Deutscher Wetterdienst

Implicit Time Integration

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State of current SI developments for nh models

- Convergence often slow for realistic cases
- SI or SI/SL not much faster than split explicit KW or RK schemes
- Desired feature: Expense of Helmholtz solver in the order of a KW step, in order that a potential increase of time step has full effect on efficiency

Plan of Lecture

- Different implicit schemes
- The principle of direct solutions for non-periodic problems
- Examples for LHI schemes in irregular boundaries
- Comparison of full and partial schemes

Implicit Approaches

Example: $\partial h / \partial t = \partial (uh) / \partial x$ u : constant field; h : dynamic field

Nonlinear : $h^{n+1} - h^{n} = dt([uh]_x{}^{n} + [uh]_x{}^{n+1})/2$

Tangent linear : $h^{n+1} - h^{n} = dt(u_x h^{n+1} + u_x h^{n} + u^{n+1} h_x + u^{n} h_x)/2$

Locally homogenised : $h^{n+1} - h^{n} = dt(uh_x{}^{n+1} + uh_x{}^{n})/2$

Organisation of the Implicit Time-Step

- The Fourier Coefficients are the same for the grid Points of a Subregion
- The Linearised Eqs. are different for each Gridpoint
- In Case of only One Subregion the Support Points of the derivative f_{0x} are the Boundary Values
- f_{0x} does not create time-Step Limitations
- GFT Returns the Grid-Point-Values after doing a Different Eigenvalue Calculation at Each Point

The SI Timestep

Subtract Large Scale Part of each function

Compute Fourier Coefficients

Choose Gridpoint

Compute next time level in Fourier Space

Transform Back

Use Result Only for Chosen Grid-Point

Do the Above for all Other Grid Points

Set Boundary Values as in Finite Difference Methods

Example:1-d Schallow Water Eq

$$\partial u / \partial t = -u \frac{\partial u}{\partial x} - \frac{\partial h}{\partial x}$$

$$\partial h / \partial t = -\partial (hu) / \partial x$$

SI Scheme :

$$u^{n+1} - u^n = -dt(u^n (u_x^{n+1} + u_x^n)/2 + (h_x^{n+1} + h_x^n)/2)$$

$$h^{n+1} - h^n = -dt(h^n (u_x^{n+1} + u_x^n)/2 + u^n (h_x^{n+1} + h_x^n)/2)$$

Definitions:

