Deutscher Wetterdienst

Influence of the higher vertical resolution on the surface layer parameterisation

- the resolved roughness layer-

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COSMO Meeting in Milan, 22-24.10.2004

The situation:

Sub grid scale structures of the earth surface

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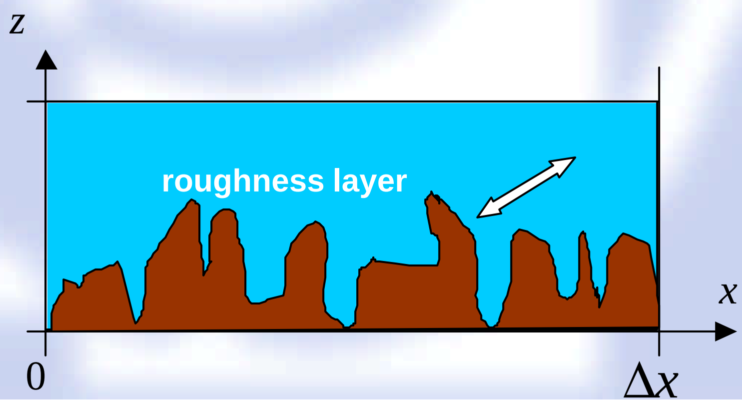
Interaction between surface elements and the surrounding air

(**roughness layer- or canopy terms**)

have to be considered in the lowest part of the atmosphere



Usually, the lowest model layer

is **much bigger** than the roughness layer

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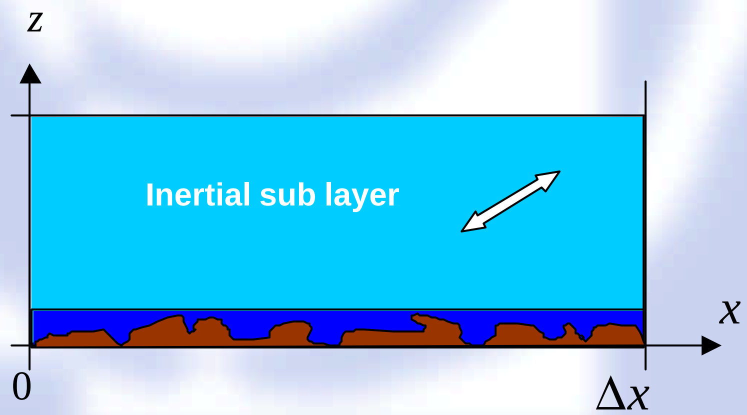
canopy terms do only appear in the **transfer scheme** at the surface, usually expressed by the help of specific roughness length values for the transported properties.

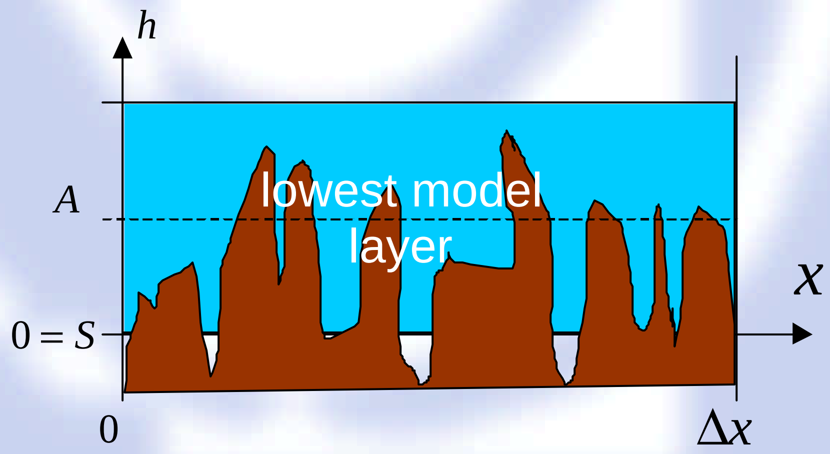


The problem with a resolved roughness layer:

- Over areas with large roughness elements (within cities, forests and mountains) the canopy is often **higher** than the depth of the lowest layer but the model equations do **not contain canopy terms so far**.

⬇

simulation errors especially over mountains



Questions:

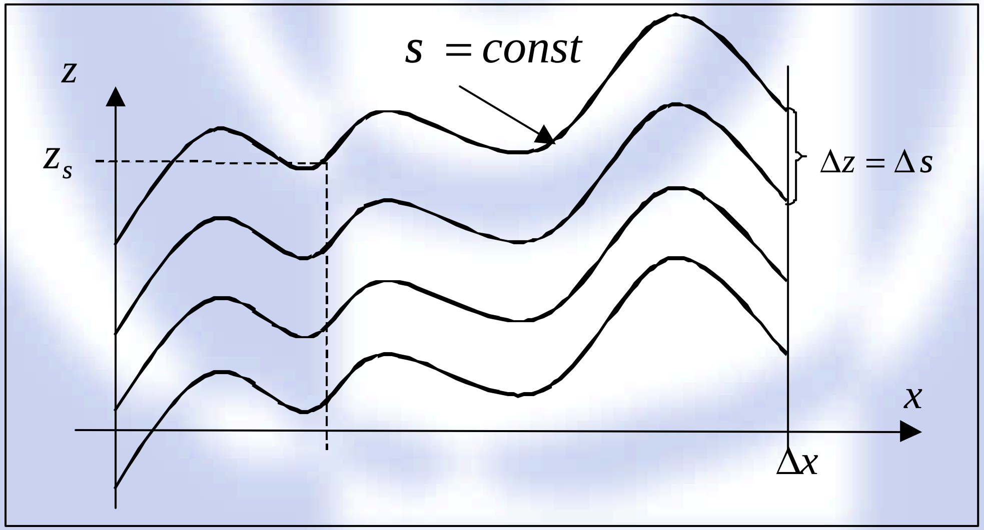
- How do **canopy terms** in principle enter to the **model equations**?
- Are there different possibilities to express these **canopy terms**?
- What is the effect of the choosen **coordinate system**?
- Is there a practical way to describe a **vertically resolved roughness layer**?

