A radiative upper boundary condition of the Klemp-Durran-Bougeault-type and its application to a compressible nonhydrostatic model -present state towards LM-implementation-

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Contents

- Simulations with a fast-mode LM toy-model demonstrating that the successful application of the KDB-RUBC in a nonhydrostatic compressible model is feasible.
- First application of the KDB-RUBC in the LM-code
- Summary and outlook

A spectral toy-model with time-difference scheme adopted from the LM fast- mode part

$$\frac{\hat{u}^{(t+1)} - \hat{u}^{(t)}}{\Delta t} = -\frac{k}{\bar{r}}\hat{p}^{(t)} + f_0 \hat{v}^{(t+1)} \quad , \qquad k = \frac{2p}{\Lambda}$$

- horizontal wave number

$$\frac{\hat{v}^{(t+1)} - \hat{v}^{(t)}}{\Delta t} = -f_0 \hat{u}^{(t+1)}$$

$$m\frac{\hat{w}^{(t+1)} - \hat{w}^{(t)}}{\Delta t} = -\frac{1}{\bar{r}}\left(b^{+}\frac{\partial \hat{p}^{(t+1)}}{\partial z} + b^{-}\frac{\partial \hat{p}^{(t)}}{\partial z}\right) - \frac{g}{\bar{r}\, c_s^2}\left(b^{+}\hat{p}^{(t+1)} + b^{-}\hat{p}^{(t)}\right) + \frac{g}{\bar{q}}\hat{q}^{(t)}$$

$$\frac{1}{c_s^2}\frac{\hat{p}^{(t+1)} - \hat{p}^{(t)}}{\Delta t} = \frac{g\,\bar{r}}{c_s^2}\left(b^{+}\hat{w}^{(t+1)} + b^{-}\hat{w}^{(t)}\right) - \bar{r}\left(b^{+}\frac{\partial \hat{w}^{(t+1)}}{\partial z} + b^{-}\frac{\partial \hat{w}^{(t)}}{\partial z}\right) + \bar{r}\, k\, \hat{u}^{(t+1)}$$

$$\frac{\hat{q}^{(t+1)} - \hat{q}^{(t)}}{\Delta t} = -\frac{N^2\bar{q}}{g}\left(b^{+}\hat{w}^{(t+1)} + b^{-}\hat{w}^{(t)}\right)$$

Method of model simplification as in Arakawa and Konor (1996) (cf. Herzog, 2004, Internal paper, prepared for a COSMO Techn.Paper)

Time-level weights to determine the degree of implicity

$$b^{+} = \frac{1}{2}(1+e), \quad b^{-} = \frac{1}{2}(1-e)$$

$$e = 0.4$$

Divergence damping in the pressure gradient term

$$\hat{p}^{(t)} := \hat{p}^{(t)} + a_d\, \Delta t\, \bar{r}\, c_s^2 \left(k\, \hat{u}^{(t)} - \frac{\partial \hat{w}^{(t)}}{\partial z}\right)$$

$a_d = 0.3$ - differing from LM default-value (=0.1)

The Radiative Upper Boundary Condition of Klemp - Durran - Bougeault
(KDB - RUBC)

Limiting assumptions for the validity of the KDB-RUBC :

- hydrostatic approximation	$m = 0$
- condition of incompressibility	$c_s^2 \rightarrow \infty$
- isothermal basic state	
- no rotation	$f_0 = 0$
- Boussinesq - approximation	$\frac{\partial}{\partial z} >> \frac{1}{H}, \; n^2 >> \frac{1}{H^2}$
- reduced frequency equation	$w \; n = \pm N \; k$

