RK-Time Integration and High-Order Spatial Discretization – a New Dynamical Core for the LMK

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Outline

- Motivation for the New Dynamical Core
- Discretization
  - “normal“ Runge-Kutta
  - TVD-Runge-Kutta (Total Variation Diminishing)
  - “Time-splitting“ - Method
  - Advection-Schemes of High Spatial Order
- Idealized Test (of Advection)
- Prognostic Treatment of Precipitation (20 February 2002)
- Numerical Experiments – the “Test Suites”
  ... and their Visualization – the “Plot Suite”
- Nesting – the “Pressure Problem”
- Shallow Convection Parameterization is Needed
- Metrics: $\sqrt{\gamma} \Rightarrow \sqrt{G}$ and
  “Symmetric Thermodynamics”: $T \Rightarrow T^* = T - T_0(z)$
- Summary and Outlook

Motivation for the New Dynamical Core

1.) Change from 3-timelevel to 2-timelevel scheme

- no time-filter is needed (Asselin-filter reduces scheme to 1st order)
- combination with positive definite advection schemes is possible (important for moisture quantities or chem. substances)

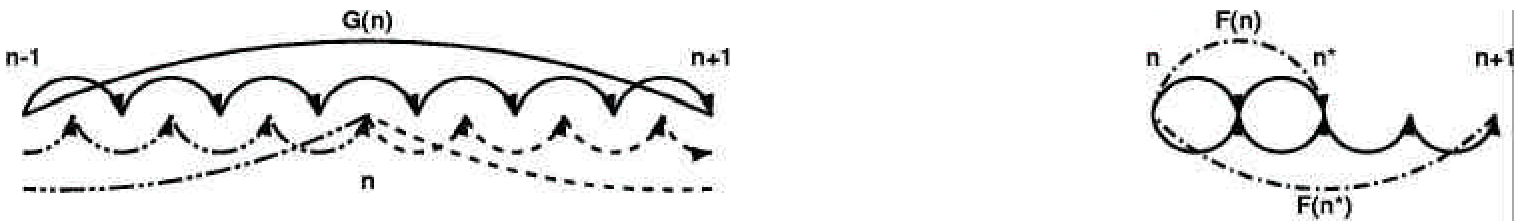


Figure 1. Gassmann (2002). left: 3-timelevel scheme (Klemp und Wilhelmson 1978), right: 2-timelevel scheme (Wicker und Skamarock 1998).

2.) Use schemes of higher temporal and spatial order

- bigger time step (Courant number of approx. 1.8)
  all in all 2-TL TVD-RK-scheme is as fast as the 3-TL Leapfrog-scheme
- better convergence due to higher spatial accuracy

Problem to Solve:

$$\frac{\partial \phi}{\partial t} = L^{slow}(\phi) + L^{fast}(\phi)$$

Computation of the Slow Tendency:

Normal 3rd-order Runge-Kutta:

$$\begin{aligned}\phi^*_{i,k} &= \phi^n_{i,k} - \frac{1}{3}\Delta t\, L^h_i(\phi^n) - \frac{1}{3}\Delta t\Big(\beta^+ L^v_k(\phi^*) + \beta^- L^v_k(\phi^n)\Big)\\ &= \phi^0_{i,k} + \frac{1}{3}\Delta t\, L^{slow}_{i,k}\Big|_0^*\end{aligned}$$

$$\begin{aligned}\phi^{**}_{i,k} &= \phi^n_{i,k} - \frac{1}{2}\Delta t\, L^h_i(\phi^*) - \frac{1}{2}\Delta t\Big(\beta^+ L^v_k(\phi^{**}) + \beta^- L^v_k(\phi^*)\Big)\\ &= \phi^0_{i,k} + \frac{1}{2}\Delta t\, L^{slow}_{i,k}\Big|_0^{**}\end{aligned}$$

$$\begin{aligned}\phi^{n+1}_{i,k} &= \phi^n_{i,k} - \Delta t\, L^h_i(\phi^{**}) - \Delta t\Big(\beta^+ L^v_k(\phi^{n+1}) + \beta^- L^v_k(\phi^{**})\Big)\\ &= \phi^0_{i,k} + \Delta t\, L^{slow}_{i,k}\Big|_0^{n+1}\end{aligned}$$

Problem to Solve:

$$\frac{\partial \phi}{\partial t} = L^{slow}(\phi) + L^{fast}(\phi)$$

Computation of the Slow Tendency:

TVD-variant of 3rd-order Runge-Kutta:

$$\phi^{*}_{i,k} = \phi^{n}_{i,k} - \Delta t \; L^{h}_{i}(\phi^{n}) - \Delta t \left(\beta^{+} L^{v}_{k}(\phi^{*}) + \beta^{-} L^{v}_{k}(\phi^{n}) \right)$$
$$= \phi^{0}_{i,k} + \Delta t \; L^{slow}_{i,k} \Big|^{*}_{0}$$

$$\phi^{**}_{i,k} = \frac{3}{4}\phi^{n}_{i,k} + \frac{1}{4}\phi^{*}_{i,k} - \frac{1}{4}\Delta t \; L^{h}_{i}(\phi^{*}) - \frac{1}{4}\Delta t \left(\beta^{+} L^{v}_{k}(\phi^{**}) + \beta^{-} L^{v}_{k}(\phi^{*}) \right)$$
$$= \phi^{0}_{i,k} + \frac{1}{4}\Delta t \; L^{slow}_{i,k} \Big|^{**}_{0}$$

$$\phi^{n+1}_{i,k} = \frac{1}{3}\phi^{n}_{i,k} + \frac{2}{3}\phi^{**}_{i,k} - \frac{2}{3}\Delta t \; L^{h}_{i}(\phi^{**}) - \frac{2}{3}\Delta t \left(\beta^{+} L^{v}_{k}(\phi^{n+1}) + \beta^{-} L^{v}_{k}(\phi^{**}) \right)$$
$$= \phi^{0}_{i,k} + \frac{2}{3}\Delta t \; L^{slow}_{i,k} \Big|^{n+1}_{0}$$

Time-Splitting Method:

After each Runge-Kutta step the fast modes are integrated forward to the desired point in time using several small time steps $\Delta\tau$ – the slow tendency is fixed. The starting point of the integration $\phi^0_{i,k}$ depends on the choosen variant of the Runge-Kutta scheme – for the first variant it is always equal to $\phi^n_{i,k}$:

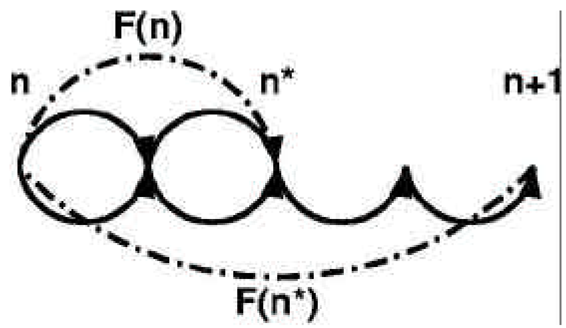
1. step:

$$\phi^{0+\Delta\tau}_{i,k} = \phi^0_{i,k} + \Delta\tau\, L^{fast}_{i,k}(\phi^0) + \Delta\tau\, L^{slow}_{i,k}\Big|^{\times}_{0}$$

remaining steps:

$$\phi^{\tau+\Delta\tau}_{i,k} = \phi^\tau_{i,k} + \Delta\tau\, L^{fast}_{i,k}(\phi^\tau) + \Delta\tau\, L^{slow}_{i,k}\Big|^{\times}_{0}$$

with $\times = *,\ **$ and $n+1$ in the individual Runge-Kutta steps.



