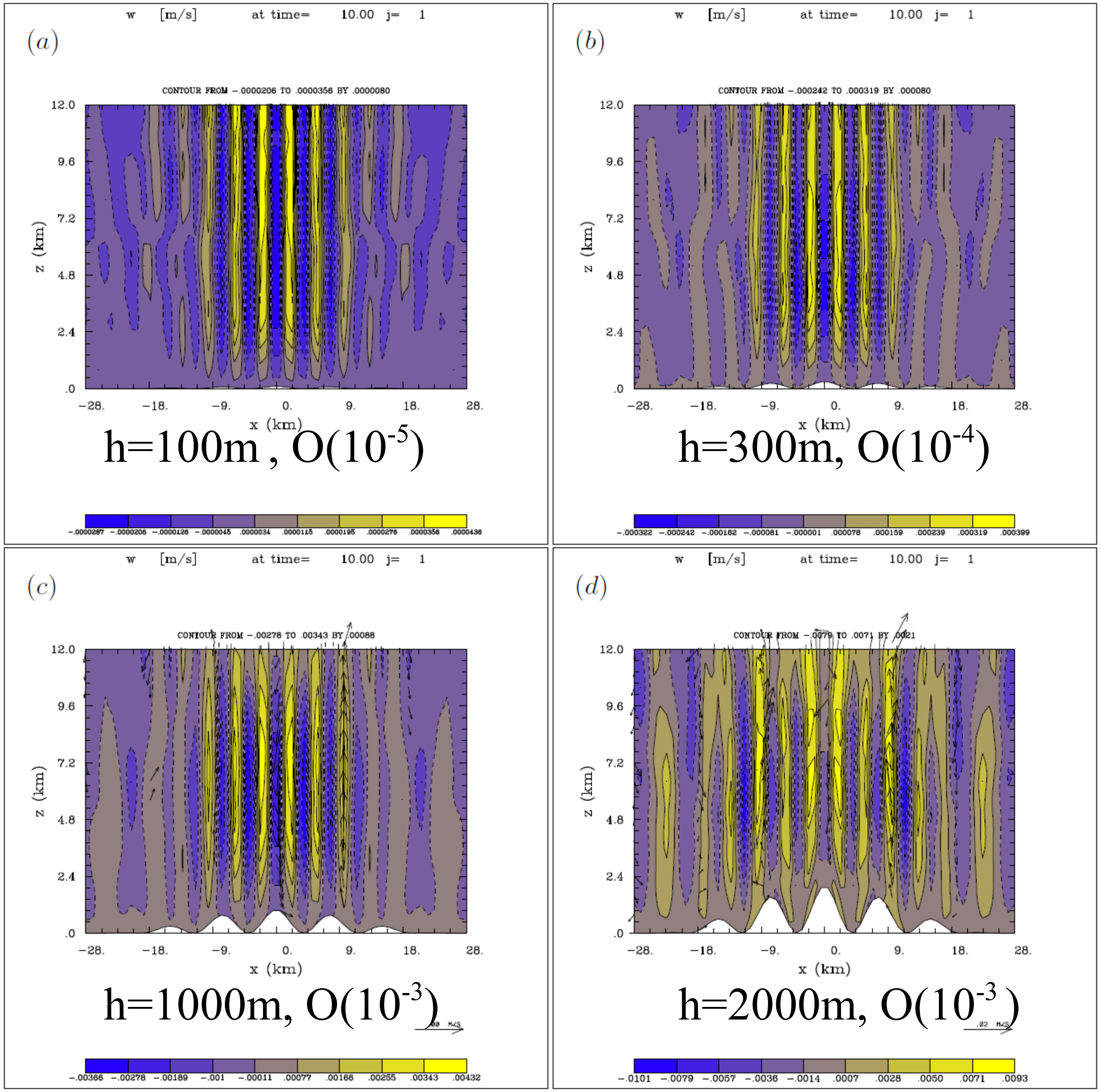
Every non-zero velocity is distortion from analytic solution

