The Water Surface Roughness Lengths for Scalars: an Observational Study

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Abstract

Data from observations in the atmospheric surface layer taken over a small fresh water lake (the Kossenblatter See field station of the German Weather Service located near the Lindenberg Meteorological Observatory, Land Brandenburg, Germany) and over the Baltic Sea (the Östergarnsholm field station of the University of Uppsala located ca. 4 km east of the Island of Gotland) are used to analyse the roughness length of the water surface with respect to potential temperature. Using data from direct flux-profile measurements and the surface layer similarity relations, we have estimated the ratio of the roughness length with respect to the wind velocity to the roughness length with respect to the temperature, or, alternatively, the increment of the temperature across the roughness layer, and tested several theoretical formulations agains data. Most formulation proposed to date suggest that the above increment increases with the increasing surface Reynolds number, $Re_0 = u_* z_{0u}/\nu$ (u_* is the surface friction velocity, z_{0u} is the roughness length with respect to wind velocity, and ν is the kinematic molecular viscosity of the air), where the power-law formulation, Re_0^n, with $n = 1/2$ was advocated by a number of authors. Empirical data demonstrate a pronounced tendency to follow the above power law and support the estimate of $n = 1/2$, although the scatter of data is large. A logarithmic dependence on Re_0 proposed by some researchers tends to underestimate data, particularly at large values of Re_0. The implications of the results from our analysis for practical applications are discussed, emphasising the utility of parameterisations for modelling purposes, most notably for numerical weather prediction, where rather stringent requirements of computational economy must be met.

Outline

- the surface-layer similarity
- scaling ideas
- comparison with data from measurements
- conclusions, future work

The surface-layer similarity

Flux profile relationships

$$u(z) = \frac{u_*}{\kappa}\left[\ln\frac{z}{z_{0u}} + \psi_u(z/L)\right],$$

$$\theta(z) - \theta_s = -\frac{\mathrm{Pr}_n}{\kappa}\frac{F_{\theta s}}{u_*}\left[\ln\frac{z}{z_{0T}} + \psi_\theta(z/L)\right],$$

where the key parameters are the roughness lengths, z_{0u} and z_{0T}.

Alternative forms

$$u_*^2 = C_m\left[u(z)\right]^2,$$

$$F_{\theta s} = -C_h\left[\theta(z) - \theta_s\right],$$

where the key parameters are the transfer coefficients, C_m and C_h, and

$$u_*^2 = \frac{1}{r_m}u(z),$$

$$F_{\theta s} = -\frac{1}{r_h}\left[\theta(z) - \theta_s\right],$$

where the key parameters are the resistances, r_m and r_h.

The Charnock relation

The Charnock (1955) formula

$$z_{0u} = C_{ch}\frac{u_*^2}{g}, \qquad C_{ch} = 0.01 - 0.1$$

Correction to the Charnock formula to account for the aerodynamically smooth regime of the flow

$$z_{0u} = C_s\frac{\nu}{u_*} + C_{ch}\frac{u_*^2}{g}, \qquad C_s = 0.1$$

Roughness length for scalars

The difference between the momentum transfer and the transfer of scalars is conveniently accounted for through

$$\frac{\mathrm{Pr}_n}{\kappa} \ln \frac{z_{0u}}{z_{0T}} = -\frac{[\theta(z_{0u}) - \theta_s]\, u_*}{F_{\theta s}} \equiv \frac{\delta\theta}{\theta_*},$$

where $\theta(z_{0u}) - \theta_s$ is the "roughness-layer temperature (scalar concentration) increment", $\theta_* \equiv -F_{\theta s}/u_*$ is the surface-layer temperature (scalar concentration) scale, and $\theta(z_{0u})$ is the aerodynamic surface temperature (scalar concentration).

Parameterisation

$$\frac{\mathrm{Pr}_n}{\kappa} \ln \frac{z_{0u}}{z_{0T}} \propto Re_0^m, \qquad Re_0 = \frac{z_{0u} u_*}{\nu},$$

where the empirical estimates of the exponent m tend towards 0.5. Theoretical values of m vary with the writer, the value of $m = 1/2$ seems to be better justified than the other values. The $\ln Re_0$ function instead of the power law has also been proposed.

Zilitinkevich, Grachev & Fairall arguments

(JAS, 2001, **58**, 320–325)

Introducing scales

$$\theta_{*\nu} = -Q_s/u_{*\nu} \quad \text{and} \quad u_{*\nu} = (\nu u_*/z_{0u})^{1/2},$$

for habitual $\theta_* = -Q_s/u_*$ and u_*, immediately yields the $Re_0^{1/2}$ dependence.