Horizontal and Vertical Operators:

$$L_i^h(\phi)^{(4\text{th})} = \frac{u_i}{12\Delta x}\left[\phi_{i-2} - 8(\phi_{i-1} - \phi_{i+1}) - \phi_{i+2}\right]$$

$$L_i^h(\phi)^{(3\text{rd})} = L_i^h(\phi)^{(4\text{th})} + \frac{|u_i|}{12\Delta x}\left[\phi_{i-2} - 4(\phi_{i-1} + \phi_{i+1}) + 6\phi_i + \phi_{i+2}\right]$$

$$L_i^h(\phi)^{(6\text{th})} = \frac{u_i}{60\Delta x}\left[-\phi_{i-3} + 9(\phi_{i-2} - \phi_{i+2}) - 45(\phi_{i-1} - \phi_{i+1}) + \phi_{i+3}\right]$$

$$L_i^h(\phi)^{(5\text{th})} = L_i^h(\phi)^{(6\text{th})} + \frac{|u_i|}{60\Delta x}\left[-\phi_{i-3} + 6(\phi_{i-2} + \phi_{i+2}) - 15(\phi_{i-1} + \phi_{i+1}) + 20\phi_i - \phi_{i+3}\right]$$

$$L_k^v(\phi)^{(2\text{nd})} = \frac{w_k}{2\Delta z}(\phi_{k+1} - \phi_{k-1})$$

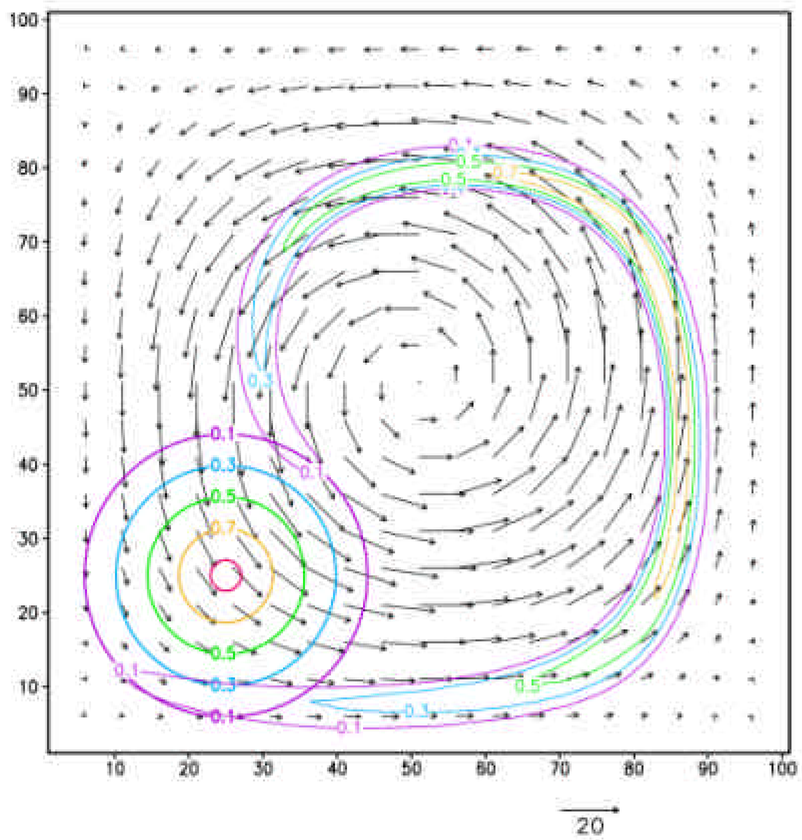
Possible Combinations – good / bad choice

- Time-Integration
  - “normal“ Runge-Kutta of 1st, 2nd or 3rd order (in time)
  - TVD-Runge-Kutta (Shu and Osher 1988) of 3rd order

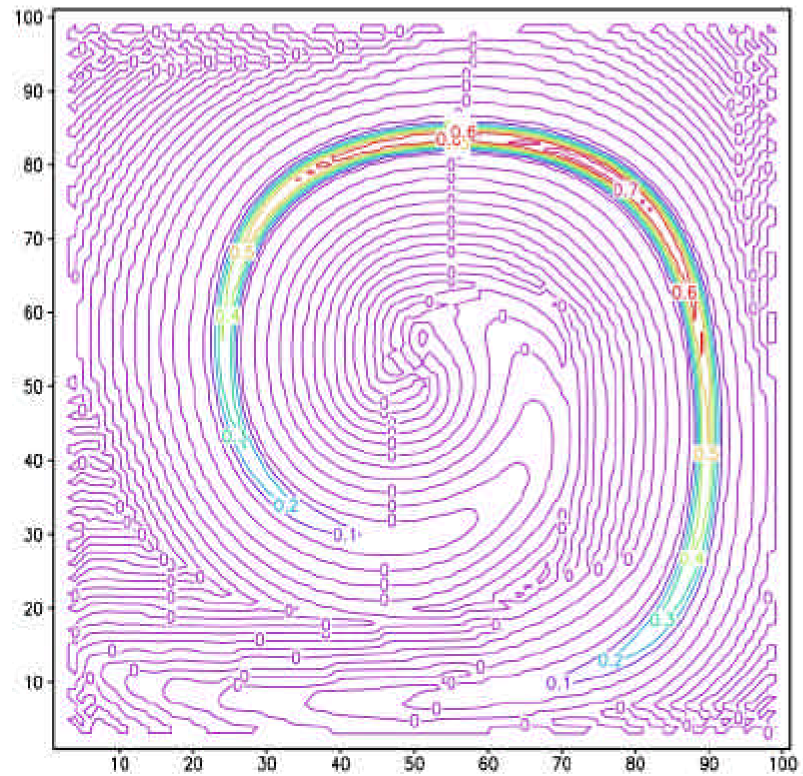
- horizontal Advection
  - upwind scheme (UP-) of 1st, 3rd or 5th order (in space)
  - centered-differences scheme (CD-) of 2nd, 4th or 6th order

