Deutscher Wetterdienst

The 3d-turbulence parameterisation scheme for LMK

Hans-Joachim Herzog, Gerd Vogel, Jochen Förstner*, Ursula Schubert

DWD, FE14 Potsdam, *Aktionsprogramm 2003

COSMO meeting

Italy, September 2004

relevant equation fragments:

$$\frac{\partial u_i}{\partial t} = \ldots - \frac{1}{r}\frac{\partial t_{ij}}{\partial x_j}$$

$$\frac{\partial X^n}{\partial t} = \ldots - \frac{1}{r}\frac{\partial Y_j^n}{\partial x_j}$$

3d-flux scheme

with

$$u_i = (u, v, w)$$

$$(X^1, X^2) = (T, q^k)$$

specified flux components:

$$t_{ij} = -rK_m^{ij}\left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i}\right)$$

$$Y_j^1 = -c_{pd}\, rpK_h^j \frac{\partial \overline{Q}}{\partial x_j}$$

$$Y_j^2 = -rpK_h^j \frac{\partial \overline{q}^k}{\partial x_j}$$

simplified anisotropic diffusion approach

$$K_m^H := K_m^{11} = K_m^{12} = K_m^{22} \qquad K_m^V := K_m^{13} = K_m^{23} = K_m^{33} \qquad \textbf{(momentum)}$$

$$K_h^H := K_h^1 = K_h^2 \qquad K_h^V := K_h^3 \qquad \textbf{(heat)}$$

aspect ratio

[Dunst (1980)]

$$K_{m.h}^H := r\frac{\sqrt{(a\cos rDl)^2}\sqrt{(a\cos jDl)^2 + (aDj)^2}}{Dz} K_{m,h}^V \approx r\frac{\sqrt{2}D}{Dz} K_{m,h}^V = aK_{m,h}^V$$

Prandtl-Kolmogorov-specification of vertical diffusion

$$K_m^V = f_m \, l (\bar{e})^{1/2} \qquad K_h^V = f_h \, l (\bar{e})^{1/2}$$

Mixing lenght Blackadar (1960):

$$l = kz \left(1 + \frac{kz}{l_\infty} \right)^{-1}$$

$$\bar{e} = \frac{1}{2} \left(\overline{u_i' u_i'} \right)$$

Turbulent kinetic energy:

Mellor & Yamada SGS-model

$$K_m^V = l^2 s_m^{3/2} \left(S_V^2 - a_n s_h N^2 \right)^{1/2}$$

$$\bar{e} = c_e^{-2/3} s_m l^2 S_V^2 \left(1 - Pr^{-1} Ri_V \right)$$

$$Pr = \frac{1}{a_n a_h}$$

coincidence at equilibrium case

$$f_m l \bar{e}^{1/2} = l^2 s_m^{2/3} \left(S_V^2 - a_n s_h N^2 \right)^{1/2}$$

$$Ri_V = \frac{N^2}{S_V^2}$$

finally gives

$$f_m = c_e^{1/3} s_m \qquad f_h = a_n c_e^{1/3} s_m s_h$$

Prognostic TKE-equation

$$\frac{\partial \bar{e}}{\partial t} + \bar{u}_j \frac{\partial \bar{e}}{\partial x_j} = d_{i3} \frac{g}{\bar{Q}_v} \overline{(u_i' Q_v')} - \overline{u_i' u_j'} \frac{\partial \bar{u}_i}{\partial x_j} - \frac{\partial \overline{(u_j' e)}}{\partial x_j} - \frac{1}{r} \frac{\partial \overline{(u_i' p')}}{\partial x_i} - e$$

$$N^2 = \frac{g}{\bar{Q}_v} \frac{\partial \bar{Q}_v}{\partial z} \qquad S^2 = \frac{1}{2}\left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right)^2 \Rightarrow \begin{aligned} S_H^2 &= 2\left(S_{11}^2 + S_{22}^2\right) + 4S_{12}^2 \\ S_V^2 &= 2S_{33}^2 + 4\left(S_{13}^2 + S_{23}^2\right) \end{aligned}$$