$$\Delta u = u^{n+1} - u^n ; \Delta h = h^{n+1} - h^n ;$$

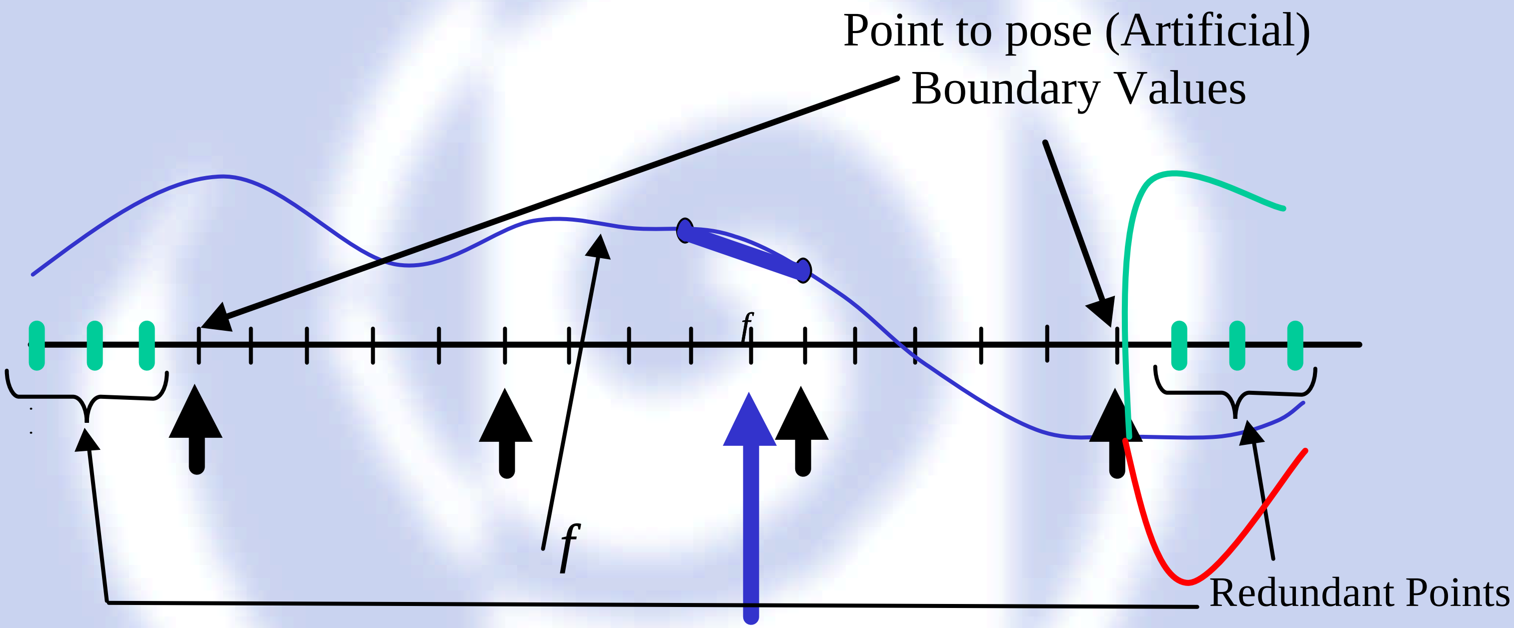
$$rsu = -u^n u_x^n - h_x^n ; rsh = u^n h_x^n - h^n u_x^n$$

SI Scheme:

$$\Delta u = rsu - dtu^n \Delta u_x - dt\Delta h_x = rsu - dtu^n \Delta u'_x - dt\Delta h'_x$$

$$\Delta h = rsh - dtu^n \Delta h_x - dth^n \Delta u_x = rsh - dt(dtu^n \Delta h'_x - dth^n \Delta u'_x$$

Boundary- and Exterior Points



•Redundant points can be included in the FT

•The result of the time-step does not depend on the continuation of the field to redundant points

Operation in Fourier Space

SI Scheme:

$$\Delta u = rsu - dt u^n \Delta u_x - dt \Delta h_x = rsu - dt u^n \Delta u'_x - dt \Delta h'_x$$

$$\Delta h = rsh - dt u^n \Delta h_x - dt h^n \Delta u_x = rsh - dt u^n \Delta h'_x - dt h^n \Delta u'_x$$

In the following rsu, rsh, u^n, h^n will be taken at some chosen gridpoint

Definition: $\tilde{f} = Ff$

Linear Equations at the chosen gridpoint:

$$\tilde{\Delta} u - dt u^n \tilde{\Delta} u_x - dt \tilde{\Delta} h_x = \tilde{r} su$$

$$-dt u^n \tilde{\Delta} h_x + \tilde{\Delta} h - dt h^n \tilde{\Delta} u_x = \tilde{r} sh$$

$\tilde{\Delta} u, \tilde{\Delta} h$ are obtained from the solution of the linear equation.

The back transformed fields $\tilde{\Delta} u, \tilde{\Delta} h = F^{-1}\tilde{\Delta} u, F^{-1}\tilde{\Delta} h$ will be taken at the chosen gridpoint only

LHI options

- Full/partial schemes: The latter partitiones the area and solves the implicit equs only on subregions.
- Blocked partial scheme: The computations are organised in such a way that only one FFT per subregion is necessary
- Ímplicit/Semi-Implicit schemes: The latter treats only the fast waves implicit.