- vertical Advection
  - implicit (*Crank-Nicolson*) with centered-differences
  - explicit of 1st, 2nd, 3rd or 4th order

Idealized Test of Advection (LeVeque 1996)



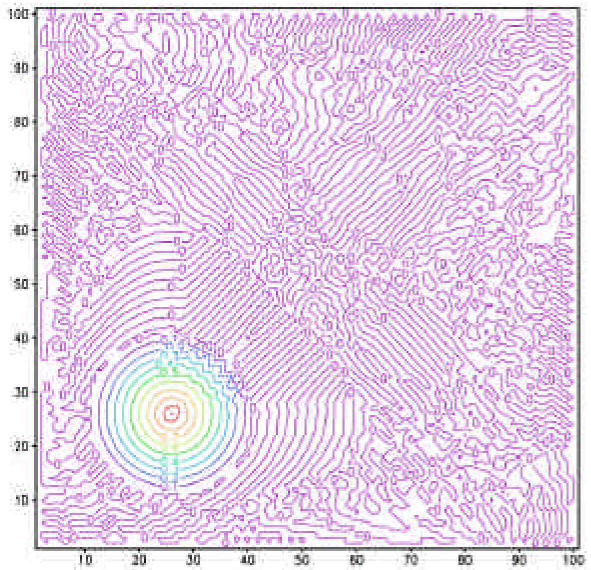
Initial field – cone with a maximum of 1.0 and...



field after first half of simulation.