The coordinate system:

- The model equations must be solved on a numerical grid.
  - We choose a regular grid, belonging to a **local Cartesian** coordinate system (x, y, z) with a physical vertical coordinate s , such that

$$z = z_s(x, y)$$

and $\partial_s z = 1$ within the boundary layer.



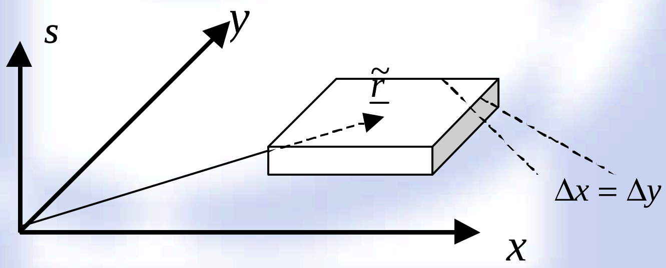
The filter operator:

- In order to apply a numerical approximation of the **derivatives in space** on the variable fields, they have to be **filtered in space** with respect to a **horizontal length scale** comparable to the horizontal grid spacing $D_0 = \Delta x = \Delta y$.

- The filter is defined as a moving average along the air containing part $Q_{|s}(\tilde{r})$ of a shallow cubic grid box in the s **-system**:

$$\overline{V}(\tilde{r}) := \frac{1}{\left|Q_{|s}(\tilde{r})\right|} \int_{s \in Q_{|s}(\tilde{r})} f_{|s}(s)\, ds^3$$ **average**,

$$\hat{V} := \frac{\overline{rV}}{\overline{r}}$$ weighted average



- Due to $\partial_s z = 1$ it is:

$$\bar{V}(\underline{r}) := \frac{1}{\left|Q_{|z}(\underline{r})\right|} \int_{s \in Q_{|z}(\underline{r})} f_{|z}(\underline{s})\, ds^3 ,$$

  if $Q_{|z} := T(Q_{|s})$ is the averaging volume in the **z-system** and $\underline{r} = T(\tilde{\underline{r}})$.

- For $\underline{s} \in Q(\underline{r})$ the according **fluctuations** are defined as follows:

$$V'(\underline{s}) := V(\underline{s}) - \bar{V}(\underline{r}) \quad \text{and} \quad V''(\underline{s}) := V(\underline{s}) - \hat{V}(\underline{r})$$

  With this definition it is: $\overline{V'} = 0 = \overline{\rho V''}$

- If $D > 0$ is a different grid spacing, we write $\bar{V}^D$ or $\hat{V}^D$ for the averages and $(V)'_D$ or $(V)''_D$ for the corresponding fluctuations.

Mathematical reason for the canopy terms:

Sub grid scale slope of the model layers or Intersections of roughness elements → Averaging and differentiation in space (flux divergence) can no longer be commuted

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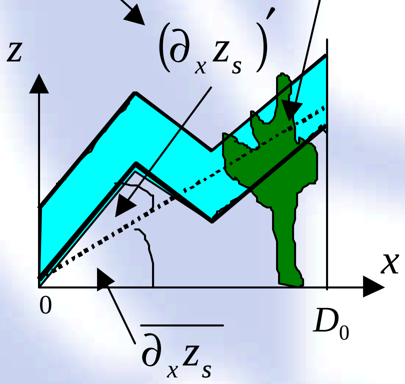
Canopy terms in the averaged model equations



The filtered budget equation:

$$\underline{\nabla} = \begin{pmatrix}\nabla_1\\ \nabla_2\\ \nabla_3\end{pmatrix} = \begin{pmatrix}\partial_x\\ \partial_y\\ \partial_z\end{pmatrix} = \begin{pmatrix}\left.\partial_x\right|_s - \partial_x z_s \partial_z\\ \left.\partial_y\right|_s - \partial_y z_s \partial_z\\ \partial_s\end{pmatrix}$$

$$\begin{Bmatrix}\left.\partial_x\right|_s - \partial_x z_s \partial_z\\ \left.\partial_y\right|_s - \partial_y z_s \partial_z\end{Bmatrix} = \left.\underline{\nabla}_h\right|_s - \underline{\nabla}_h z_s \partial_z$$