# 4. Results



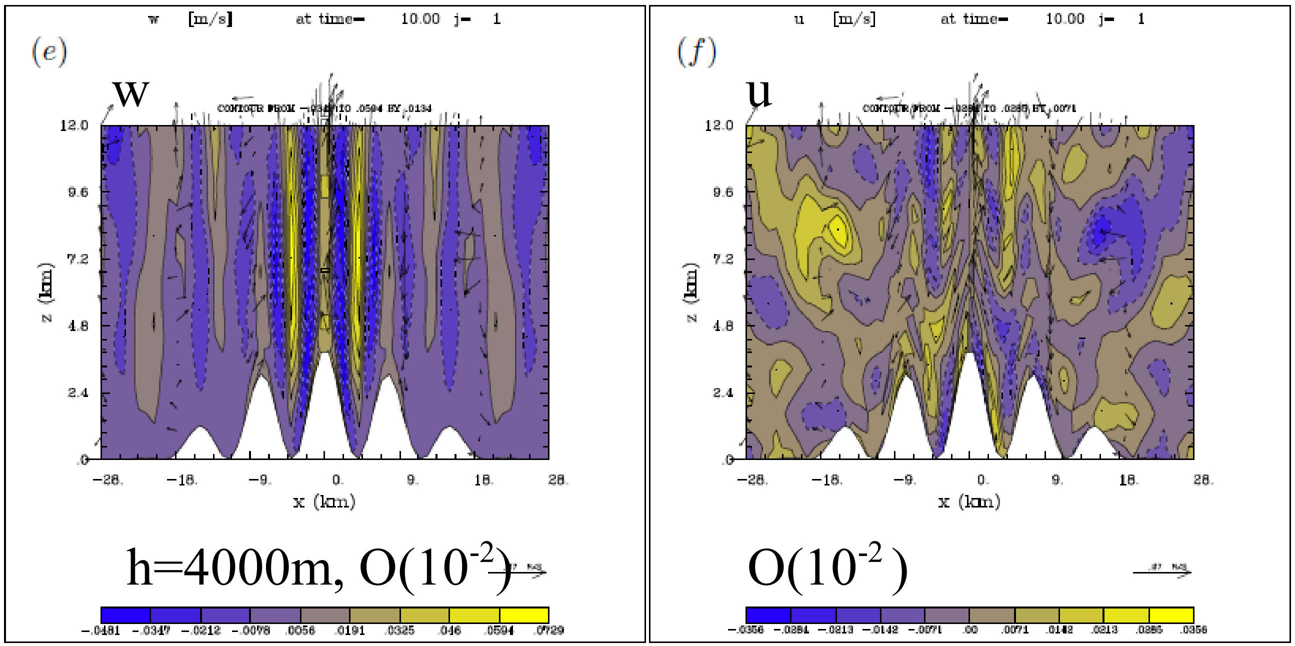

Vertical velocity after 10h for gaussian mountain

# 4. Results




Vertical velocity after 10h for Schar mountain

# 4. Results




Vertical velocity after 10h for Schar mountain

# 4. Results

Shear flow

specified as in Schar et al. (2003)

$$u(z) = u_0 \begin{cases} 1 & for \quad z_2 \leq z \\ sin^2(0.5\pi(z - z_1)/(z_2 - z_1)) & for \quad z_1 \leq z \leq z_2 \\ 0 & for \quad z \leq \quad z_1 \end{cases}$$