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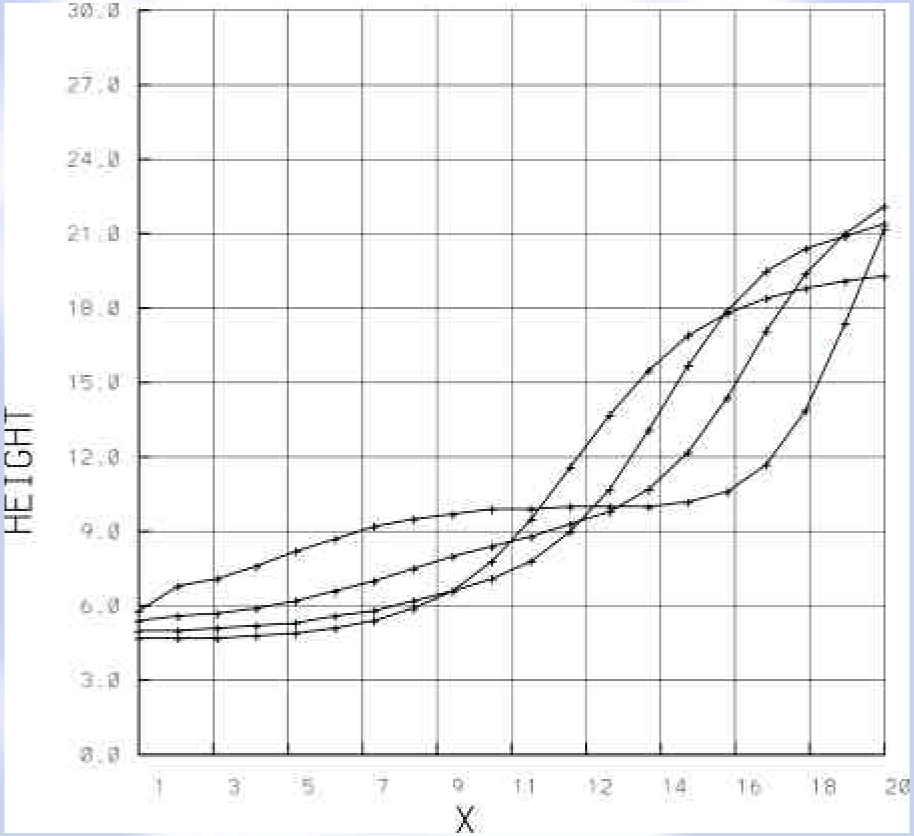
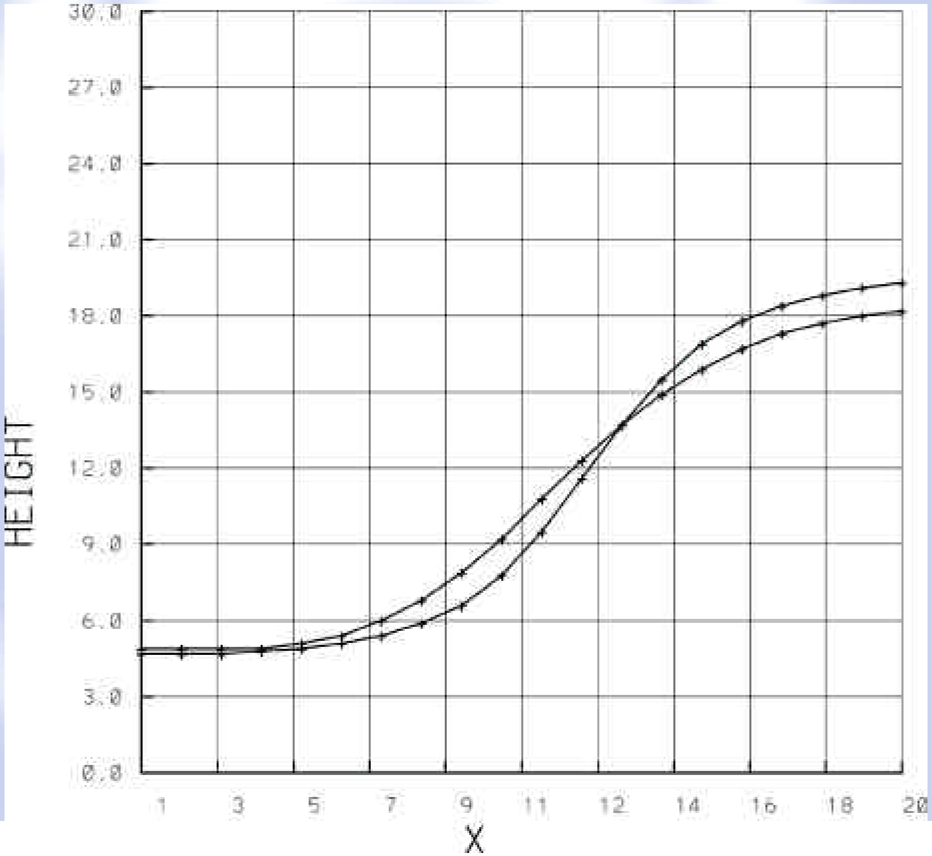
1-d schock (height field) wave at different times

