The 3d-turbulence parameterisation scheme for LMK

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relevant equation fragments:

\[ \frac{\partial u_i}{\partial t} = \ldots - \frac{1}{\rho} \frac{\partial \tau_{ij}}{\partial x_j} \]
\[ \frac{\partial X^n}{\partial t} = \ldots - \frac{1}{\rho} \frac{\partial Y^n_j}{\partial x_j} \]

3d-flux scheme

with

\[ u_i = (u, v, w) \]
\[ (X^1, X^2) = (T, q^1) \]

specified flux components:

\[ \tau_{ij} = -\rho K^i_m \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \]
\[ Y^1_j = -c_{pd} \rho \pi K^i_h \frac{\partial \Theta}{\partial x_j} \]
\[ Y^2_j = -\rho \pi K^i_h \frac{\partial q^k}{\partial x_j} \]

simplified anisotropic diffusion approach

\[ K^H_m := K^{11}_m = K^{12}_m = K^{22}_m \quad K^V_m := K^{13}_m = K^{23}_m = K^{33}_m \quad \text{(momentum)} \]
\[ K^H_h := K^1_h = K^2_h \]
\[ K^V_h := K^3_h \quad \text{(heat)} \]

aspect ratio

[Dunst (1980)]

\[ K^H_{m,h} := r \sqrt{(a \cos \Delta \phi)^2 \sqrt{(a \cos \Delta \phi)^2 + (a \Delta \phi)^2}} \]
\[ K^V_{m,h} \approx r \frac{\sqrt{2 \Delta}}{\Delta z} \]
\[ K^V_{m,h} = \alpha K^V_{m,h} \]
Prandtl-Kolmogorov-specification of vertical diffusion

\[
K^V_m = \phi_m l(\bar{\epsilon})^{1/2} \quad K^V_h = \phi_h l(\bar{\epsilon})^{1/2}
\]

Mixing length

\[ l = \kappa_z \left( l + \frac{\kappa_z}{l_\infty} \right)^{-1} \quad \bar{\epsilon} = \frac{1}{2} \left( \frac{u_i u_i}{u_i} \right) \]

Turbulent kinetic energy:

Mellor & Yamada SGS-model

\[
K^V_m = l^2 \sigma_m^{1/2} \left( S_v^2 - \alpha_n \sigma_h N^2 \right)^{1/2}
\]

\[
\bar{\epsilon} = c_e^{-2/3} \sigma_m l^2 S_v^2 \left( l - Pr^{-1} Ri_v \right)
\]

coincidence at equilibrium case

\[
\phi_m l^{1/2} = l^2 \sigma_m^{2/3} \left( S_v^2 - \alpha_n \sigma_h N^2 \right)^{1/2}
\]

\[
Ri_v = \frac{N^2}{S_v^2}
\]

finally gives

\[
\phi_m = c_e^{1/3} \sigma_m \quad \phi_h = \alpha_n c_e^{1/3} \sigma_m \sigma_h
\]
Prognostic TKE-equation

\[ \frac{\partial \bar{e}}{\partial t} + u_j \frac{\partial \bar{e}}{\partial x_j} = \delta_{ij} \frac{g}{\Theta_v} \frac{\partial (u_i \Theta_v)}{\partial x_j} - u_i \frac{\partial u_j}{\partial x_j} - \frac{\partial (u_j e)}{\partial x_j} - \frac{1}{\rho} \frac{\partial (u_i p)}{\partial x_i} - \varepsilon \]

\[ N^2 = \frac{g}{\Theta_v} \frac{\partial \Theta_v}{\partial z} \]

\[ S^2 = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)^2 \]

\[ S_H^2 = 2 \left( S_{11}^2 + S_{22}^2 \right) + 4 S_{12}^2 \]

\[ S_V^2 = 2 S_{33}^2 + 4 \left( S_{13}^2 + S_{23}^2 \right) \]

\[ \frac{\partial \bar{e}}{\partial t} + u_j \frac{\partial \bar{e}}{\partial x_j} = -K^V H N^2 + K_m S_H^2 + K_m S_V^2 + 2 \left( \frac{\partial}{\partial x_1} \left( K_m \frac{\partial \bar{e}}{\partial x_1} \right) + \frac{\partial}{\partial x_2} \left( K_m \frac{\partial \bar{e}}{\partial x_2} \right) \right) + 2 \frac{\partial}{\partial z} \left( K_m \frac{\partial \bar{e}}{\partial z} \right) - c_{\varepsilon} \frac{\bar{e}^2}{l} \]

(1) forward time-stepping
(2): Crank Nicholson (for vertical advection and diffusion terms)
(3): improved horizontal advection scheme (Förstner, 2004)
Surface weather chart (12 UTC)

07.05.2004

LMK integration area

- Advection of warm air from east to Northern Germany (500hPa)
- Advection of cold air from south-west to Southern Germany in the lower atmosphere
LMK run 07.05.2004 00 + 15 UTC

diagnostic TKE (1D)  
prognostic TKE (3D)

TKE [m²/s²]  
44m above ground
LMK run  07.05.2004 00+15UTC

3h precipitation sum (diagn. 1D TKE)

3h precipitation sum (progn. 3D TKE)

Qualitative Radar scan (superposed 13, 14, 15 UTC)
3h - precipitation field over Southern Germany and the Alps
CONCLUSIONS

- A 3D-turbulence parameterisation scheme with prognostic TKE-equation is now implemented in the LMK code and is running reasonably well. Tests of the preliminary physical approach need to follow next. In this line controlled experiments are planned.

- In parallel this new scheme will be envisaged with in-depth validations. This includes the diversity of available measurements of vertical turbulent flux profiles from observatory capacity in the DWD (Lindenberg).

- As an immediate next step the formulation of the scheme has to be extended to conservative variables taking into account the presence of clouds.