horizontal Cartesian gradient operator

vertical gradient operator

$$f_j^f := \begin{cases} \rho \overline{f'' v_j''} - \rho \kappa^f \nabla_j f , & f \text{ is a scalar} \\ \left(\rho \overline{v_i'' v_j''} - \delta_{ij} p_d\right) - \rho \nu \nabla_j v_i , & f = v_i \text{ is a velocity component} \end{cases}$$

component in j-direction of the **non advective** flux for a budget variable f

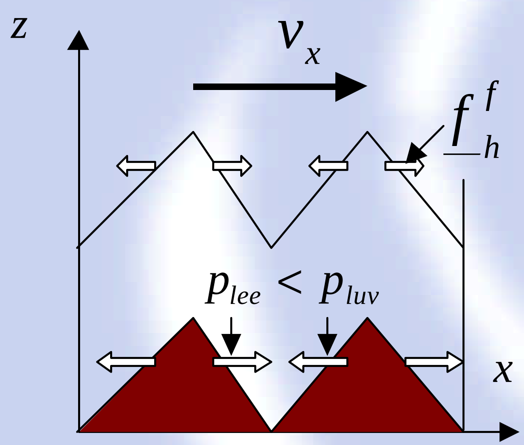
$p_d := \frac{\rho}{3} \sum_j v_j''^2$ dynamic pressure

p_s static pressure

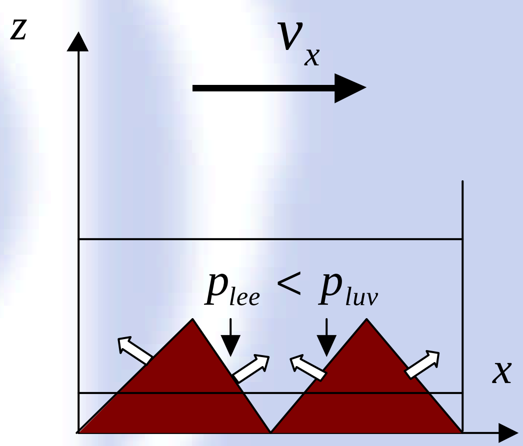
$p := p_s + p_d$ total pressure

	Grid scale divergence of advective fluxes	Pure atmospheric correction	Sub grid scale transf. (or slope correlation) correction	Intersection correction	
$\overline{Q^f} - \partial_t \overline{\rho f}$ =	$\underline{\overline{\nabla}} \cdot \left(\bar{\rho} \hat{f} \hat{\underline{v}}\right)$	$+\underline{\overline{\nabla}} \cdot \overline{\underline{f}^f}$	$-\partial_z \overline{\underline{f}_h^f \cdot (\underline{\nabla}_h z_s)'}$	$+\overline{\underline{\nabla} \cdot \underline{f}^{f'}}$	For all budget variables f
		Grid scale divergence of the mean non advective (turbulent and laminar) flux	**Vertical divergence of the non advective fluxes through the surface**	-S **Outflow into intersected bodies and flux divergence by volume reduction**	
		$\overline{\underline{\nabla} \cdot \underline{f}^f}$			
		$+\nabla_i \bar{p}$	$-\partial_z \overline{p(\nabla_i z_s)'}$	$+\overline{\nabla_i p'}$	Only if $f = v_i$ is a velocity component and $\overline{\underline{\nabla} \cdot \underline{v}} = 0$
		Gradient in i-dir. of mean **pressure**	**Form drag in i-direction**		

Some Illustration of the canopy terms:

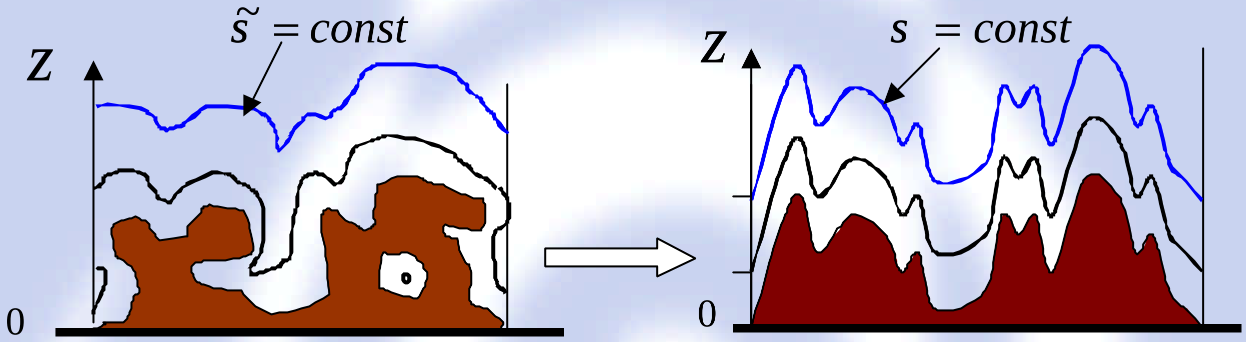


No intersection correction



No transformation correction

A new concept: The "orographic approximation" **OA**:



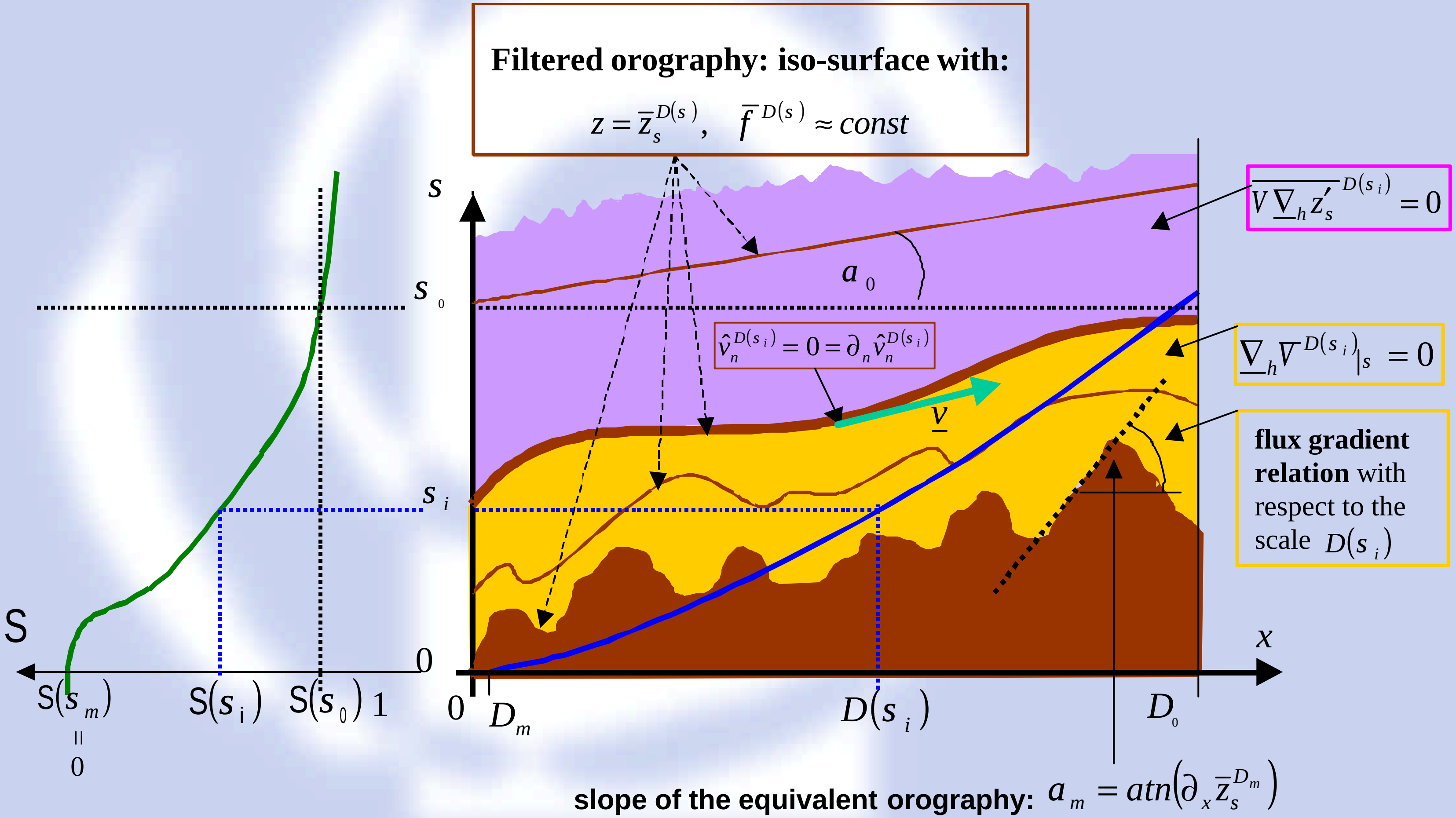
- For any real surface (**RS**) of the earth, there exists an "**equivalent orographic surface**" (**EO**), which has the following meaning:
- There exists a mapping from the s -surfaces belonging to the **EO** towards corresponding $\tilde{s}$ -surfaces belonging to the **RS** with the following properties:

- The $\tilde{s}$ -surfaces cover the **RS**
- The $\tilde{s}$ -surfaces do not intersect
- The $\tilde{s}$ -surfaces have the same magnitude like the corresponding s -surfaces
- The including air volume between two s -surfaces is the same like that between the two corresponding $\tilde{s}$ -surfaces
- The average of all model variables along the air between two s -surfaces is the same as that one belonging to the corresponding $\tilde{s}$ -surfaces

- **No** intersection terms must be considered
- But the sub grid transformation terms have to be **parametrized.**

The general boundary layer approximation **GBA**:

A **substitute** of the horizontal boundary layer approximation (HBA)