Physical radiative condition

$$\overline{\left(\hat{p} \; \hat{w} \right)}_{top} > 0$$

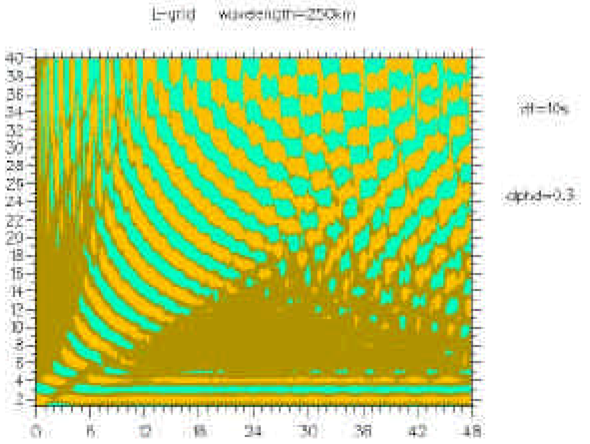
KDB-RUBC

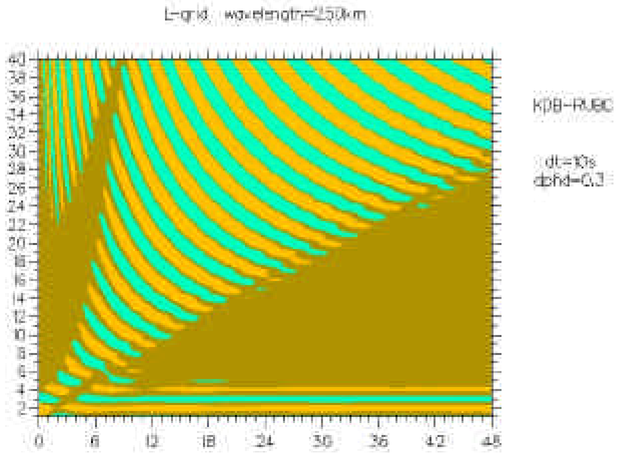
$$\hat{p}_{top}(t) = \frac{\left(\bar{r} \; N \right)_{top}}{k} \; \hat{w}_{top}(t)$$

lid-condition

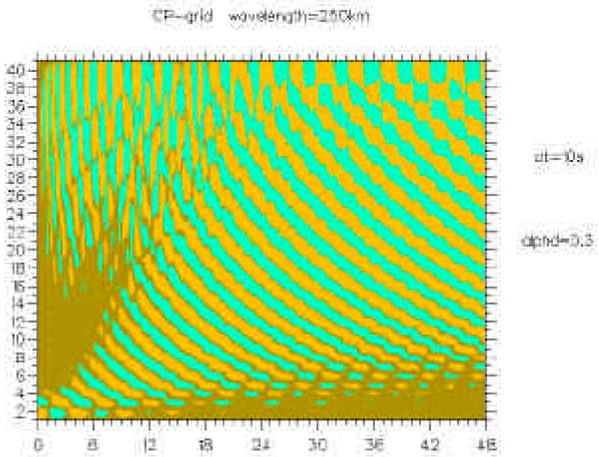
KDB-RUBC

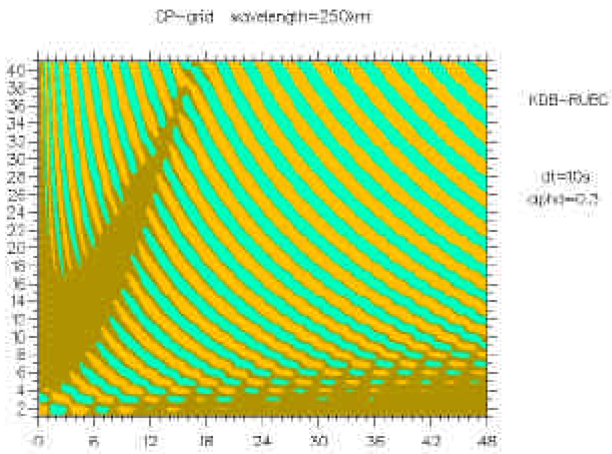
Process of dispersion from defined initial perturbance of potential temperature chosen as in Arakawa und Konor(1996). Simulation over 48 h. Spectral perturbation amplitude associated to horizontal wavelength L=250km.





L-grid





CP-grid

Simulation experiments with the <u>fast-mode LM toy-model</u> have shown that the KDB-RUBC can be implemented successfully in a nonhydrostatic compressible model with the given semi-implicit time-scheme, from which vertically propagating acoustic waves are effectively filtered out and horizontally propagating acoustic waves are sufficiently suppressed due to a divergence damping approach.

We have found out <u>two equally successful methods</u> how to implement the KDB-RUBC:

Method 1: A judicious idea from D.R. Durran (1999) to incorporate this RUBC in the vertically implicit time-scheme. It is functioning with our toy-model, but needs reformulations of the given algorithm with lid-condition.

Method 2: A direct method, which is easily implemented without interventions in the given model algorithm and is independent of a given time-scheme. It operates satisfactory, too, and leads to results equivalent to the Durran-method.

Further strategy : - stepwise generalisation in the LM-world
- first step ⟶ repetition of the given experimental set-up for the toy-model now with the LM

First experiment with the LM

1. Horizontal integration domain: 84 x 84 gridpoints, Δx=2.8km
2. Number of vertical layers: 35 (ke = 35)
3. Double-periodic continuation in the horizontal (lperi=.true.)
4. Slow tendencies are set equal to zero in subroutine fast_waves
5. Physics and Rayleigh-damping deleted
6. Isothermal basic state (polytropic state also possible)
7. Definition of initial state - u = v = w = p‘= 0
   - temperature disturbance

$$T'^{(0)}_{i,k} = A^{(0)}_{k} \sin\left(\frac{2\pi n}{N_l - 4}(i-1) \right), N_l = 84$$

$$A^{(0)}_{k=20} = -0.5K \,,\; A^{(0)}_{k=21} = +0.5K$$

$$n = 2,4,\ldots\left(\Lambda = 112, 56, \ldots km \right)$$

8. Application of KDB-RUBC after Method 2, now including discrete Fourier-transform and its inverse operation, with adaptation to the C-grid staggering. (We failed with Method 1 !)