$$\frac{\partial \bar{e}}{\partial t} + \underbrace{\bar{u}_j \frac{\partial \bar{e}}{\partial x_j}}_{\textbf{(2,3)}} = \underbrace{-K_h^V N^2 + K_m^H S_H^2 + K_m^V S_V^2}_{\textbf{(1)}} + 2\underbrace{\left(\frac{\partial}{\partial x_1}\left(K_m^H \frac{\partial \bar{e}}{\partial x_1} \right) + \frac{\partial}{\partial x_2}\left(K_m^H \frac{\partial \bar{e}}{\partial x_2} \right) \right)}_{\textbf{(1)}} + 2\underbrace{\frac{\partial}{\partial z}\left(K_m^V \frac{\partial \bar{e}}{\partial z} \right)}_{\textbf{(2)}} - \underbrace{c_e \frac{\bar{e}^{3/2}}{l}}_{\textbf{(1)}}$$

Equation integrated by a Runge-Kutta time scheme (3rd-order normal/TVD variant)

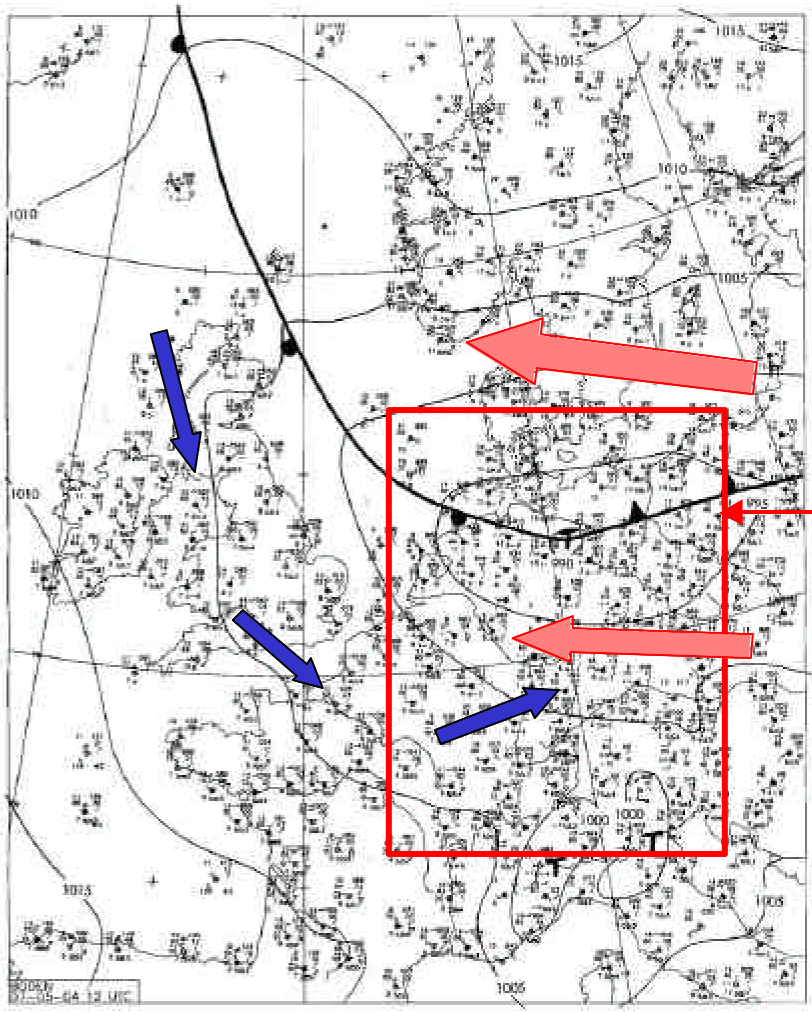
(1): forward time-stepping

(2): Crank Nicholson (for vertical advection and diffusion terms)

(3): improved horizontal advection scheme (Förstner, 2004)

Surface weather chart (12 UTC)