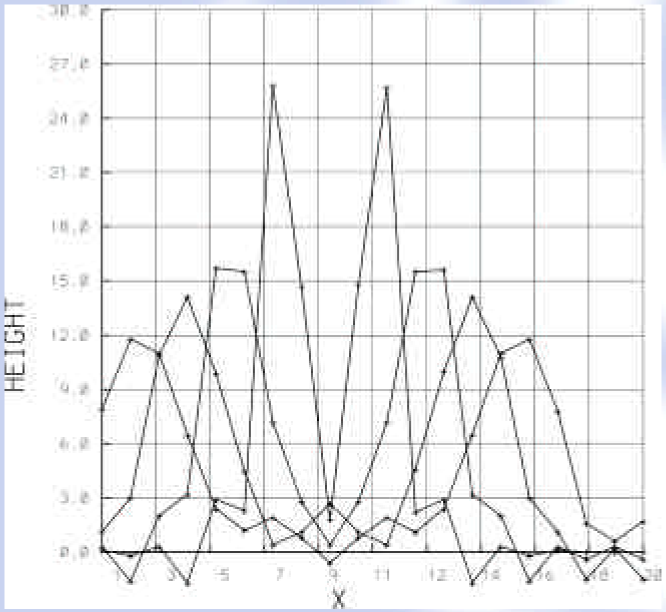


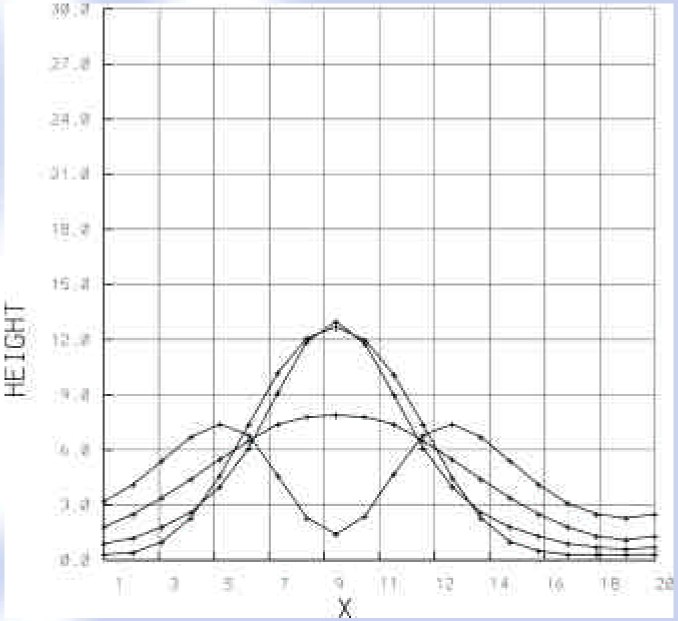
Explit (lax Wendroff) and implicit solutions at time n=100