The KDB -RUBC - operations

1. p'_{top} be extrapolated hydrostatically from k=1 to k'=1

2. **Fourier-transform** $(\hat{p}'_{k,l})_{top} := FT(p'_{i,j})_{top}$

3. **KDB - RUBC :**

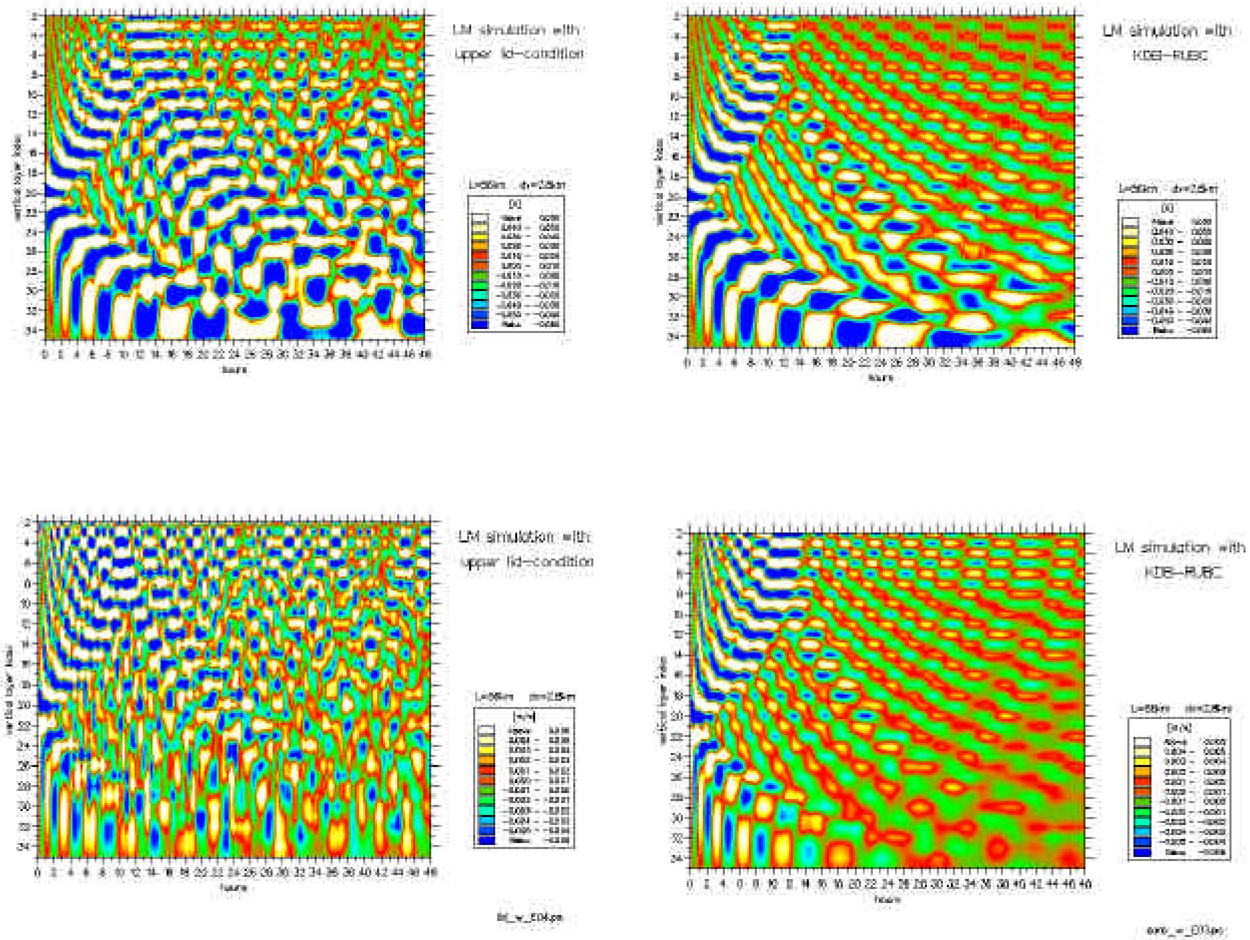
$$(\hat{w}_{k,l})_{top} := \frac{\sqrt{k'^2 + l'^2}}{(N\bar{\rho})_{top}} (\hat{p}'_{k,l})_{top}$$

mit $k' = \dfrac{2\sin(k\frac{\pi}{N_\lambda})}{a\cos\varphi_0 \Delta\lambda}$, $l' = \dfrac{2\sin(l\frac{\pi}{N_\varphi})}{a\Delta\varphi}$, $k \in [0, N_\lambda - 1]$, $l \in [0, N_\varphi - 1]$