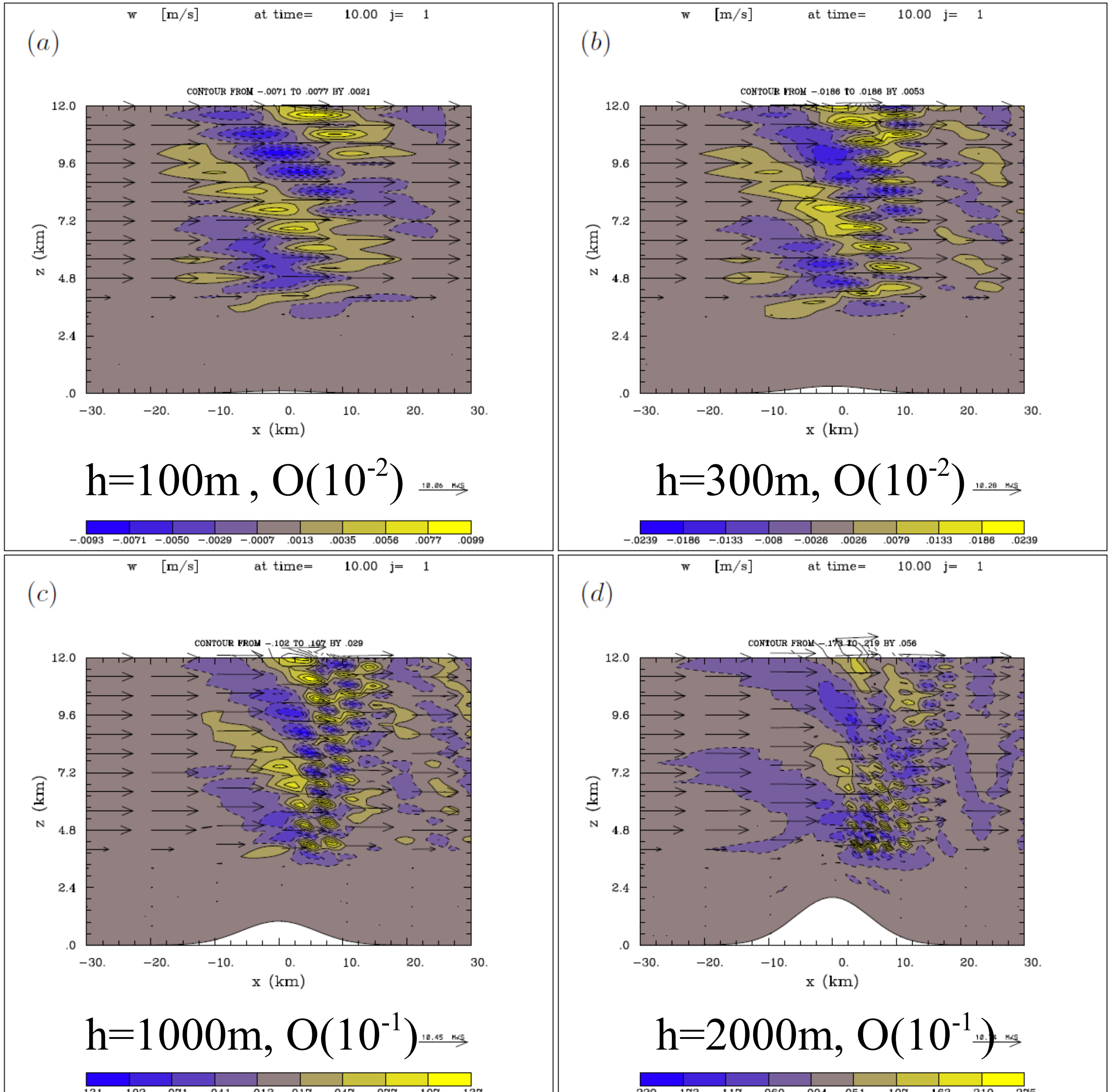
$u_o = 10 \mathrm{m}/s$

$z_1 = 4\mathrm{km}$

$z_2 = 5\mathrm{km}$

- open boundary conditions
- lateral absorbers (20km) in order to prevent wave reflection
- absorber in upper part of domain (13-20km)