Formulations for practical use

$$\frac{Pr_n}{\kappa} \ln \frac{z_{0u}}{z_{0T}} \equiv \frac{\delta\theta}{\theta_*} = \begin{cases} -2 & \text{at} \quad Re_0 \leq 0.1 \\ 4.0 Re_0^{1/2} - 3.2 & \text{at} \quad Re_0 \geq 0.1 \end{cases}$$

$$\frac{Sc_n}{\kappa} \ln \frac{z_{0u}}{z_{0q}} \equiv \frac{\delta q}{q_*} = \begin{cases} -3 & \text{at} \quad Re_0 \leq 0.1 \\ 4.0 Re_0^{1/2} - 4.2 & \text{at} \quad Re_0 \geq 0.1 \end{cases}$$

Alternative arguments

Resistance r_{h0} of the roughness layer $0 \le z \le z_{0u}$

$$r_{h0} u_* = \delta\theta/\theta_*, \qquad r_{h0} = \int_0^{z_{0u}} \frac{dz}{K_h}$$

Interpolation formula for the temperature conductivity

$$K_h = \chi_h + \left(C_0' u_* z_{0u} - \chi_h\right) \left(z/z_{0u}\right)^m$$

Then, $\zeta = \left(C_0' Pr_m Re_0\right)^{1/m} (z/z_{0u})$,

$$r_{h0} u_* = \frac{Pr_m Re_0}{\left(C_0' Pr_m Re_0 - 1\right)^{1/m}} \int_0^{\left(C_0' Pr_m Re_0 - 1\right)^{1/m}} \frac{d\zeta}{1+\zeta^m}$$

Asymptotic behaviour at $Re_0 \gg 1$

$$r_{h0} u_* \propto Pr_m^{1-\frac{1}{m}} Re_0^{1-\frac{1}{m}} \propto Re_0^n, \qquad n = 1 - \frac{1}{m}.$$

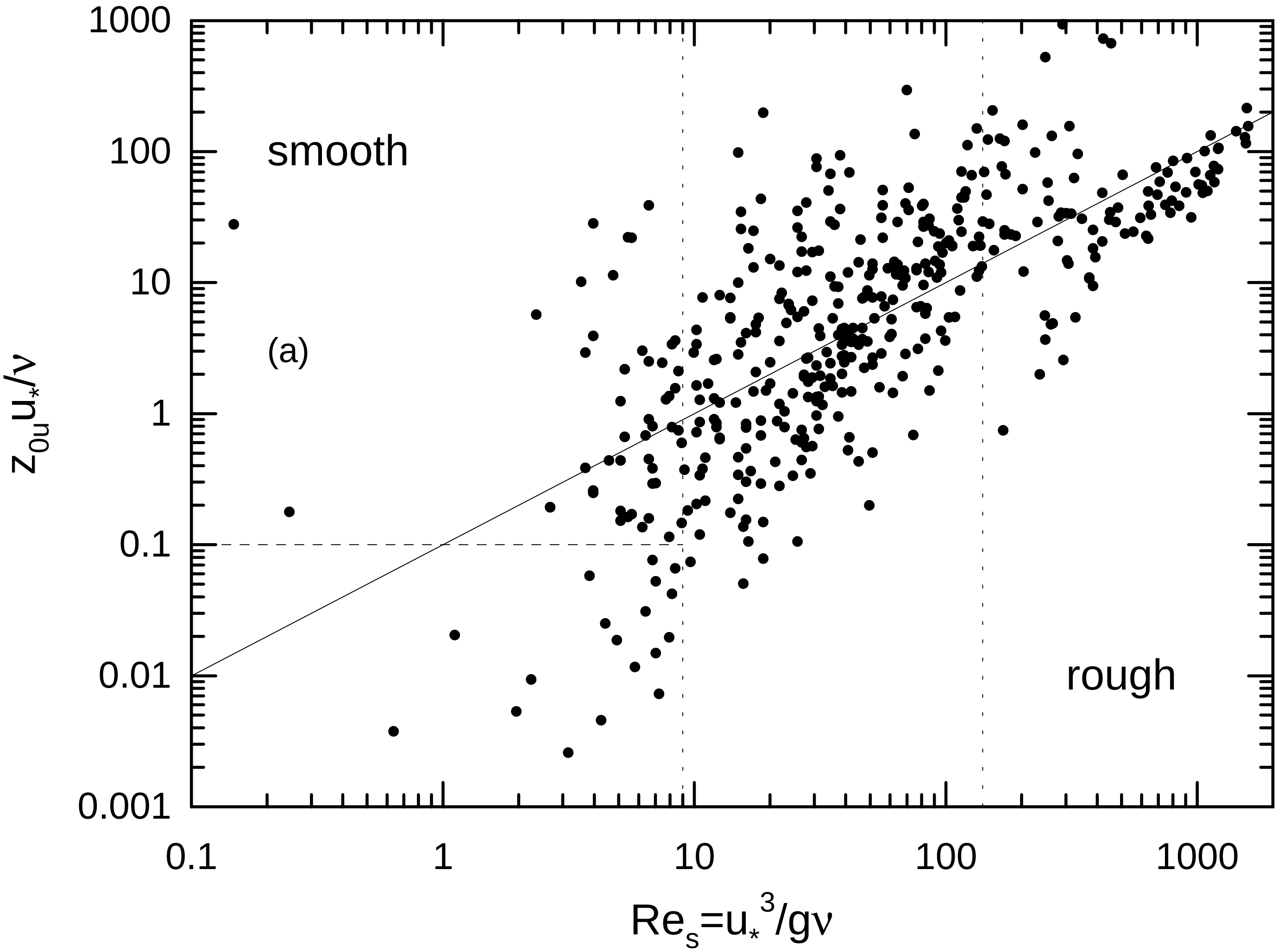
The linear profile, $m = 1$, gives the log-law, $\delta\theta/\theta_* \propto \ln Re_0$.