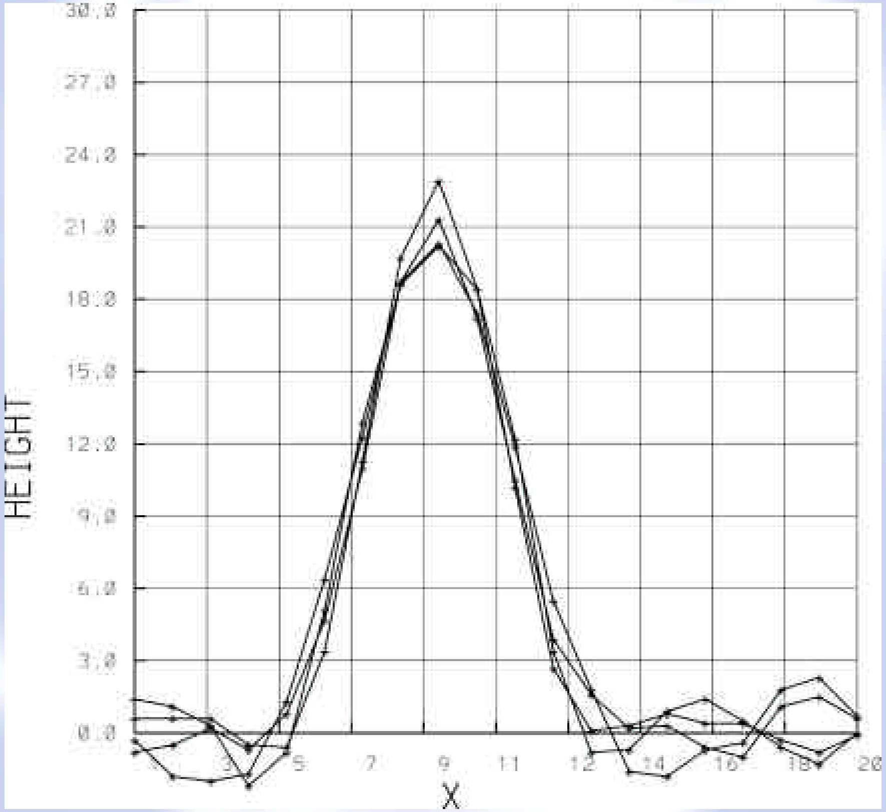
Propagation of gravitational wave

N=24,47,70,93 N=231,254,277,300

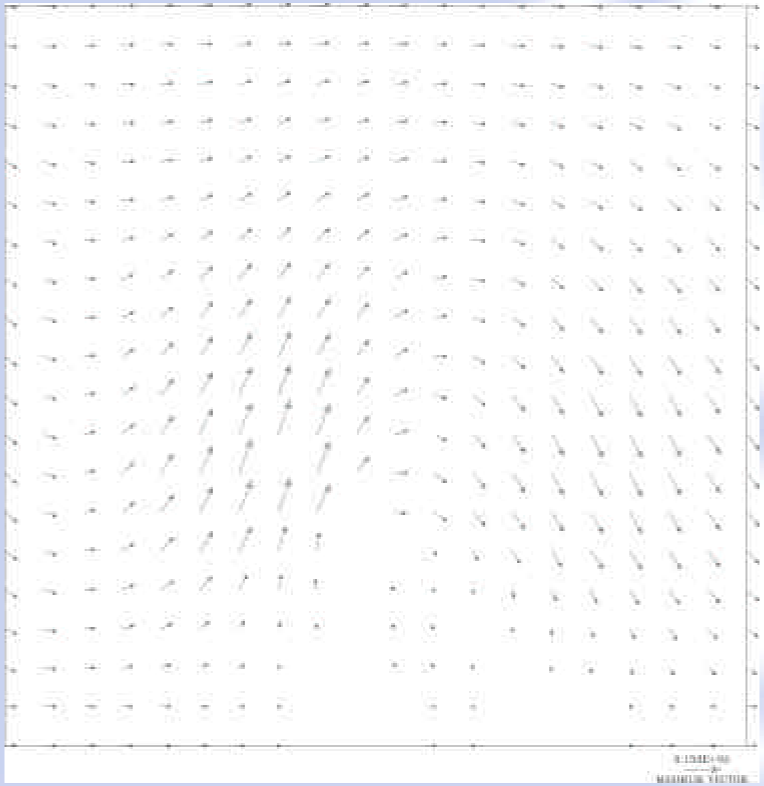


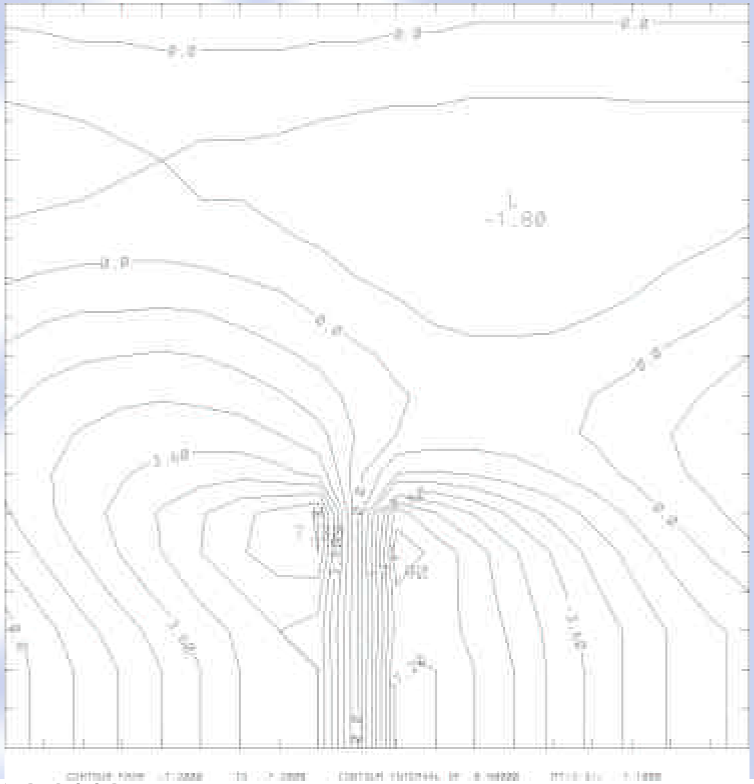


Gravitationalwave, for h0=160000,n=51,101,151,200



Barotropic flow around solid wall, two sided boundary conditions u0=10 m/sec