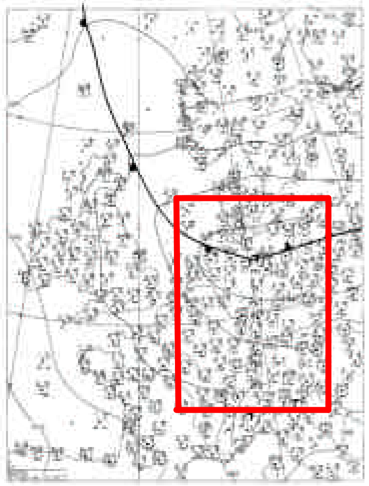
07.05.2004



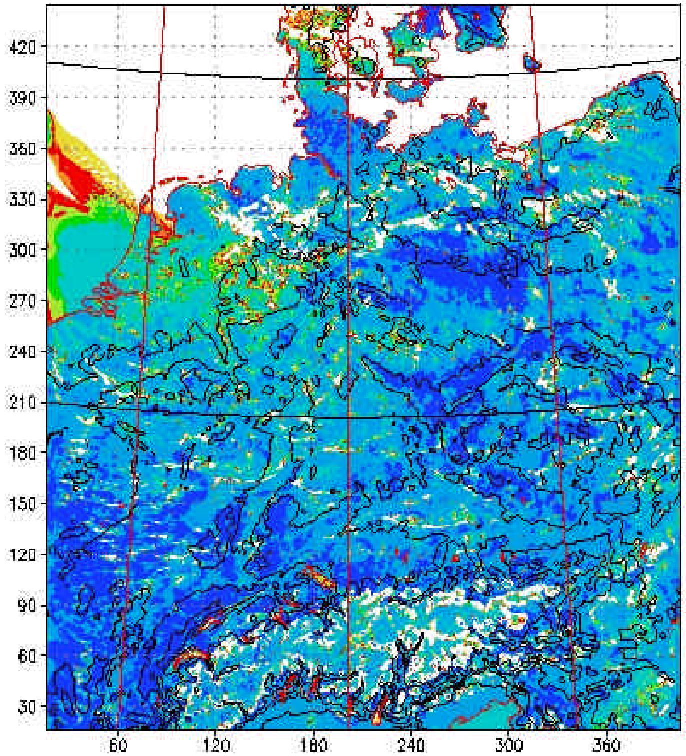
LMK integration area

- Advection of warm air from east to Northern Germany (500hPa)
- Advection of cold air from south-west to Southern Germany in the lower atmosphere

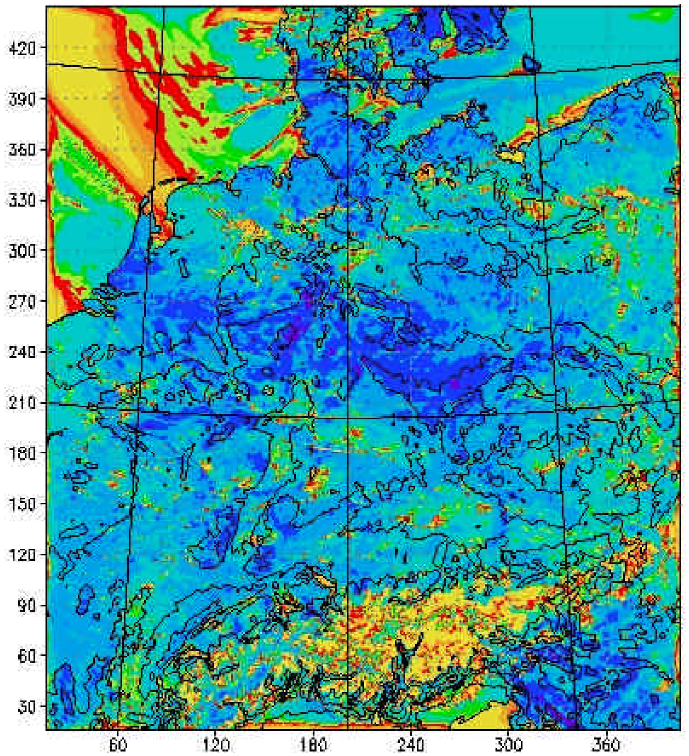
LMK run 07.05.2004 00 + 15 UTC



diagnostic TKE (1D)



prognostic TKE (3D)