Conclusions

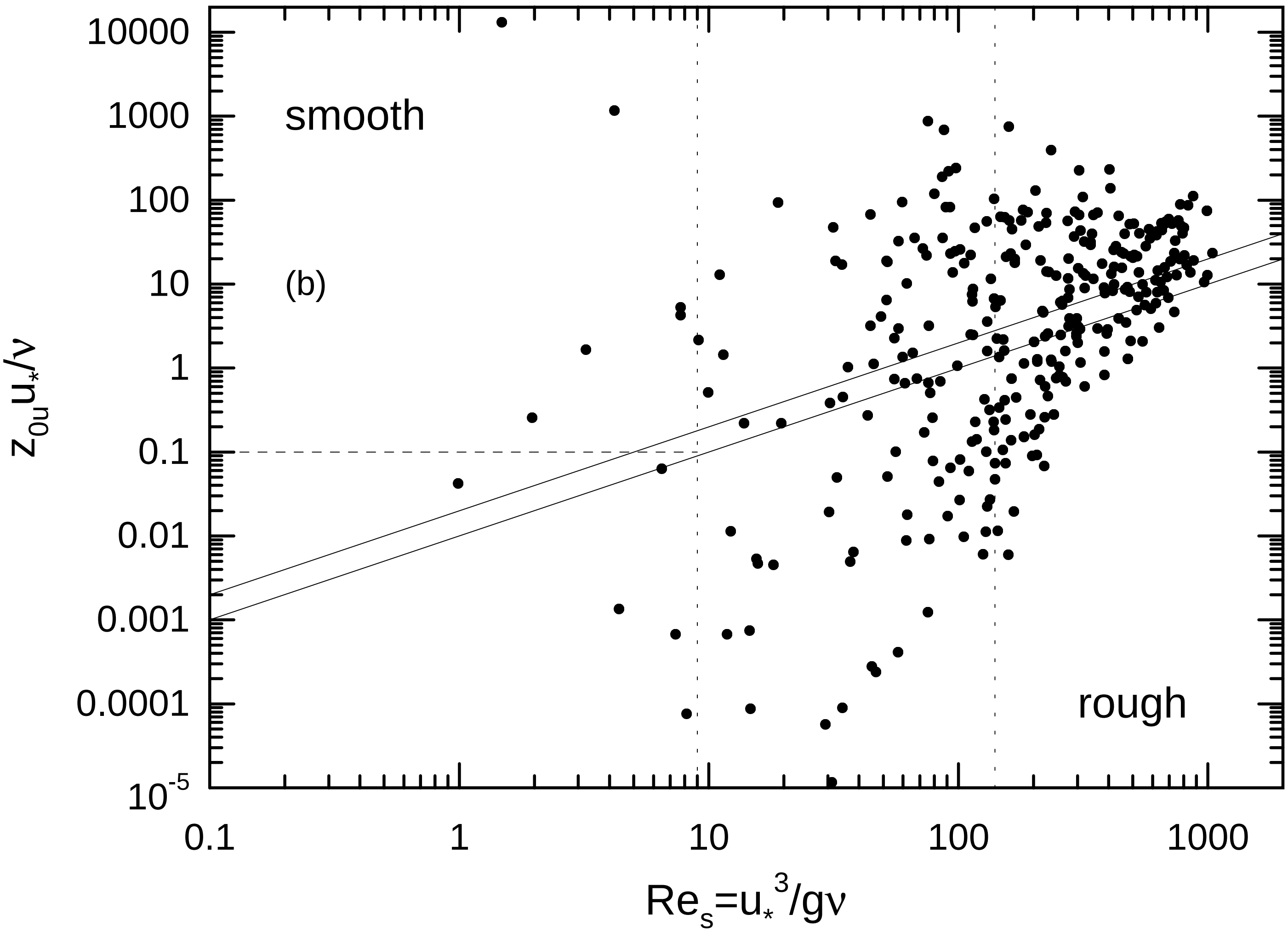
- The 1/2 power law dependence of $\delta\theta/\theta_*$ on Re_0 is consistent with theoretical arguments and empirical data.
- A recipe based on the $Re_0^{1/2}$ power law is usable.