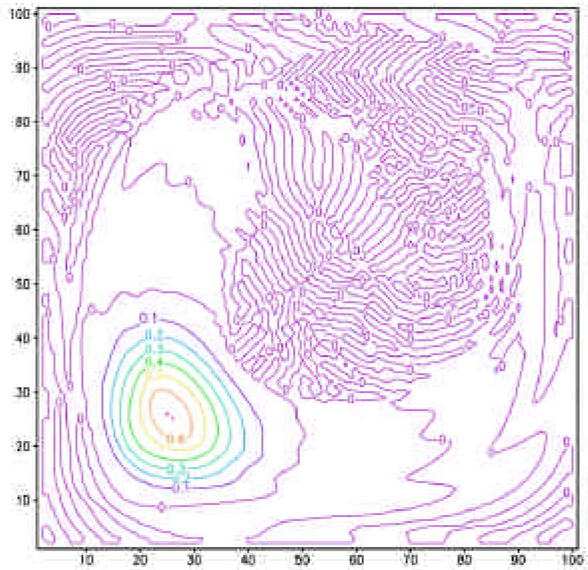
Centered-Differences Schemes

RK-3rd / CD-4th



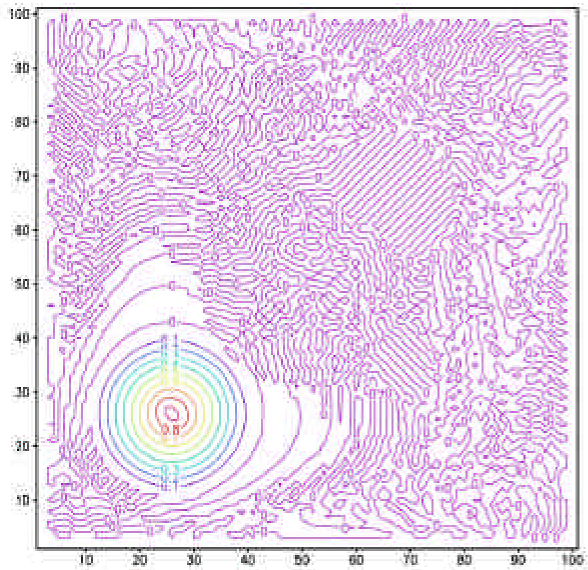
670 time steps

RK-3rd / CD-4th + HD



550 time steps

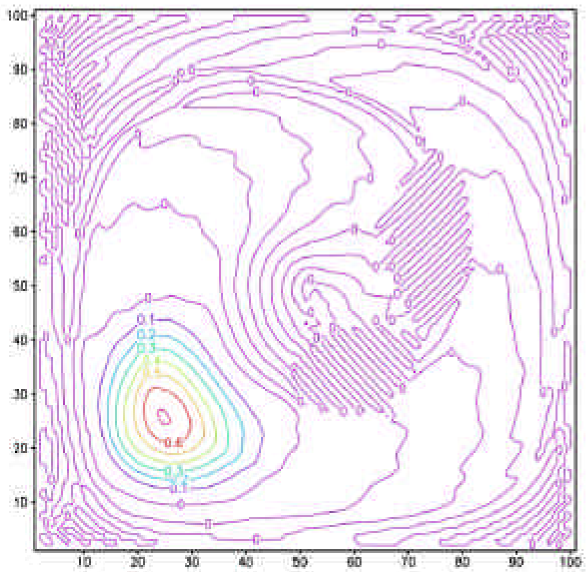
TVD-RK-3rd / CD-4th



450 time steps

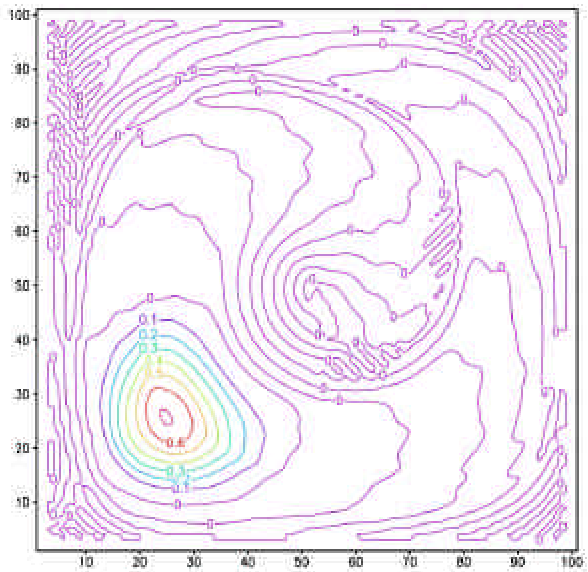
Upwind Schemes

RK-3rd / UP-5th



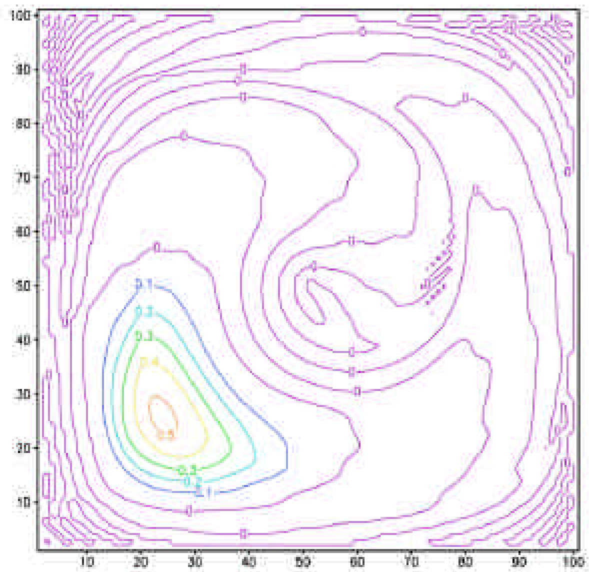
380 time steps

TVD-RK-3rd / UP-5th



380 time steps

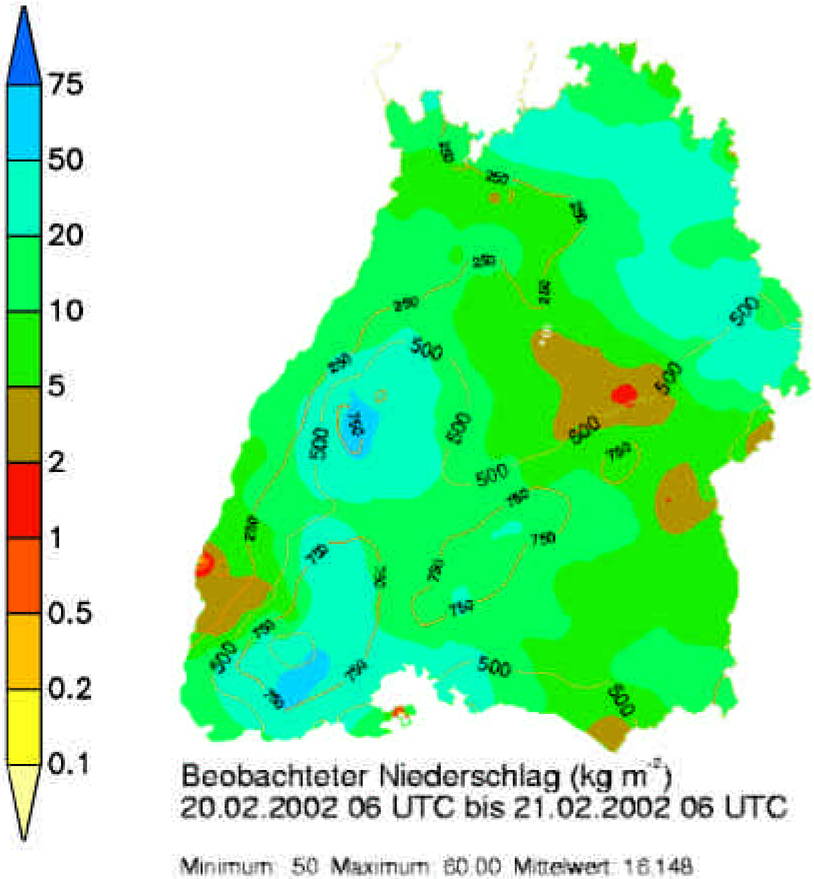
TVD-RK-3rd / UP-3rd



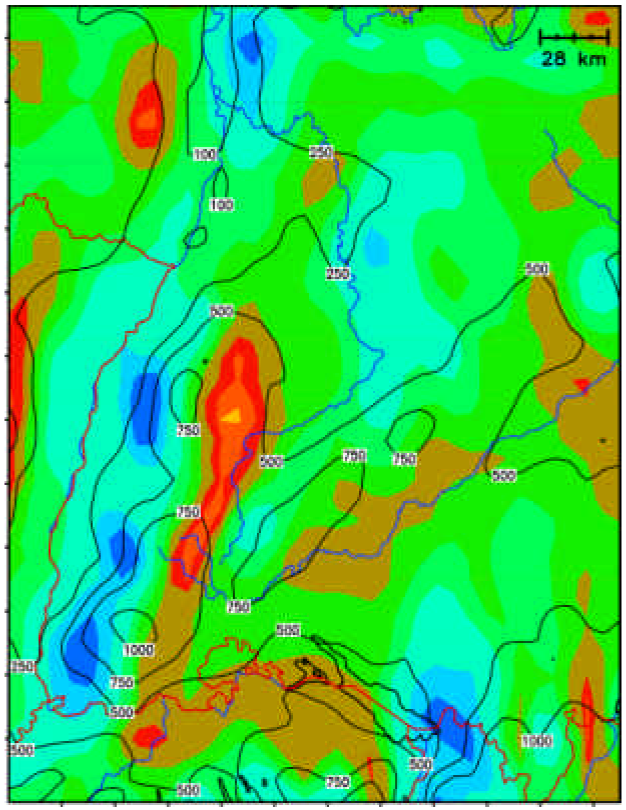
310 time steps

Precipitation 20.2.-21.2.2002 (06-30 UTC)