The surface area function (SAF):

$$s^2(s) = \overline{\left|\nabla_h\left(\bar{z}_s^{D(s)}\right)'\right|^2}$$ **Variance of the surface slope**

$$s_0^2 = \left|\nabla_h(\bar{z}_s)\right|^2 = \tan^2(a_0)$$

$$(S^f)^2 = 1 + s_0^2 + a^f s^2$$

$$a^f = \begin{cases} 1 & \textit{for scalars} \\ a^{v_i} & \textit{for momentum} \end{cases}$$

drag parameter

$$S := \sqrt{1 + s_0^2 + s^2} = \sqrt{1 + \overline{\tan^2(a)}} = \sqrt{\overline{\left(\frac{1}{\cos^2 a}\right)}}$$ **SAF**

$$\boxed{\overline{\nabla \cdot f^f} = \partial_z\left[S^2 \cdot \overline{f_3^f}\right]}$$

averaged non advective flux divergence for **scalars**

drag flux density

$$\boxed{\overline{\nabla \cdot f^{v_i}} + \overline{\nabla_i p} = \nabla_i \bar{p} + \partial_z\left[S^2 \cdot \overline{f_3^{v_i}} - \overline{p(\partial_i z_s)'}\right]}$$

averaged stress vector divergence for **momentum**

$$(S^{v_i})^2 \cdot \overline{f_3^{v_i}}$$

specific SAF for momentum including **form drag**

Roughness layer architecture:

- **decomposition of the orography** in **spectral interval modes**:
  - the i-th mode includes all wavelengths in: $\left]D_{i+1}, D_i\right]$

D_i horizontal scale ⟷ s_i blending height

↕ land use groups ↕ model half levels

$$s_i^2(s) = \overline{\left|\nabla_h \left(\overline{\overline{z}_s^{D_{i+1}}}^{D(s)}\right)'_{D_i}\right|^2}$$

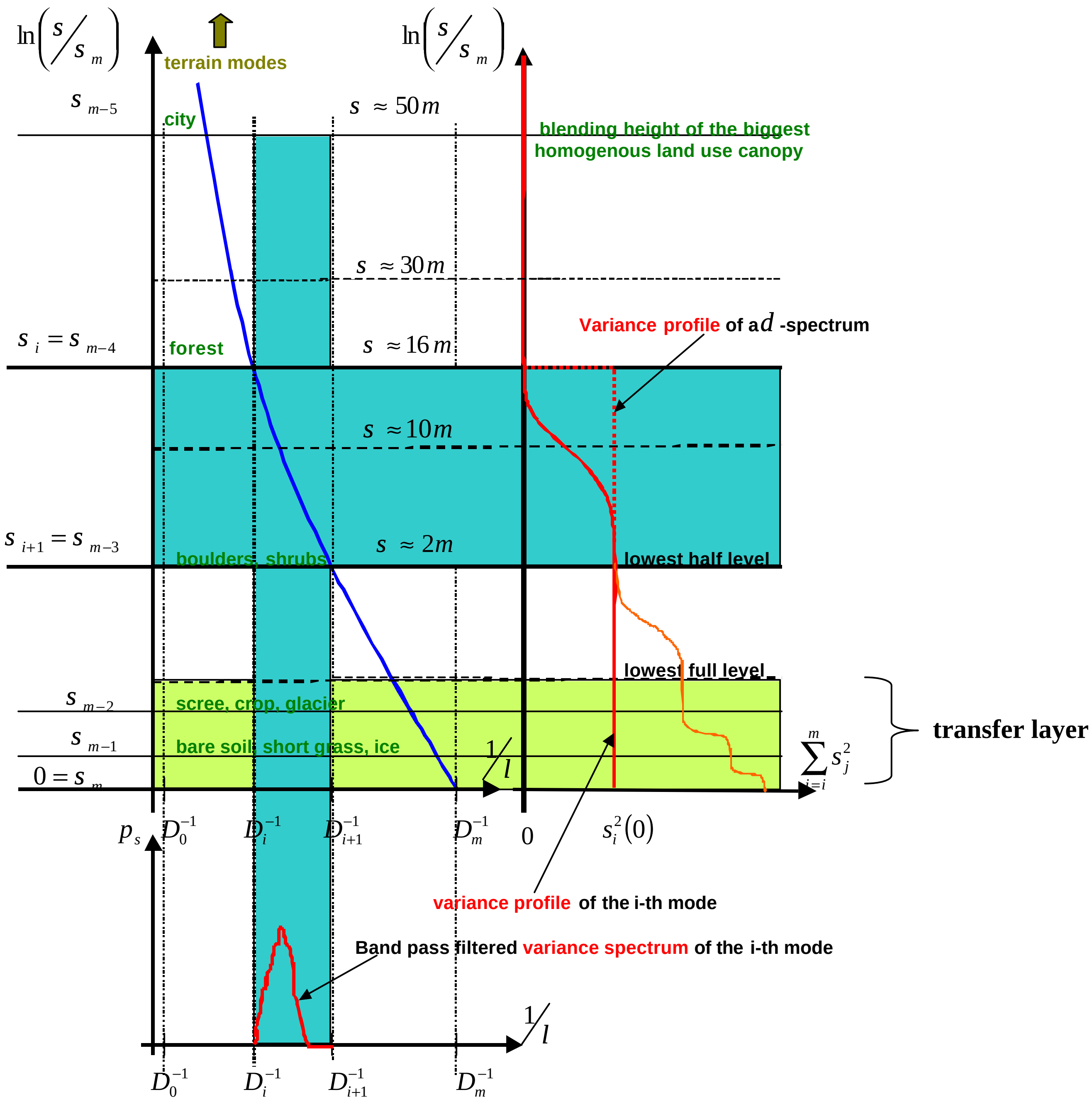
slope variance profile function

- **simple vertical profiles** of the slope variance profile function of each mode:

step function ⟷ δ-variance spectrum of the mode

- It holds the relation: $s^2(s) = \sum_{i=1}^{m} s_i^2(s)$

The canopy architecture with spectral decomposition in orographic modes:



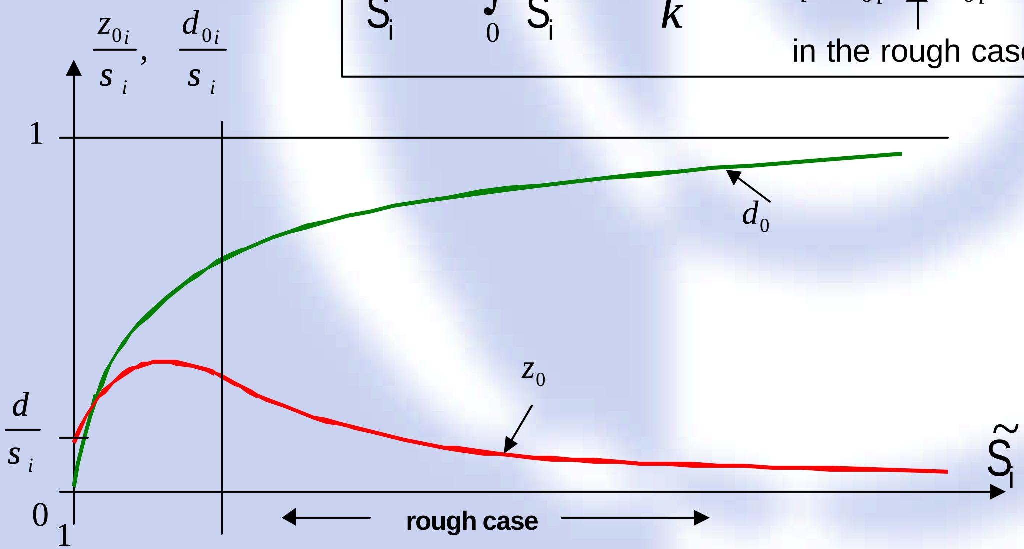
Relations to available external parameters

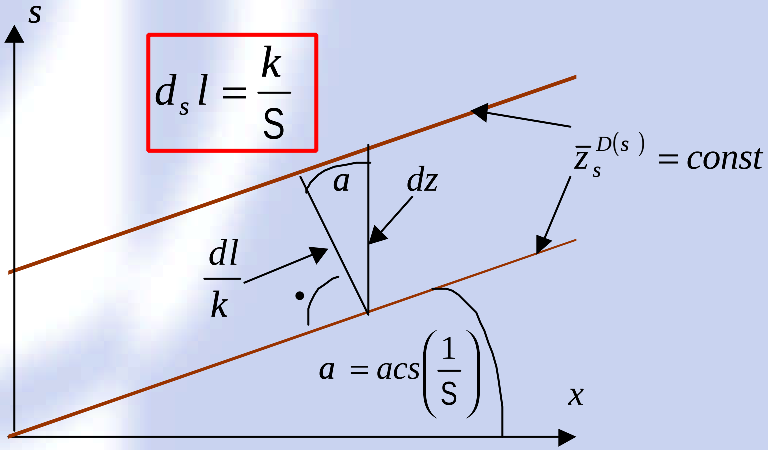
- Define the **horizontal surface area function** $(S_i)^2 := 1 + s_i^2$ **for a single mode i**.

- Combination of the **GBA**-results with the logarithmic wind profile above a roughness layer leads to the following relations between the **blending height** s_i **, the roughness length** z_{0i} **, the displacement height** d_{0i} and the **mean dynamic surface area index** $\tilde{S}_i$:

$$\frac{s_i}{\tilde{S}_i} := \int_0^{s_i} \frac{ds}{S_i} = \frac{l(s_i)}{k} = s_i - d_{0i} \approx z_{0i}$$

in the rough case





Extension for the turbulence scheme in GBA:

- Substitution of the vertical turbulent fluxes in the shear terms of the 2-nd order equations by those multiplied with the appropriate **surface area function** (**effective vertical fluxes**)

- **Mellor/Yamada-Scheme** with HBA replaced by the **GBA**, where the gravity vector assumed to be normal to the mean orography.