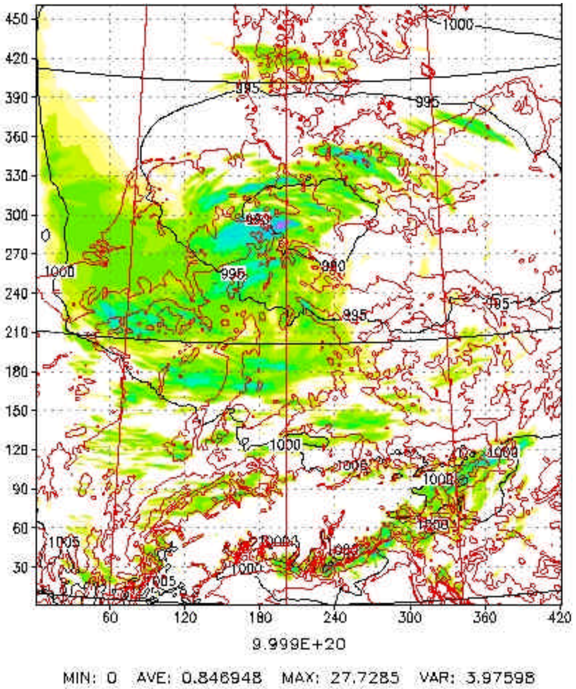
TKE [m²/s²]
44m above
ground

Deutscher Wetterdienst

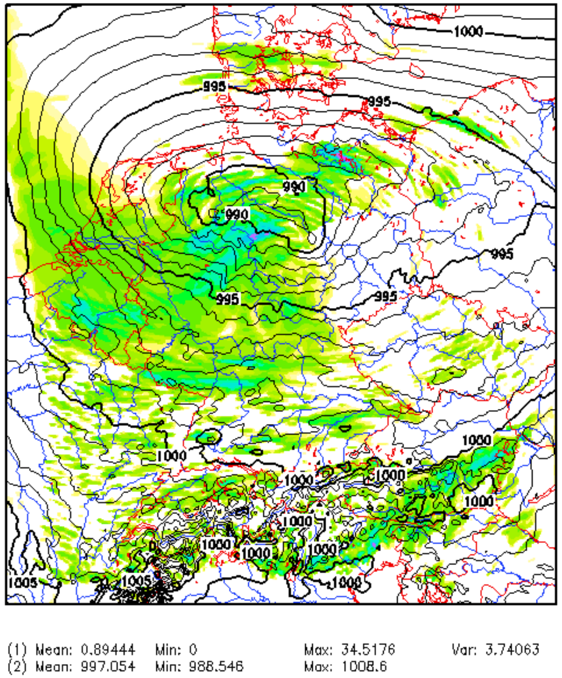
DWD

LMK run 07.05.2004 00+15UTC

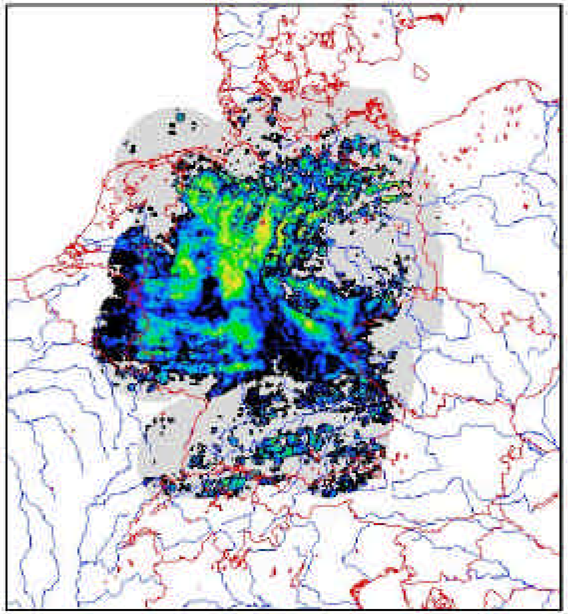
3h precipitation sum
(diagn. 1D TKE)