# 4. Results




Vertical velocity after 10h for Schar mountain

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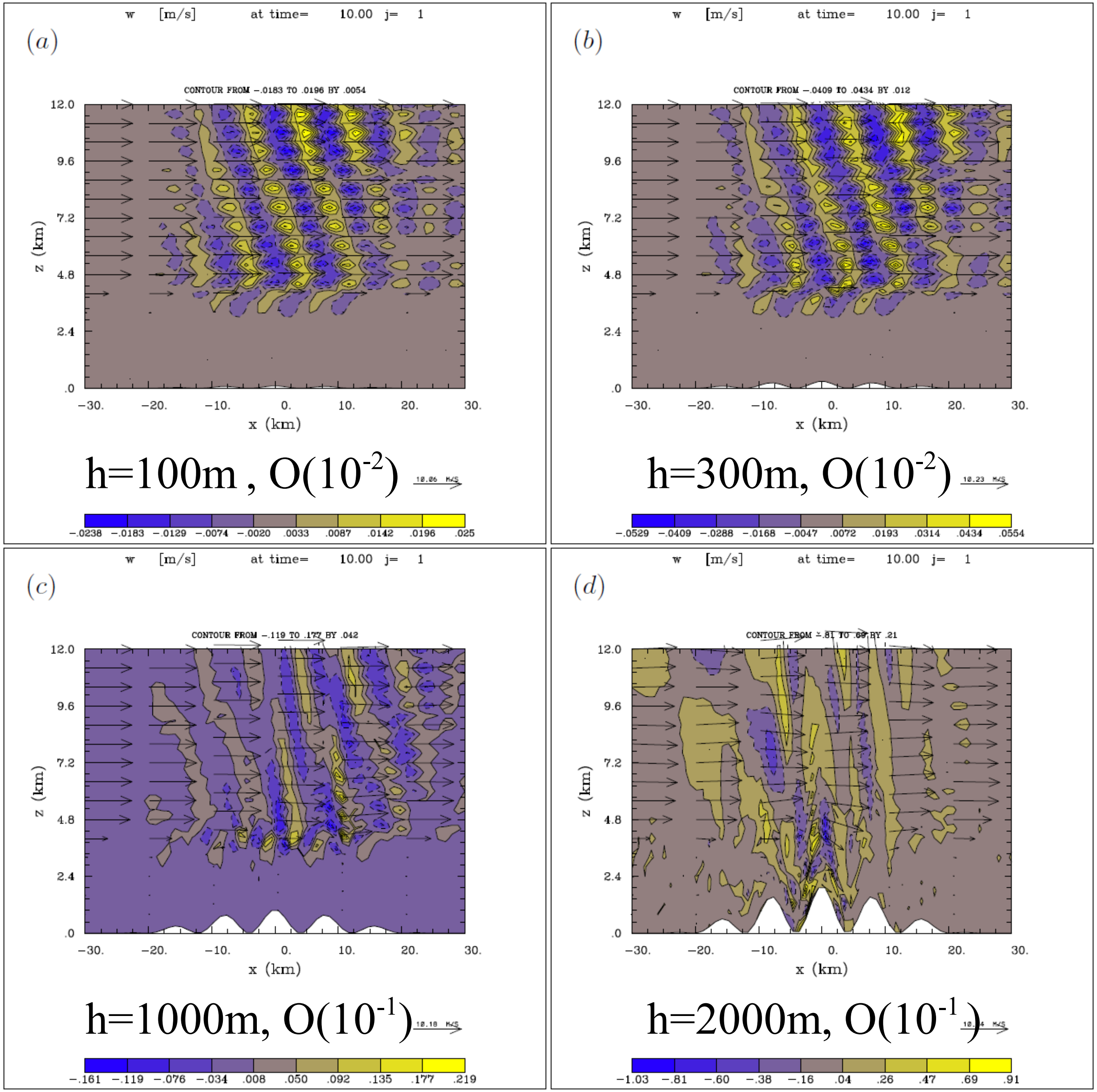


Vertical velocity after 10h for Schar mountain

# 4. Results

## Comparison of maximum values of vertical velocity after 10h of simulations

| | NO FLOW | | SHEAR FLOW | |
|---|---|---|---|---|
| | gauss | Schär | gauss | Schär |
| $h$ $[m]$ | $w$ $[m/s]$ | $w$ $[m/s]$ | $w$ $[m/s]$ | $w$ $[m/s]$ |
| 0 | $\mathcal{O}(10^{-13})$ | | | |
| 100 | $1 \cdot 10^{-5}$ | $4.4 \cdot 10^{-5}$ | $9.9 \cdot 10^{-3}$ | $2.5 \cdot 10^{-2}$ |
| 300 | $2.5 \cdot 10^{-5}$ | $4.0 \cdot 10^{-4}$ | $2.4 \cdot 10^{-2}$ | $5.5 \cdot 10^{-2}$ |
| 500 | $4.4 \cdot 10^{-5}$ | $1.1 \cdot 10^{-3}$ | $4.8 \cdot 10^{-2}$ | $1.2 \cdot 10^{-1}$ |
| 1000 | $1.5 \cdot 10^{-4}$ | $4.3 \cdot 10^{-3}$ | $1.4 \cdot 10^{-1}$ | $2.2 \cdot 10^{-1}$ |
| 2000 | $8.5 \cdot 10^{-4}$ | $9.3 \cdot 10^{-3}$ | $2.8 \cdot 10^{-1}$ | $9.1 \cdot 10^{-1}$ |
| 4000 | $5.5 \cdot 10^{-3}$ | $7.3 \cdot 10^{-2}$ | - | - |

# 4. Conclusion

• Artifitial fluctuations in the vertical velocity field due to terrain-following coordinate transformation depend on steepness of the slopes and are negligible for the atmosphere at rest

• For idealized shear flow vertical velocity fluctuations were of the order of mm/s to cm/s, growing up to almost 1m/s for extremally steep Schar mountain

• No computational problems were reported during model runs, even for very steep orography