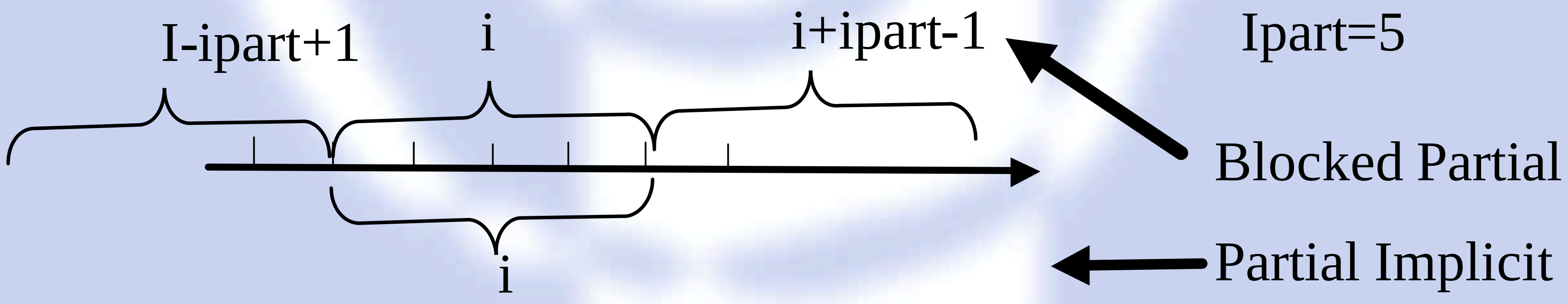
Partial implicit scheme

$$\Delta u = rsu - dt\,u^n \Delta u_x - dt\,\Delta h_x = rsu - dt\,u^n \Delta u'_x - dt\,\Delta h'_x$$

$$\Delta h = rsh - dt\,u^n \Delta h_x - dt\,h^n \Delta u_x = rsh - dt\,u^n \Delta h'_x - dt\,h^n \Delta u'_x$$