4. **Inverse Fourier - transform** $(w_{i,j})_{top} := FT^{-1}(\hat{w}_{k,l})_{top}$

lid-condition **KDB-RUBC**

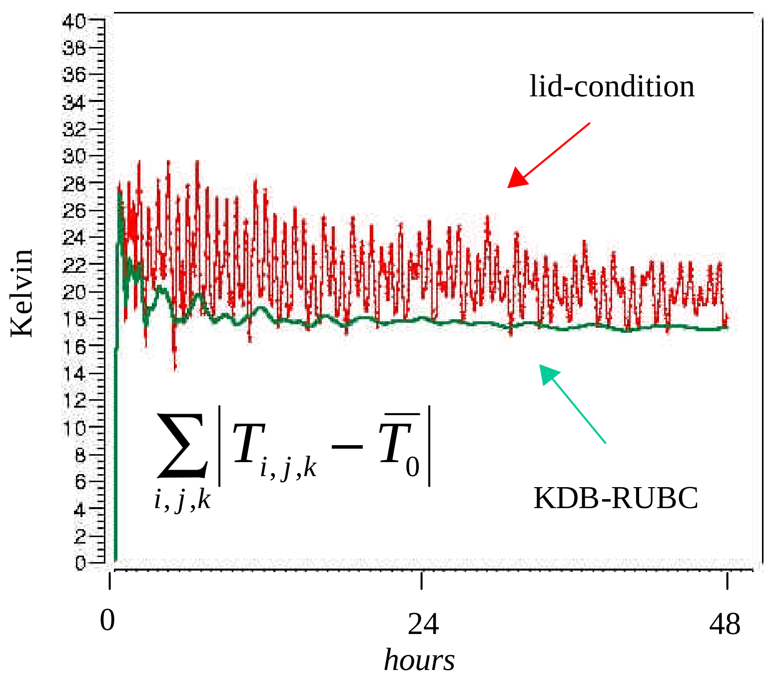
Time-height cross-section of perturbation amplitude for T‘ and w , respectively, at zonal gridpoint i = 11, associated to horizontal wavelength L =56km.

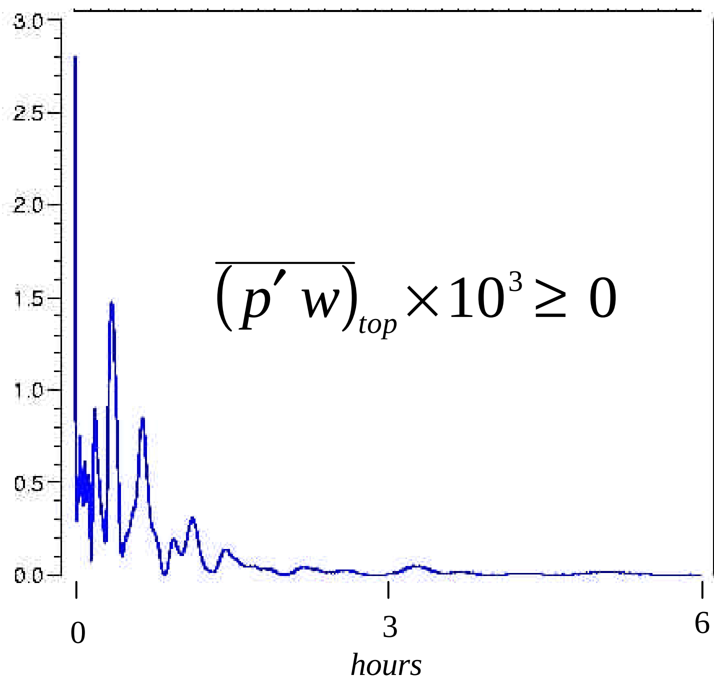


T‘

w

Pressure gradient term error test with LMK
for isothermal atmosphere at rest with bell-shaped mountain





$$a / H = 6\Delta s / 1000m \;, \quad \Delta s = 2.8km$$

Summary and outlook for further work

- On principle, we were able to show that the KDB-RUBC is applicable in a model-type like the LM, and it works in the right way.
- What follows is a straightforward engineering work
- Diverse generalisations of experimentation towards real cases ...
- Replacing FT by FFT
- Introducing an aperiodic integration domain
- Taking advantage from experiences with DM / SM / HRM
- Co-operation with *MeteoSwiss* : real-case - studies and optimisation !

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Deutscher Wetterdienst

Further investigations concerning CP - grid advantages versus L - grid shortcomings

by

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A report about is unfortunately not possible here,
but a COSMO Technical Report is being prepared instead.

A note to the COSMO meeting
in Milano, Italia , Sept. 2004