  - Same kind of flux gradient representation, **but** :

    - coefficients in the linear equation for S^H and S^M are modified by the **surface area functions**
    - shear term in the TKE-equation contains the **specific surface area function** for **momentum** expressing **wake turbulence** production

- The resistance law in the **constant flux layer** can also be generalised by the help of the surface area functions

Conclusions:

- As averaging and differentiation in space can not be commuted near the surface, additional roughness layer terms arise and have to be considered in a shallow lowest model layer.
- In the orographic approximation, model layers are not intersected by roughness elements. Then roughness layer terms can be explained purely by coordinate transformation and have the form of correlations between sub grid scale model layer slopes and the model variables.
- In the general boundary layer approximation, roughness layer effects can be expressed by consideration of simple surface area functions.
- The subgrid scale orography can be decomposed in spectral modes, each with an own blending height and an own surface area index.
- The surface area index and the blending height can be related to the roughness length and the displacement height of the surface structure.
- The second order turbulence closure can be generalised for the roughness layer.
- First implementation, testing (and parameter tuning) will be done using the single column framework.

↓ poster

Thank you

for your

attention!!

The model equations:

♦ 5 prognostic variables:

$f =$	$v_i \quad , i = 1,2,3$	Velocity components (momentum concentration)
	q_w	Mass fraction of the total water content
	q_l	liquid water potential temperature

♦ Prognostic budget equations:

$$\partial_t (r f) + \boxed{\nabla \cdot \underline{F}^f} = Q^f$$

general budget

$$F_j^f = r f v_j - c_j^f$$

j-component of the total flux density for the property f

- with the non advective molecular flux density components:

$$c_j^f = \begin{cases} -r\ k^f \partial_j f & \text{for scalars } f = q_l\,,\, q_w \\ -r\,u\left(\partial_i v_j + \partial_j v_i\right) + d_{ij}\,p & \text{for momentum } f = v_i \end{cases}$$

k^f : molecular diffusion coeff. $\qquad u$: kinimatic viscosity

- and the source terms:

$$Q^f = \begin{cases} -\nabla \cdot \underline{S} \big/ p\, c_p \approx 0 \quad (\text{for moist adiab. idealizati on}) & ,\, f = q_l \quad (\text{conservative}) \\ 0 & ,\, f = q_w \quad \text{conservative} \\ -d_{i3}\, r\, g - 2\, r\left(\underline{\Omega} \times \underline{v}\right)_i & ,\, f = v_i \quad \text{not conservative} \end{cases}$$

$\underline{S}$: radiation flux density $\qquad \underline{\Omega}$: angular vector of the earth

g : gravity of the earth $\qquad p$: Exner factor

c_p : heat capacity of the air at constant pressure

- The transformation T from the s -system into the z -system with respect to a scalar field V can be described by the following relation:

$$(x,y,s)\in\Re^3 \xrightarrow[T]{} (x,y,z_s(x,y))\in\Re^3 \xrightarrow[V_{|z}]{} V\in\Re$$

$$(x,y,s) \xrightarrow[V_{|s}]{} V$$

- Derivatives in space transform as follows:

$$\partial_j V_{|z} = \partial_j V_{|s} - \underline{\partial_z s \, \partial_s V_{|s} \, \partial_j z_s} \quad , j\in(1,2) \hat{=} (x,y)$$

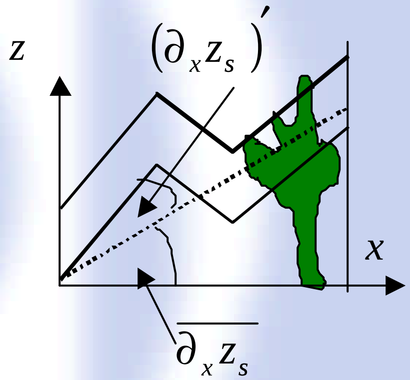
$$\partial_z V_{|z} = \partial_z s \, \partial_s V_{|s}$$

The problem of the filtered horizontal gradients

- If roughness elements are intersecting a grid box, **derivation in space** and **averaging** can **not** be **commuted**:

$$\overline{\partial_j V_{|s}} = \partial_j \overline{V_{|s}} + \underbrace{\overline{\partial_j V'_{|s}}}$$

consists of a tem due to the volume reduction and a integral along the surfaces of intersected bodies



- The **averaged derivative** along a horizontal coordinate of the Cartesian z **-system**:

$$\overline{\partial_j V}_{|z} = \underbrace{\partial_j \overline{V}_{|s} - \partial_z s\, \partial_s \overline{V}_{|s}\, \overline{\partial_j z_s}}_{\overline{\partial_j}\,\overline{V}_{|z}} \underbrace{- \partial_z s\, \partial_s \left[\overline{V'_{|s} \left(\partial_j z_s\right)'} \right]}_{\partial_z \left[\overline{V'_{|z} \left(\partial_j z_s\right)'} \right]} + \underbrace{\boxed{\overline{\partial_j V'_{|z}}}}_{} \quad , j = x, y$$

$\overline{\partial_j}\,\overline{V}_{|z}$: transformed grid scale gradient

$\boxed{\partial_z \left[\overline{V'_{|z} \left(\partial_j z_s\right)'} \right]}$: sub grid scale transformation correction

$\boxed{\overline{\partial_j V'_{|z}}}$: sub grid scale intersection correction

Former solutions in LM

- They **all** refer to coordinates following the mean orography.