Artificial Boundary Condition

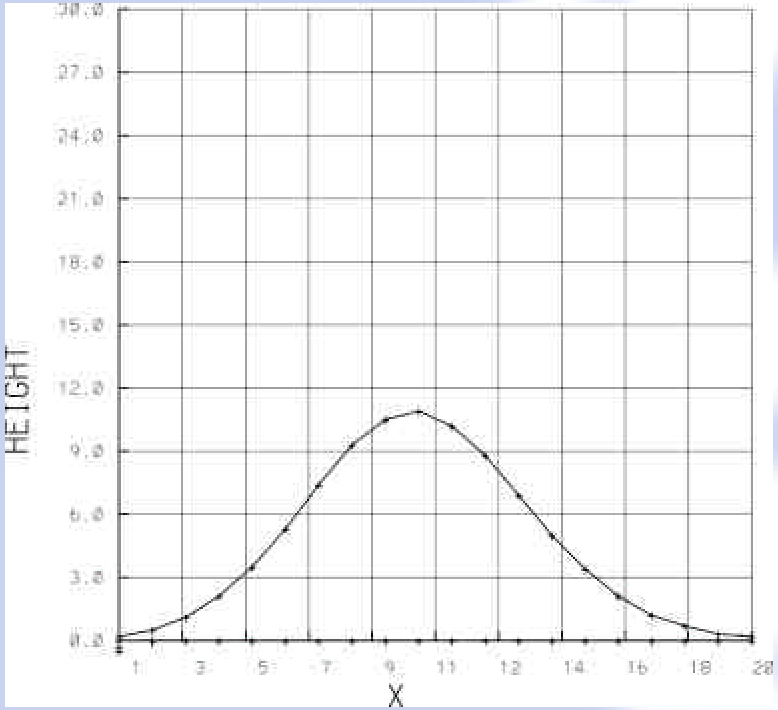
$$\Delta h(i - ipart/2) = \Delta h(i + ipart/2); \Delta u(i - ipart/2) = \Delta u(i + ipart/2)$$