Observation



Operational LM

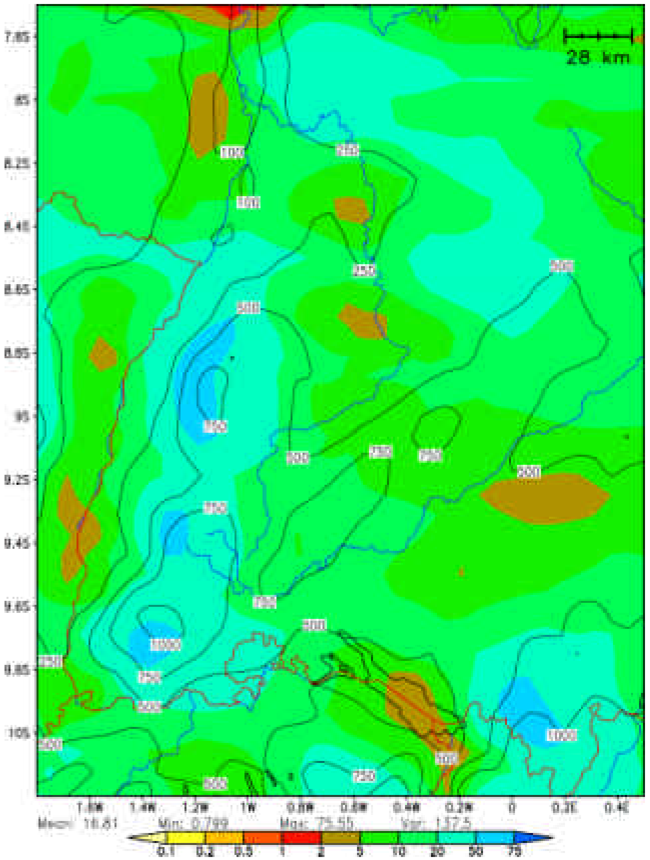


Δx, Δy = 7 km / Δt = 40 s

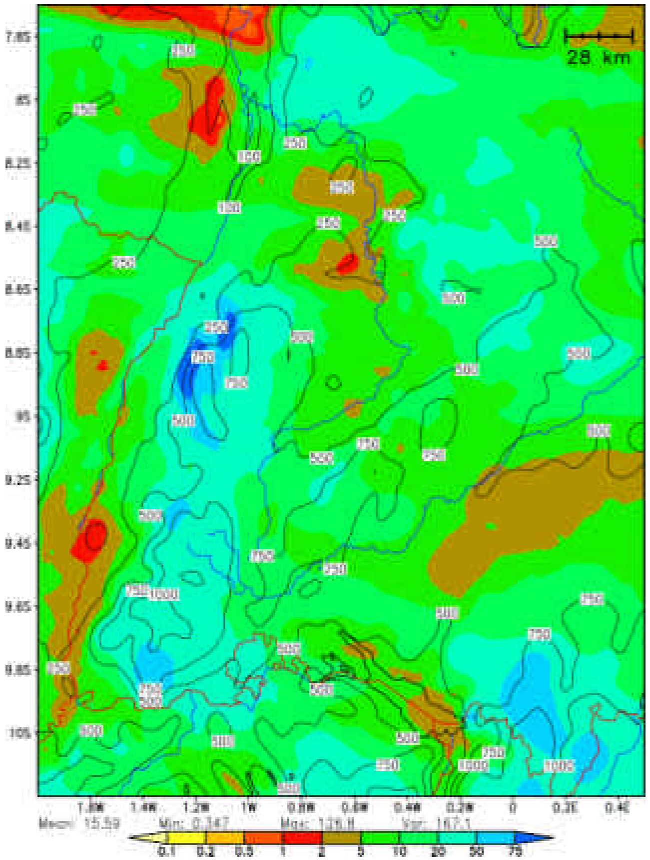
without progn. precipitation

TVD-RK-3rd / UP-5th

Prognostic Precipitation



Δx, Δy = 7 km / Δt = 72 s



Δx, Δy = 2.8 km / Δt = 36 s

2 x Euler-forward

in each

(Runge-Kutta-)

time step

using

positive definite

advection schemes

Mean values (7 km):
2 x E-F: 16.8 kg m^{-2}
1 x S-L: 14.9 kg m^{-2}

Numerical Experiments – the “Test Suites”

- Test Suites 1.3 - 1.5 (finished):
  LMK-domain, TVD-RK-3rd / UP-5th, 30 s, 2.8 km, winter and summer periods, no data assimilation.
  Problems: “pressure jump” at lateral boundaries, need for shallow conv. param.

- Test Suite 2.1 (planned):
  Data assimilation (without LHN), boundary fields with balanced pressure, preliminary version of shallow convection parameterization.

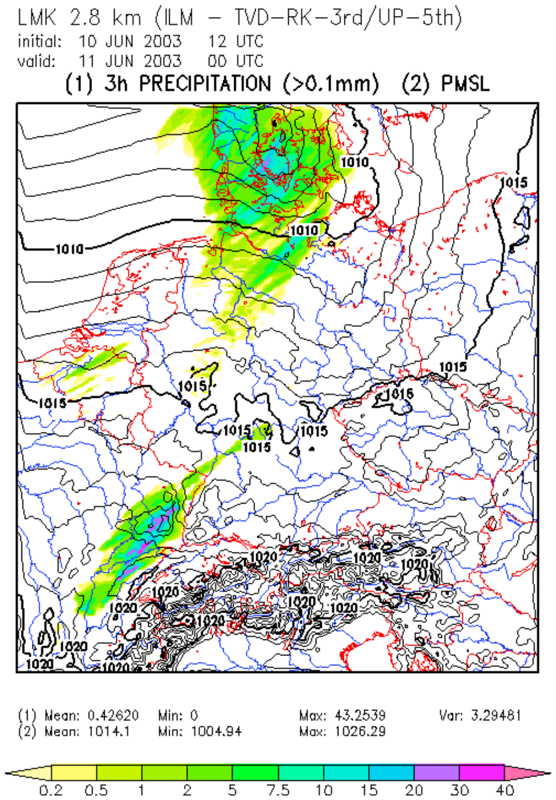
Visualization – the “Plot Suite”

Scripts (Perl and ksh) and templates (Förstner, Doms, Klink, Hanisch) to...
1. retrieve data... 2. visualize using GrADS... 3. generate HTMLs and JavaScript-animations... 4. put all in the intranet of DWD.

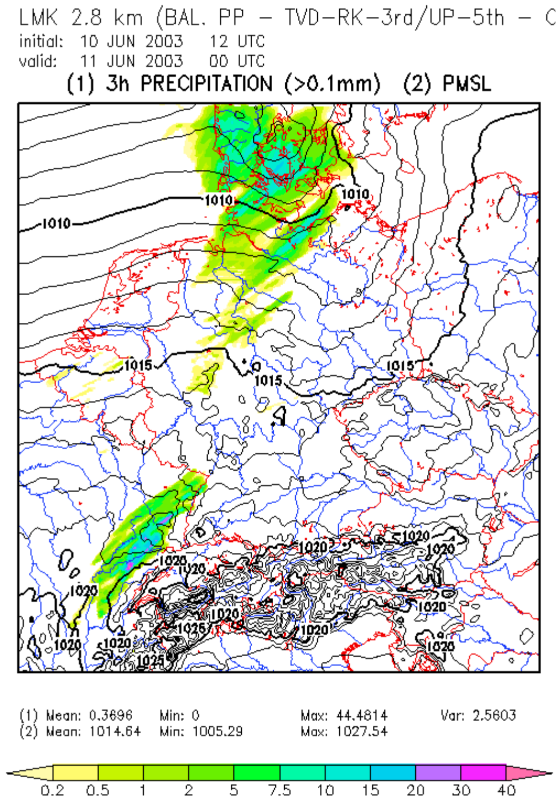
Compare two model results:
LMK-, LM- or LME-domain / numerical experiment- or routine-data.

