Influence of the higher vertical resolution on the surface layer parameterisation - the resolved roughness layer-

Matthias Raschendorfer, DWD, Offenbach

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The situation:

Sub grid scale structures of the earth surface

Interaction between surface elements and the surrounding air

(roughness layer- or canopy terms)

have to be considered in the lowest part of the atmosphere
Usually, the lowest model layer is much bigger than the roughness layer. Canopy terms do only appear in the transfer scheme at the surface, usually expressed by the help of specific roughness length values for the transported properties.
The problem with a resolved roughness layer:

- Over areas with **large roughness elements** (within cities, forests and mountains) the canopy is often **higher** than the depth of the lowest layer but the model equations do not contain **canopy terms** so far.

**Simulation errors** especially over mountains

\[ S = 0 \] lowest model layer

\[ h \]

\[ A \]

\[ 0 = S \]
Questions:

• How do **canopy terms** in principle **enter to** the **model equations**?

• Are there **different possibilities to express** these **canopy terms**?

• What is the **effect** of the chosen **coordinate system**?

• Is there a **practical way to describe** a **vertically resolved roughness layer**?
The coordinate system:

- The model equations must be solved on a **numerical grid**.
  - We choose a **regular grid**, belonging to a **local Cartesian** coordinate system 
    \((x, y, z)\) with a physical vertical coordinate \(\sigma\), such that 
    \[ z = z_\sigma(x, y) \]
    and 
    \[ \partial_\sigma z = 1 \]
    within the boundary layer.
The filter operator:

- In order to apply a **numerical approximation** of the **derivatives in space** on the variable fields, they have to be **filtered in space** with respect to a **horizontal length scale** comparable to the **horizontal grid spacing** \( D_0 = \Delta x = \Delta y \).

- The filter is defined as a **moving average** along the air containing part \( Q_{|\sigma}(\vec{r}) \) of a **shallow cubic** grid box in the \( \sigma \)-**system**:

\[
\bar{\xi}(\vec{r}) := \frac{1}{\left| Q_\sigma(\vec{r}) \right|} \int_{s \in Q_\sigma(\vec{r})} \Phi_s(s) \, ds
\]

For the **average**, \( \bar{\xi} := \frac{\rho \bar{\zeta}}{\bar{\rho}} \) and for the **weighted average**.

\[
\bar{\zeta} := \frac{\rho \bar{\theta}}{\bar{\rho}}
\]
Due to $\partial_\sigma z = 1$ it is:

$$\bar{\zeta}(r) := \frac{1}{|Q_z(r)|} \int_{s \in Q_z(r)} \phi_z(s) ds^3,$$

if $Q_z := T(Q_\sigma)$ is the averaging volume in the \textit{z-system} and $r = T(\mathbf{r})$.

For $s \in Q(r)$ the according \textbf{fluctuations} are defined as follows:

$$\zeta'(s) := \zeta(s) - \bar{\zeta}(r) \quad \text{and} \quad \zeta''(s) := \zeta(s) - \bar{\zeta}'(r).$$

With this definition it is:

$$\bar{\zeta}' = 0 = \rho \bar{\zeta}''.$$

If $D > 0$ is a different grid spacing, we write $\zeta^D$ or $\hat{\zeta}^D$ for the averages and $(\zeta)'_D$ or $(\bar{\zeta})''_D$ for the corresponding fluctuations.
Mathematical reason for the canopy terms:

- Sub grid scale slope of the model layers
- Intersections of roughness elements

Averaging and differentiation in space (flux divergence) can no longer be commuted

Canopy terms in the averaged model equations
The filtered budget equation:

\[
\overline{\nabla} = \begin{pmatrix}
\nabla_1 \\
\nabla_2 \\
\nabla_3
\end{pmatrix} = \begin{pmatrix}
\frac{\partial}{\partial x} \\
\frac{\partial}{\partial y} \\
\frac{\partial}{\partial z}
\end{pmatrix} = \begin{pmatrix}
\frac{\partial}{\partial x} \sigma - \frac{\partial x \sigma}{\partial z} \\
\frac{\partial}{\partial y} \sigma - \frac{\partial y \sigma}{\partial z} \\
\frac{\partial}{\partial \sigma}
\end{pmatrix} = \nabla_h \sigma \begin{pmatrix}
-\frac{\partial z \sigma}{\partial x} \\
-\frac{\partial z \sigma}{\partial y} \\
-\frac{\partial \sigma}{\partial z}
\end{pmatrix}
\]

horizontal Cartesian gradient operator

vertical gradient operator

\[
f^\phi_j := \begin{cases}
\rho \phi \hat{v}_j - \rho k^\phi \nabla_j \phi, & \text{\(\phi\) is a scalar component in } j \text{-direction of the non advective flux for a budget variable } \phi
\\
(\rho v_j \hat{v}_j - \delta_{ij} p_d) - \rho \nu \nabla_j v_i, & \phi = v_i \text{ is a velocity component}
\end{cases}
\]