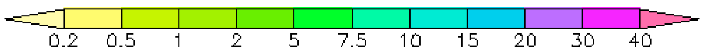


3h precipitation sum
(progn. 3D TKE)



Qualitative Radar scan
(superposed 13, 14, 15 UTC)

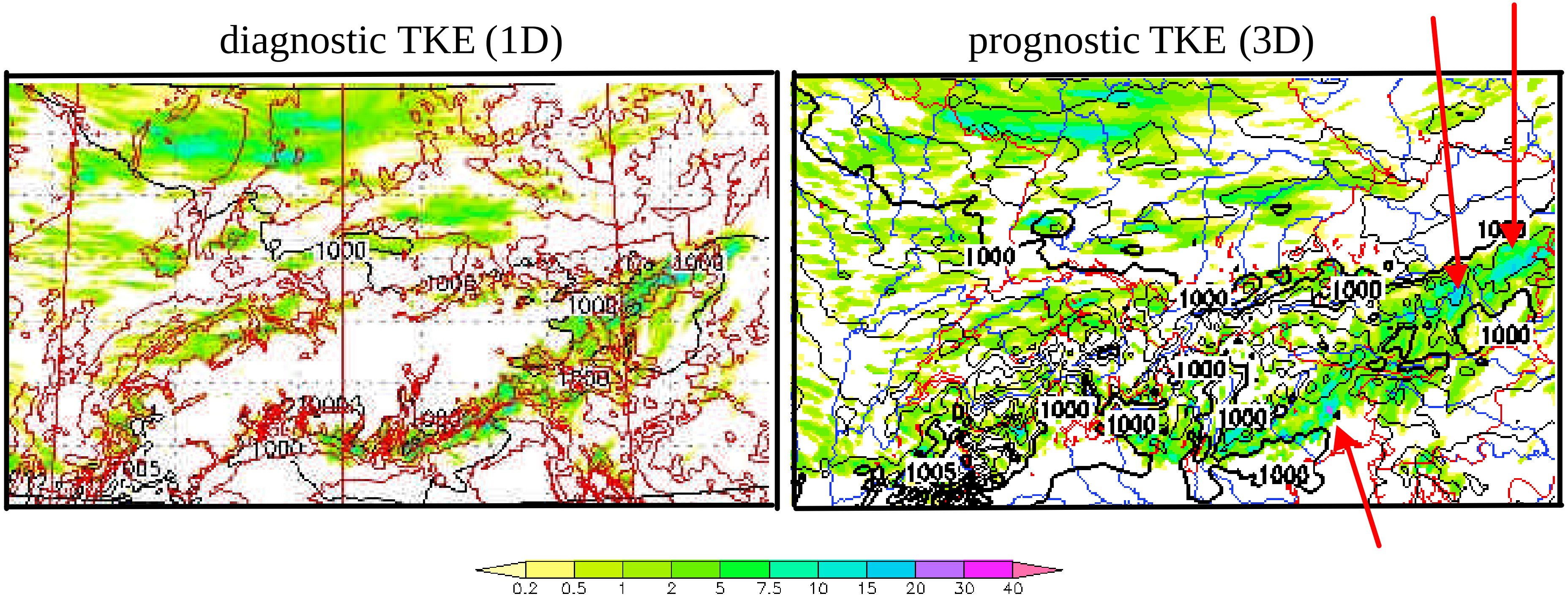




Deutscher Wetterdienst

LMK run 07.05.2004 00+15 UTC (zoom)

diagnostic TKE (1D) prognostic TKE (3D)



3h - precipitation field over Southern Germany and the Alps

CONCLUSIONS

- A 3D-turbulence parameterisation scheme with prognostic TKE-equation is now implemented in the LMK code and is running reasonably well. Tests of the preliminary physical approach need to follow next. In this line controlled experiments are planned.

- In parallel this new scheme will be envisaged with in-depth validations. This includes the diversity of available measurements of vertical turbulent flux profiles from observatory capacity in the DWD (Lindenberg).

- As an immediate next step the formulation of the scheme has to be extended to conservative variables taking into account the presence of clouds.