Nesting – the “Pressure Problem”

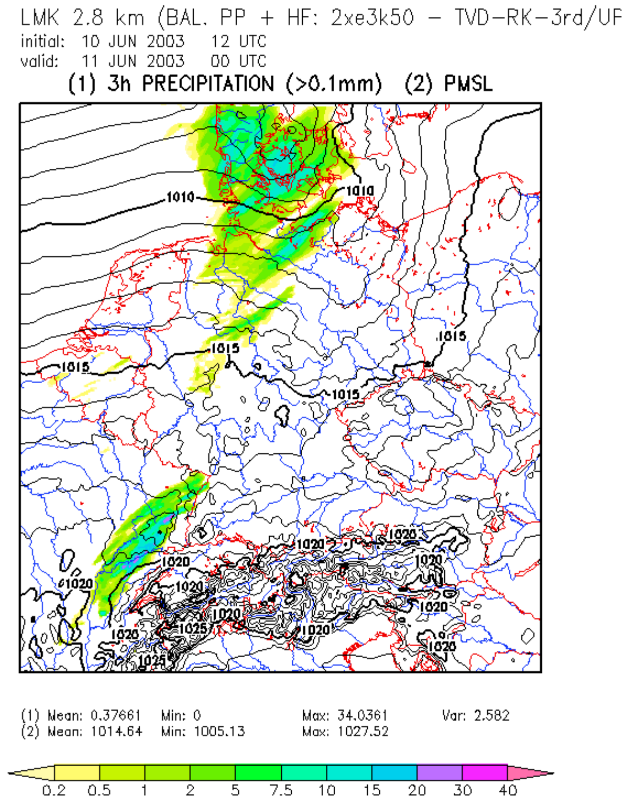
- Problem: „pressure jump“ of approx. 1 hPa at lateral boundaries.
- Solution (G. Doms) – via LM2LM:
  1. interpolate pressure perturbation p* on lowest model layer (as before)...
  2. calculate hydrostatically balanced p* fields on the model layers above...
  3. apply digital low-pass filter in horizontal directions (J. Förstner) (to reduce noise correlated with Δz).
- By-products:
  - explicit formulation of lateral boundary relaxation using a COS function (width of relaxation layer e.g. 50 km).
  - radiative lateral boundary conditions in fast-waves solver (for idealized test cases – yet to be tested).



"pressure problem"



hydr. balanced p*



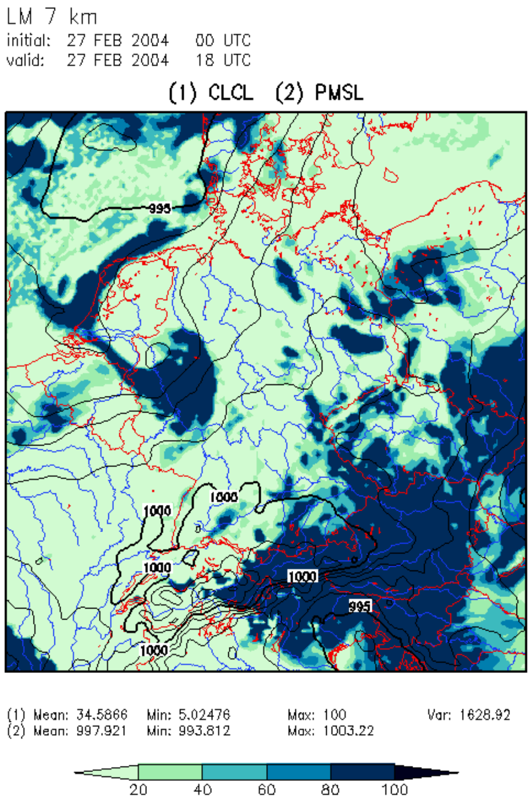
hydr. balanced p* +
digital low-pass filter

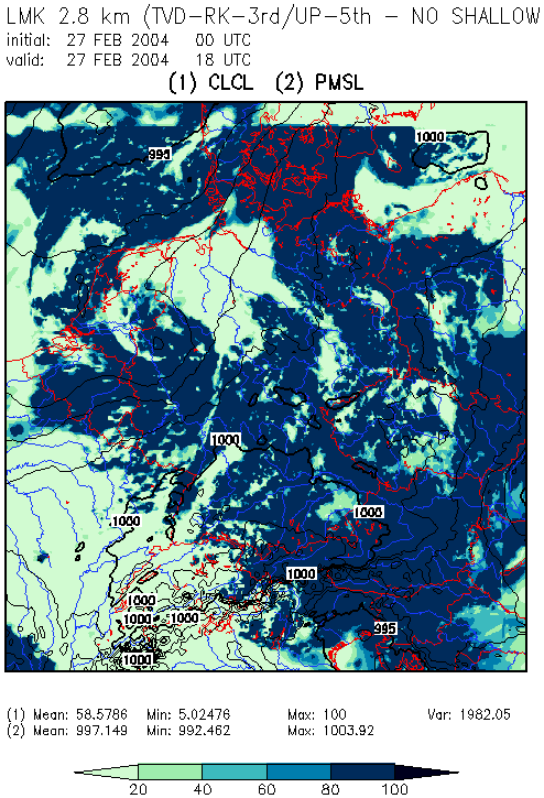
Deutscher Wetterdienst

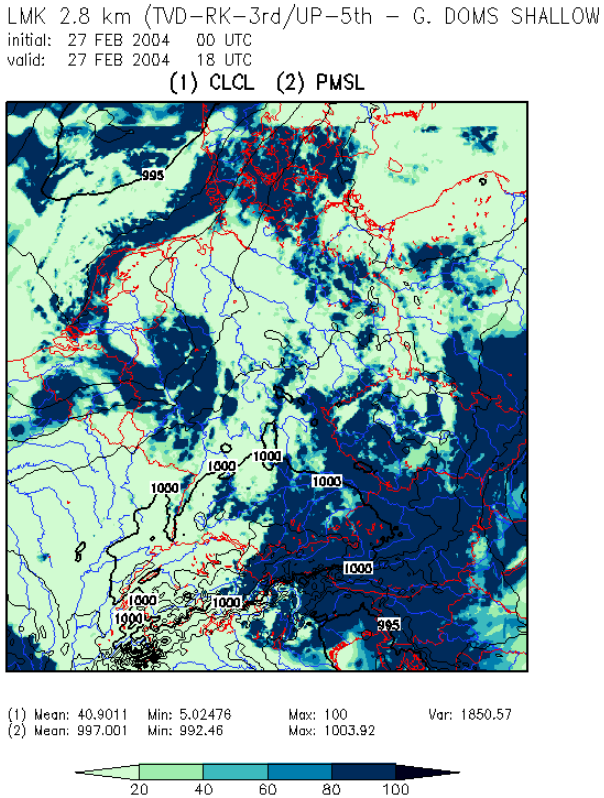
DWD

Shallow Convection Parameterization is Needed

- G. Doms: based on Tiedtke scheme, no deep convection, no convective precipitation, no downdrafts, consider only shallow clouds (see figure on the right).
- D. Mironov: extended formulation of entrainment / detrainment (test phase).