Grid scale divergence of advective fluxes
Pure atmospheric correction
Sub grid scale transf. (or slope correlation) correction
Intersection correction

For all budget variables \(\phi\)

\[
\overline{Q^\phi} - \partial_t \overline{\rho \phi} = \nabla \cdot (\overline{\rho \hat{\phi} \hat{v}}) + \nabla \cdot f^\phi - \partial_z \overline{f^\phi \cdot (\nabla_h z_\sigma)' + \nabla \cdot f^\phi'}
\]

\[
\overline{Q^\phi} - \partial_t \overline{\rho \phi} = \nabla \cdot (\overline{\rho \hat{\phi} \hat{v}}) + \nabla \cdot f^\phi - \partial_z \overline{f^\phi \cdot (\nabla_h z_\sigma)' + \nabla \cdot f^\phi'}
\]

Grid scale divergence of the mean non advective (turbulent and laminar) flux
Vertical divergence of the non advective fluxes through the surface
Outflow into intersected bodies and flux divergence by volume reduction

Gradient in i-dir. of mean pressure
Form drag in i-direction

Only if \(\phi = v_j\) is a velocity component and \(\nabla \cdot \nu = 0\)

\[
d_{\text{dynamic}} := \frac{\overline{\rho}}{3} \sum_j v_j^2
\]

\[
d_{\text{static}}
\]

\[
p := p_s + p_d
\]
Some Illustration of the canopy terms:

No intersection correction

No transformation correction
A new concept: The "orographic approximation" OA:

- For any real surface (RS) of the earth, there exists an "equivalent orographic surface" (EO), which has the following meaning:
  - There exists a mapping from the $\sigma$ -surfaces belonging to the EO towards corresponding $\tilde{\sigma}$ -surfaces belonging to the RS with the following properties:
    - The $\tilde{\sigma}$ -surfaces cover the RS.
    - The $\tilde{\sigma}$ -surfaces do not intersect.
    - The $\tilde{\sigma}$ -surfaces have the same magnitude like the corresponding $\sigma$ -surfaces.
    - The including air volume between two $\sigma$ -surfaces is the same like that between the two corresponding $\tilde{\sigma}$ -surfaces.
    - The average of all model variables along the air between two $\sigma$ -surfaces is the same as that one belonging to the corresponding $\tilde{\sigma}$ -surfaces.

- No intersection terms must be considered.
- But the sub grid transformation terms have to be parametrized.
The general boundary layer approximation GBA:

A substitute of the horizontal boundary layer approximation (HBA)

Filtered orography: iso-surface with:

\[ z = z^{D(\sigma)}_{\sigma}, \quad \Phi^{D(\sigma)} \approx \text{const} \]

slope of the equivalent orography: \( \alpha_m = \text{atan}(\partial_z z^{D(\sigma)}_{\sigma}) \)

\[ \frac{\nabla_h z_{\sigma}^{D(\sigma)}}{\text{flux gradient relation with respect to the scale } D(\sigma)} = 0 \]

\[ \frac{\nabla_h z_{\sigma}^{D(\sigma)}}{\text{flux gradient relation with respect to the scale } D(\sigma)} = 0 \]
The surface area function (SAF):

\[ s^2(\sigma) = \left( \nabla_h \left( z_\sigma^D(\sigma) \right) \right)^2 \]

Variance of the surface slope

\[ s_0^2 = \left| \nabla_h \left( z_\sigma \right) \right|^2 = \tan^2(\alpha_0) \]

\[ s = \sqrt{1 + s_0^2 + s^2} = \sqrt{1 + \tan^2(\alpha)} = \sqrt{\frac{1}{\cos^2(\alpha)}} \]

SAF

\[ \nabla \cdot f^\Phi = \partial_z \left( S^2 \cdot f_3^\Phi \right) \]

averaged non advective flux divergence for scalars

drag flux density

\[ \nabla \cdot f^{\ve_i} + \nabla_i p = \nabla_i \bar{p} + \partial_z \left( S^2 \cdot f_3^{\ve_i} - p(\partial_i z_\sigma) \right) \]

divergence for momentum

\[ (S^{\ve_i})^2 \cdot f_3^{\ve_i} \]

specific SAF for momentum including form drag

\[ \alpha^\Phi = \left\{ \begin{array}{ll} 1 & \text{for scalars} \\ \alpha^{v_i} & \text{for momentum} \end{array} \right. \]

drag parameter
Roughness layer architecture:

- **decomposition of the orography in spectral interval modes:**
  - the $i$-th mode includes all wavelengths in: $]D_{i+1}, D_i]$

\[
D_i \quad \text{horizontal scale} \quad \leftrightarrow \quad \sigma_i \quad \text{blending height} \quad \uparrow \quad \text{land use groups} \quad \downarrow \quad \text{model half levels}
\]

\[
s_i^2(\sigma) = \left[ \nabla_h \left( \frac{D_{i+1}}{D_i} D(\sigma) \right) \right]_{D_i}^2
\]

**simple vertical profiles** of the slope variance profile function of each mode:

\[
\text{step function} \quad \leftrightarrow \quad \delta\text{-variance spectrum of the mode}
\]

- It holds the relation: $s^2(\sigma) = \sum_{i=1}^{m} s_i^2(\sigma)$
The canopy architecture with spectral decomposition in orographic modes:

- Band pass filtered variance spectrum of the i-th mode
- Variance profile of the i-th mode
- Variance profile of a $\delta$-spectrum
- Blending height of the biggest homogenous land use canopy
- $\sigma = 50m$
- $\sigma = 30m$
- $\sigma = 16m$
- $\sigma = 10m$
- $\sigma = 2m$
- $\sigma = 0$

Symbols:
- $\sigma_m$
- $\sigma_i$
- $\sigma_{i+1}$
- $D_0$, $D_1$, $D_2$, $D_3$
- $s^2_0$, $s^2_1$

Transfer layer:
- $\sum_{i=0}^{m} s^2_i$
Relations to available external parameters

- Define the **horizontal surface area function** \((S_i)^2 := 1 + s_i^2\) for a single mode \(i\).

- Combination of the GBA-results with the logarithmic wind profile above a roughness layer leads to the following relations between the **blending height** \(\sigma_i\), the roughness length \(z_{0i}\), the displacement height \(d_{0i}\) and the **mean dynamic surface area index** \(\widetilde{S}_i\):

\[
\frac{z_{0i}}{\sigma_i}, \frac{d_{0i}}{\sigma_i}, \frac{\sigma_i}{\widetilde{S}_i} := \int_0^{\sigma_i} \frac{d\sigma}{\widetilde{S}_i} = \frac{l(\sigma_i)}{\kappa} = \sigma_i - d_{0i} \approx z_{0i}
\]

*in the rough case*
Extension for the turbulence scheme in GBA:

- Substitution of the **vertical turbulent fluxes** in the **shear terms** of the 2-nd order equations by those multiplied with the appropriate **surface area function** (effective vertical fluxes)

- **Mellor/Yamada-Scheme** with **HBA replaced by the GBA**, where the **gravity vector** assumed to be normal to the mean orography.

  - Same kind of **flux gradient representation**, but:
    - **coefficients** in the linear equation for \( S^H \) and \( S^M \) are modified by the **surface area functions**
    - **shear term** in the TKE-equation contains the **specific surface area function** for **momentum** expressing **wake turbulence** production

- **The resistance law** in the **constant flux layer** can also be generalised by the help of the **surface area functions**
Conclusions:

• As averaging and differentiation in space cannot be commuted near the surface, additional roughness layer terms arise and have to be considered in a shallow lowest model layer.

• In the orographic approximation, model layers are not intersected by roughness elements. Then roughness layer terms can be explained purely by coordinate transformation and have the form of correlations between sub grid scale model layer slopes and the model variables.

• In the general boundary layer approximation, roughness layer effects can be expressed by consideration of simple surface area functions.

• The subgrid scale orography can be decomposed into spectral modes, each with an own blending height and an own surface area index.

• The surface area index and the blending height can be related to the roughness length and the displacement height of the surface structure.

• The second order turbulence closure can be generalised for the roughness layer.

• First implementation, testing (and parameter tuning) will be done using the single column framework.
Thank you for your attention!!
The model equations:

♦ 5 prognostic variables:

\[ \phi = \begin{array}{c|c}
V_i & i = 1, 2, 3 \\hline
q_w & \text{Mass fraction of the total water content} \\hline
\theta_l & \text{liquid water potential temperature}
\end{array} \]

- Velocity components (momentum concentration)

♦ Prognostic budget equations:

\[ \partial_t (\rho \phi) + \nabla \cdot F^\phi = Q^\phi \]

- General budget

\[ F_j^\phi = \rho \phi v_j - c_j^\phi \]

- J-component of the total flux density for the property \( \phi \)
• with the non advective molecular flux density components:

\[ c_j^\phi = \begin{cases} 
- \rho \ k^\phi \partial_j \phi & \text{for scalars } \phi = \theta_i, q_w \\
- \rho \nu (\partial_i v_j + \partial_j v_i) + \delta_{ij} p & \text{for momentum } \phi = v_i 
\end{cases} \]