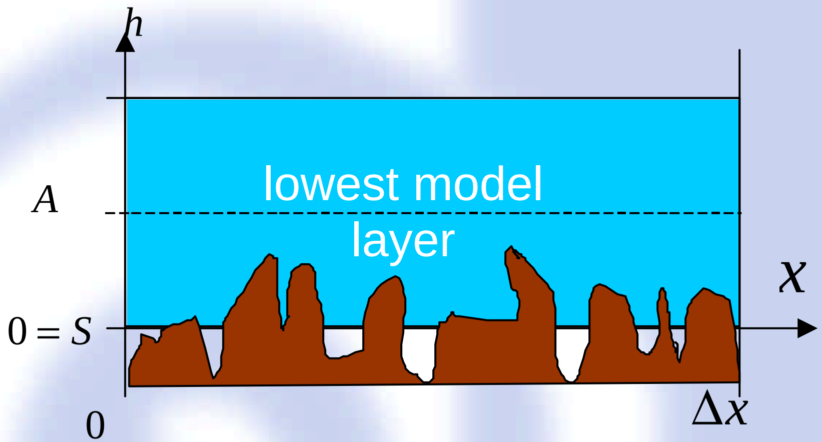
  ➡ **No** sub grid scale **transformation correction** needed

- The **old scheme** and even the **operational configuration** of the **new scheme** do **not** contain any sub grid scale **intersection correction** as well

  ➡ **No** canopy term is included at all

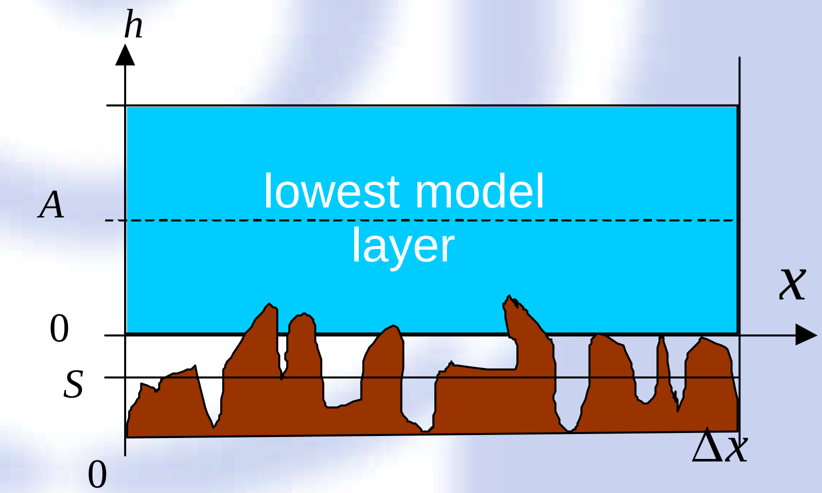
- The **old scheme:**

  The canopy height is assumed to be **small** compared to the depth of the lowest model layer



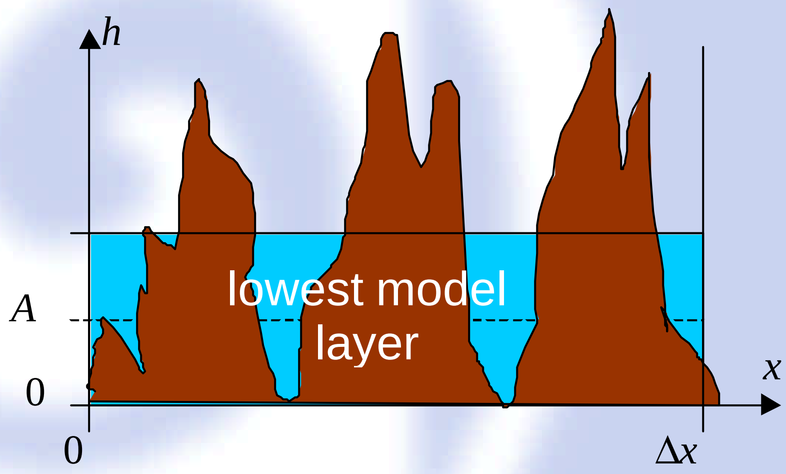
- The **operational scheme:**

  The canopy is assumed to be **below** the lowest model layer



- The up to now **incomplete extension version** of the operational scheme contains **intersection terms** at least in the momentum budget:

  - The **canopy** is **resolved** by model layers
  - The intersection terms have to be **parameterized**
  - For **each** inner canopy layer:
    - **external parameters**
    - a **soil model**



The general boundary layer approximation GBA:

A **substitute** of the horizontal boundary layer approximation

- For any level $s > 0$ there exists a strictly in s monotonic growing horizontal scale $D(s) > 0$, which is the **largest** scale that has for any model variable V the following properties:
  - For any **smaller** scale $x \leq D(s)$ it is: $\overline{V \nabla_h z_s'}^{x} = 0$
  - For any **larger** scale $X \geq D(s)$ it is : $\nabla_h \overline{V}^{X}|_s = 0$.
- Is v_n the velocity component normal to the idealised iso-surface $S(s)$ given by $z = \overline{z}_s^{D(s)}$, it is: $\hat{v}_n^{D(s)} = 0 = \partial_n \hat{v}_n^{D(s)}$.
- It holds a **flux gradient relation** for all sub grid scale fluxes with respect to the scale $D(s)$ **.**