Metrics: $$\sqrt{\gamma} \equiv \frac{\partial p_0}{\partial \zeta} \quad \Longrightarrow \quad \sqrt{G} \equiv -\frac{\partial z}{\partial \zeta} = \frac{1}{g\rho_0}\sqrt{\gamma}$$

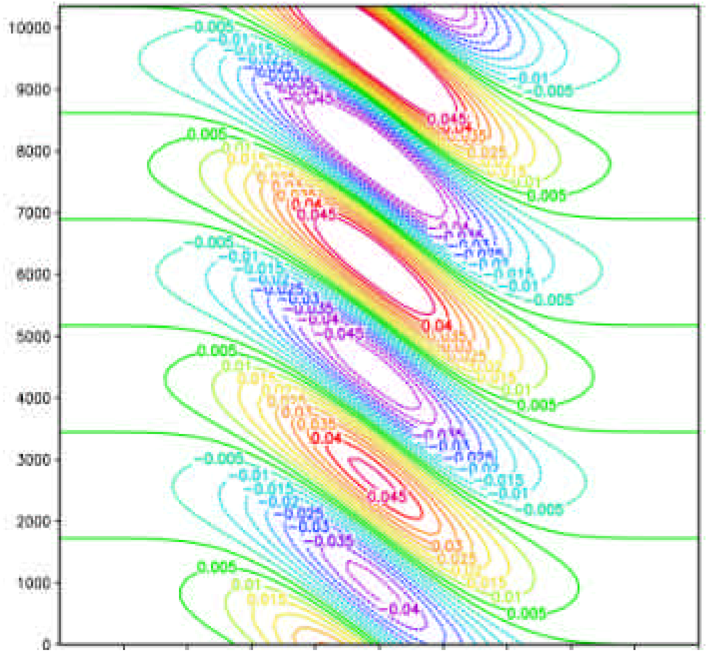
"Symmetric Thermodynamics": $T \quad \Longrightarrow \quad T^* = T - T_0(z)$

$$\vec{v} \cdot \vec{\nabla} T_0 = -\frac{\mathrm{d}T_0}{\mathrm{d}\ln p_0}\frac{g\rho_0}{p_0} w = \frac{\mathrm{d}T_0}{\mathrm{d}z} w$$

Advection of Reference Temperature T_0 in the Fast-Waves Solver...

$$w^{(\nu+1)} = \ldots + \Delta\tau g \frac{\overline{\rho_0}^{\zeta}}{\overline{\rho^n}^{\zeta}} \left[\overline{\left(\frac{\beta^+}{T^n}\right) T^{*(\nu+1)}}^{\zeta} + \overline{\left(\frac{\beta^-}{T^n}\right) T^{*(\nu)}}^{\zeta} \right]$$

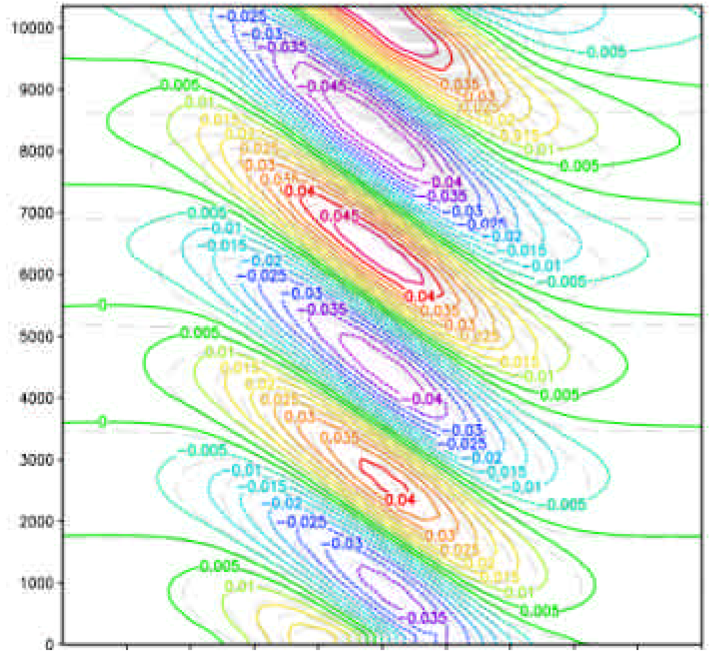
$$T^{*(\nu+1)} = \ldots - \Delta\tau \frac{\mathrm{d}T_0}{\mathrm{d}z} \left(\beta^+ \overline{w}^{\zeta(\nu+1)} + \beta^- \overline{w}^{\zeta(\nu)} \right)$$

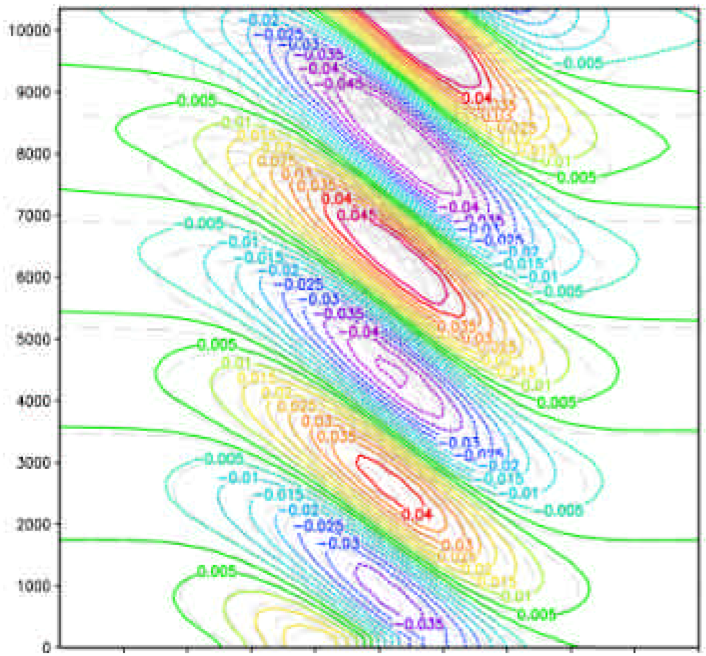


analytic solution

(2D, isotherm, h = 100m)