\( k^\phi \): molecular diffusion coeff. \( \nu \): kinematic viscosity

• and the source terms:

\[ Q^\phi = \begin{cases} 
- \nabla \cdot \frac{S}{\pi c_p} = 0 & (\text{for moist adiab. idealization}) \quad , \phi = \theta_i \quad (\text{conservative}) \\
0 & , \phi = q_w \quad \text{conservative} \\
- \delta_{i3} \rho \ g - 2 \rho (\Omega \times v)_i & , \phi = v_i \quad \text{not conservative}
\end{cases} \]

\( S \): radiation flux density \( \Omega \): angular vector of the earth
\( g \): gravity of the earth \( \pi \): Exner factor
\( c_p \): heat capacity of the air at constant pressure
• The **transformation**  $T$  from the $\sigma$ -system into the $z$ -system with respect to a scalar field $\zeta$ can be described by the following relation:

\[
(x, y, \sigma) \in \mathbb{R}^3 \xrightarrow{T} (x, y, z_\sigma(x, y)) \in \mathbb{R}^3 \quad \zeta \in \mathbb{R}
\]

• **Derivatives in space** transform as follows:

\[
\begin{align*}
\partial_j \zeta \big|_z &= \partial_j \zeta \big|_\sigma - \partial_z \sigma \partial_\sigma \zeta \big|_\sigma \partial_j z_\sigma, \quad j \in \{1, 2\} = (x, y) \\
\partial_z \zeta \big|_z &= \partial_z \sigma \partial_\sigma \zeta \big|_\sigma
\end{align*}
\]
The problem of the filtered horizontal gradients

- If roughness elements are intersecting a grid box, derivation in space and averaging can not be commuted:

$$\partial_j \zeta_{|\sigma} = \partial_j \zeta_{|\sigma} + \partial_j \zeta'_{|\sigma}$$

consists of a term due to the volume reduction and an integral along the surfaces of intersected bodies.
The averaged derivative along a horizontal coordinate of the Cartesian $\mathcal{Z}$-system:

$$\frac{\partial}{\partial_j} \zeta_{||z} = \frac{\partial}{\partial_j} \zeta_{||\sigma} - \frac{\partial}{\partial_z} \sigma \frac{\partial}{\partial_j} \zeta_{||\sigma} \frac{\partial}{\partial_j} \zeta_{||\sigma} - \frac{\partial}{\partial_z} \sigma \frac{\partial}{\partial\sigma} \left[ \zeta'_{||\sigma} \left( \frac{\partial}{\partial_j} \zeta_{||\sigma} \right)' \right] + \frac{\partial}{\partial_j} \zeta'_{||z}, \ j = x, y$$

- transformed grid scale gradient
- sub grid scale transformation correction
- sub grid scale intersection correction
- grid scale correction
Former solutions in LM

- They all refer to coordinates following the mean orography.
  - No sub grid scale transformation correction needed

- The old scheme and even the operational configuration of the new scheme do not contain any sub grid scale intersection correction as well
  - No canopy term is included at all
• The **old scheme:**

The canopy height is assumed to be **small** compared to the depth of the lowest model layer.

• The **operational scheme:**

The canopy is assumed to be **below** the lowest model layer.

\[ S = 0 \]

\[ x \Delta x \]
The up to now **incomplete extension version** of the operational scheme contains **intersection terms** at least in the **momentum budget**:

- The **canopy** is **resolved** by model layers.
- The **intersection terms** have to be **parameterized**.
- For each inner canopy layer:
  - **external parameters**
  - a **soil model**
The general boundary layer approximation GBA:

A substitute of the horizontal boundary layer approximation

• For any level $\sigma > 0$ there exists a strictly in $\sigma$ monotonic growing horizontal scale $D(\sigma) > 0$, which is the largest scale that has for any model variable $\xi$ the following properties:

  - For any smaller scale $x \leq D(\sigma)$ it is: $\xi \nabla_h z_{\sigma}^x = 0$
  - For any larger scale $X \geq D(\sigma)$ it is: $\nabla_h \xi x |_{\sigma}^X = 0$.

• Is $V_n$ the velocity component normal to the idealised iso-surface $S(\sigma)$ given by $z = z_{\sigma}^{D(\sigma)}$, it is:

  $\hat{V}_n^{D(\sigma)} = 0 = \partial_n \hat{V}_n^{D(\sigma)}$.

• It holds a flux gradient relation for all sub grid scale fluxes with respect to the scale $D(\sigma)$. 