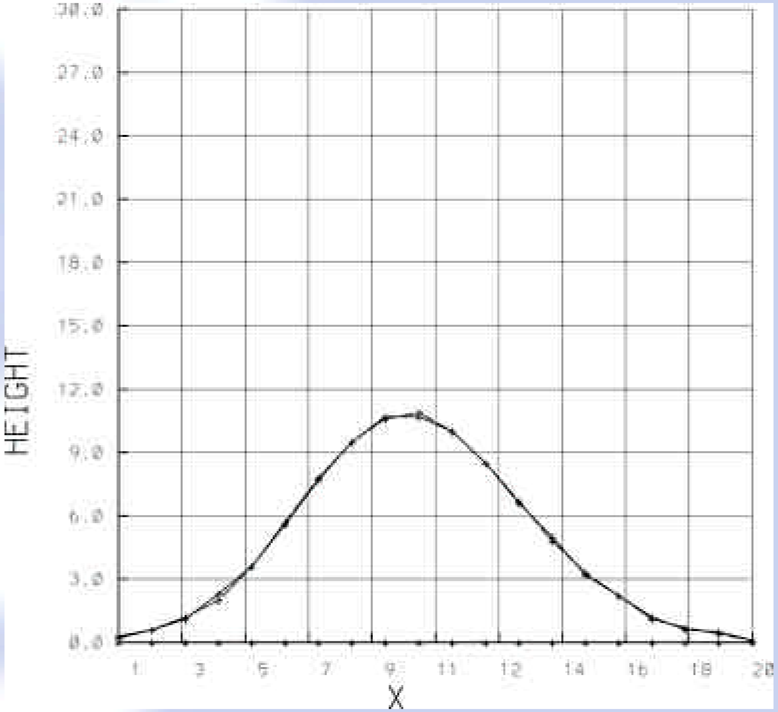
Partial implicit, 1-d Rossby wave

area=20dx; dt=400

Initial



Forecast ipart=5, Ipart=11



Partial and Blocked Partial Imlicit

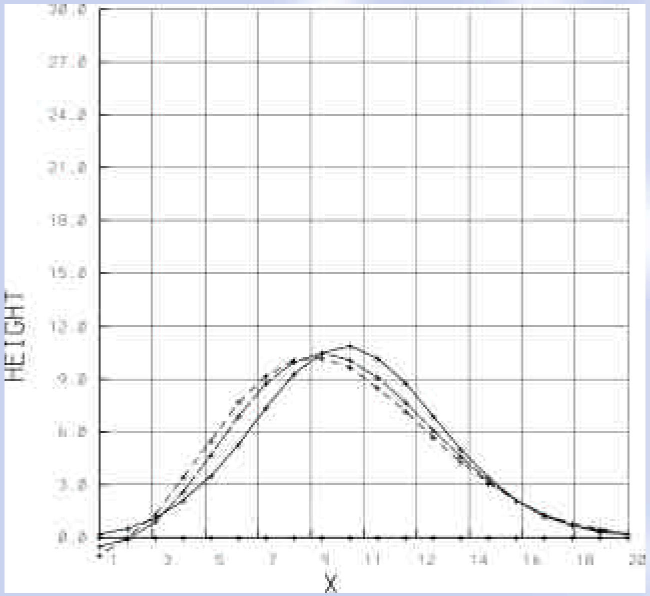
area=200 dx

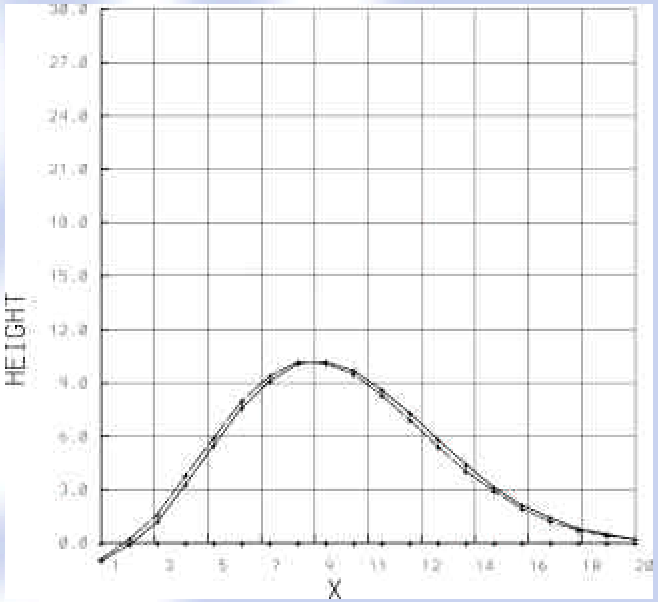
Partial Implicit,

ipart=11,dt=400 and dt=800sec

Blocked Partial Implicit

(5-,5), dt=800 sec





Conclusions

- A direct si- method was proposed
- The method is based on a generalised Fourier Transform
- The generalised FT is potentially as efficient as the FT (fast FT)
- The method is efficient for increased spatial order
- 1-d and 2-d tests have been performed

Direct Methods for Locally Homogenized SI

- The LH - is the Most Common SI Method
- The Equations of Motion are homogeneously linearised at each Grid Point
- At Each Grid-Point a Problem of Constant Coefficients is Defined
- For Each Grid-Point The Associated Linear Problem Can be Solved Using an FT and a Linear Problem Specific to Each Grid-Point
- The GFT (Generalised FT) Computes the Results of the **Different FTs** Using **One generalised Transform**
- The numerical cost of GFT is Simlar to that of an FT
- A Fast GFT exists similar to Fast FT