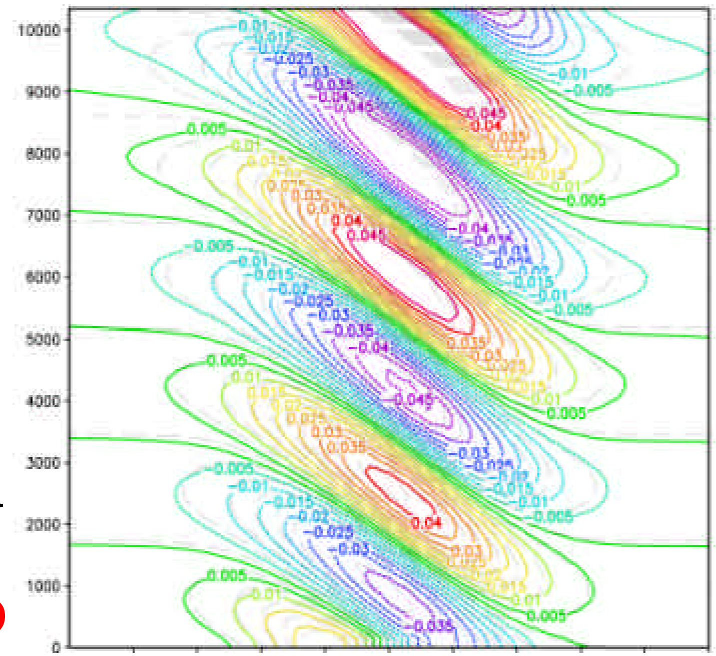
Leapfrog

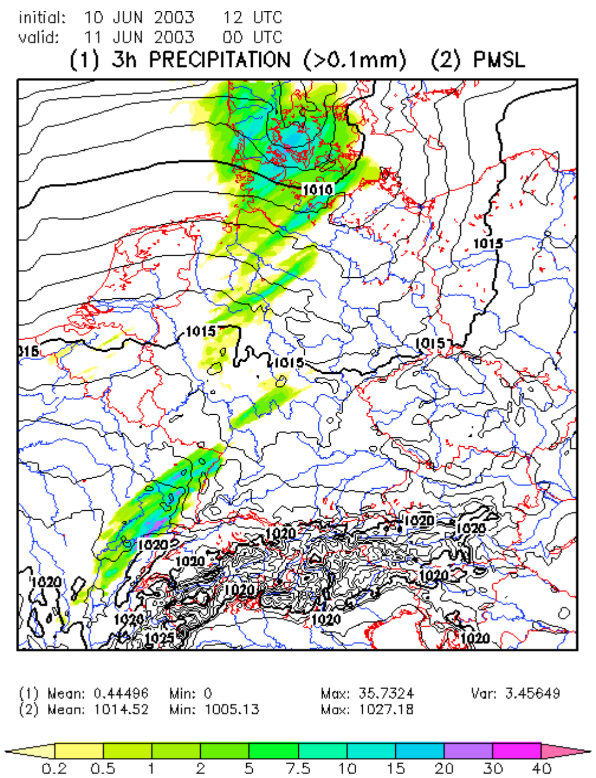




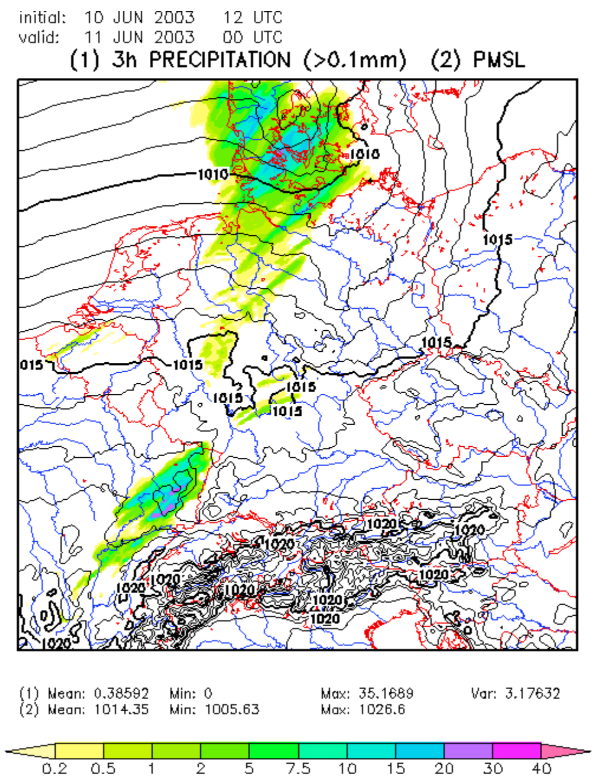
TVD-Runge-Kutta

TVD-Runge-Kutta and symmetr. TD

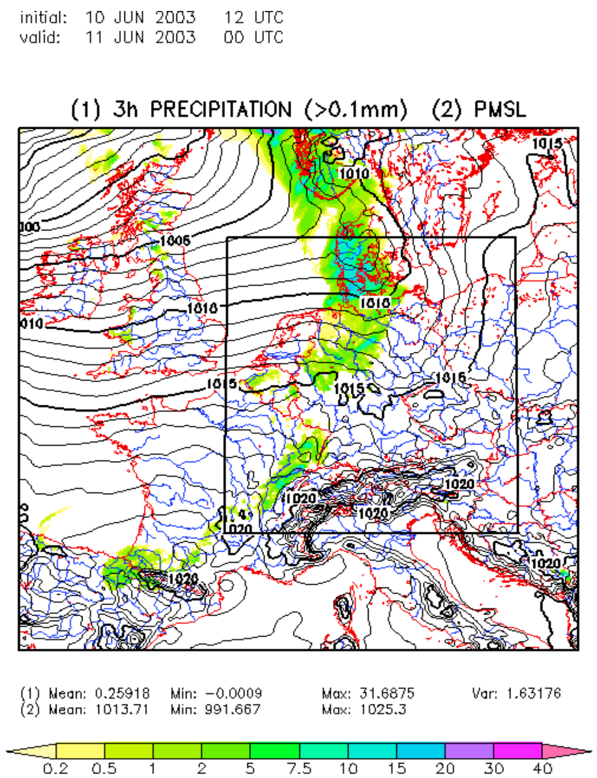




LMK-domain
Coarse: LM unsymmetr. TD
Nested: LMK symmetr. TD



LMK-domain
Coarse: LM symmetr. TD
Nested: LMK symmetr. TD



LM-domain
Coarse: GME
Nested: LM symmetr. TD

Summary and Outlook

- Fast Dynamical Core with TVD-RK-3rd / UP-5th Scheme.
- Stable Test Suites – problems at least partially solved.
- New Metrics and ...

- “Symmetric Thermodynamics” – to be evaluated and tested.
  - necessary for correct treatment of gravity waves in the fast-modes solver?!
  - mountain flow for non-isotherm atmosphere
  - falling cold bubble (Straka et al. 1993)
- 3D-Turbulence (WG3: H.-J. Herzog and G. Vogel) – to be evaluated...
  - optional 1D-treatment, prognostic treatment of TKE (optional diagnostic), dry turbulence formulation... straightforward implementation!
  - correct metrics for / relevance of the horizontal diffusion terms?!
  - parameterizations (of the operational version) of moist turbulence (incl. subscale clouds), subscale thermal circulation, ...
- Implementation of Dynamical LBC (Gassmann, NL 4).
- Implementation of Alternative Schemes for Positive Definite Advection of Moisture Quantities and TKE.
- Documentation / COSMO Technical Report.

Thanks Günther!