Future work

- Testing against other data sets.
- Implementation and testing in numerical models.



The roughness length with respect to the wind velocity made dimensionless with the depth scale of the viscous sublayer, $z_{0u}u_*/\nu$, versus the surface Reynolds number, $Re_s = u_*^3/g\nu$. Symbols show data from measurements taken over Kossenblatter See, Land Brandenburg, Germany, 6, 13 and 15 August and 12 October 1999. Solid line shows the Charnock formula with $C_{ch} = 0.1$. Dashed line shows the smooth regime approximation with $C_s = 0.1$. Vertical dotted lines are $Re_s = 9$ and $Re_s = 140$.



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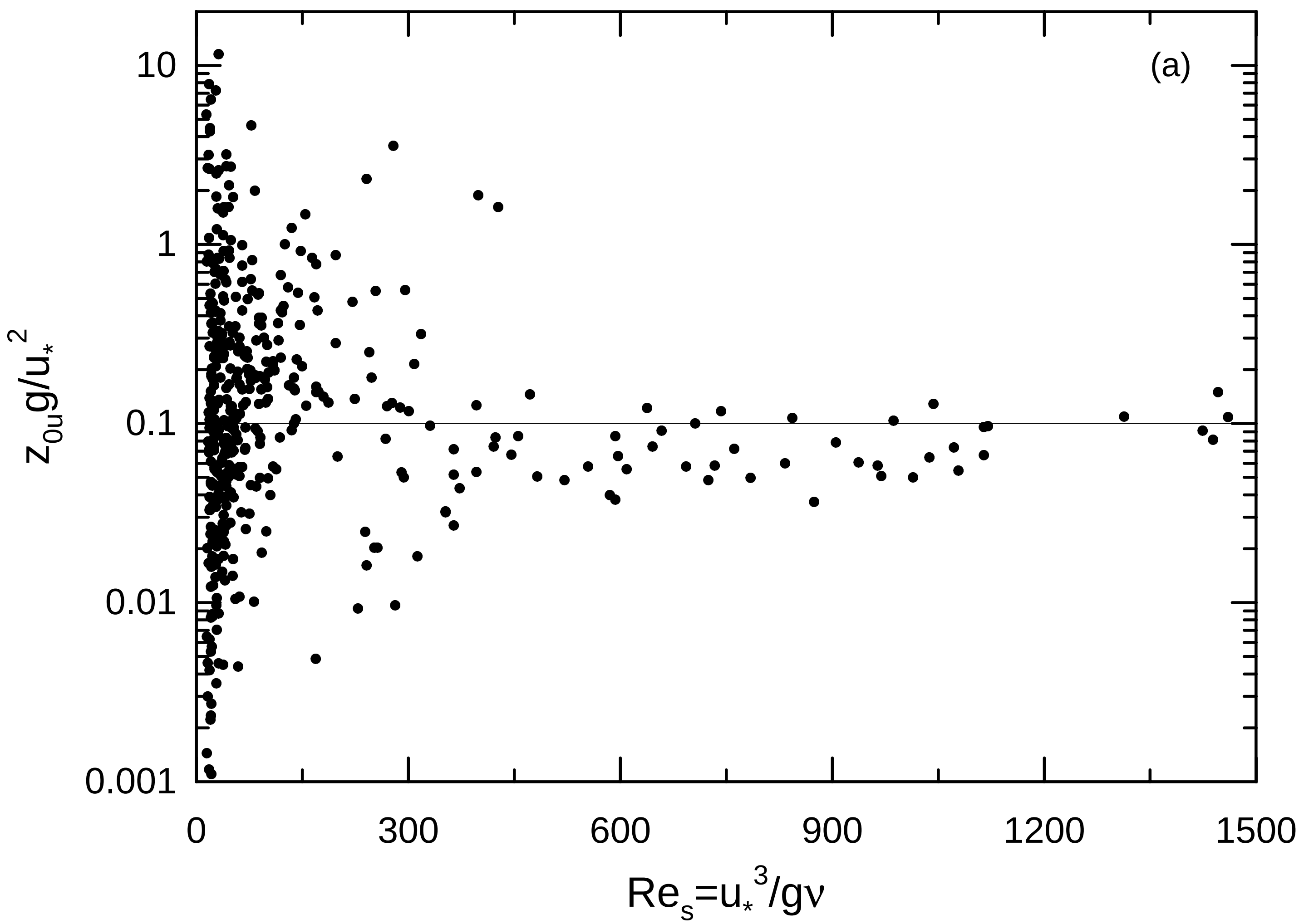
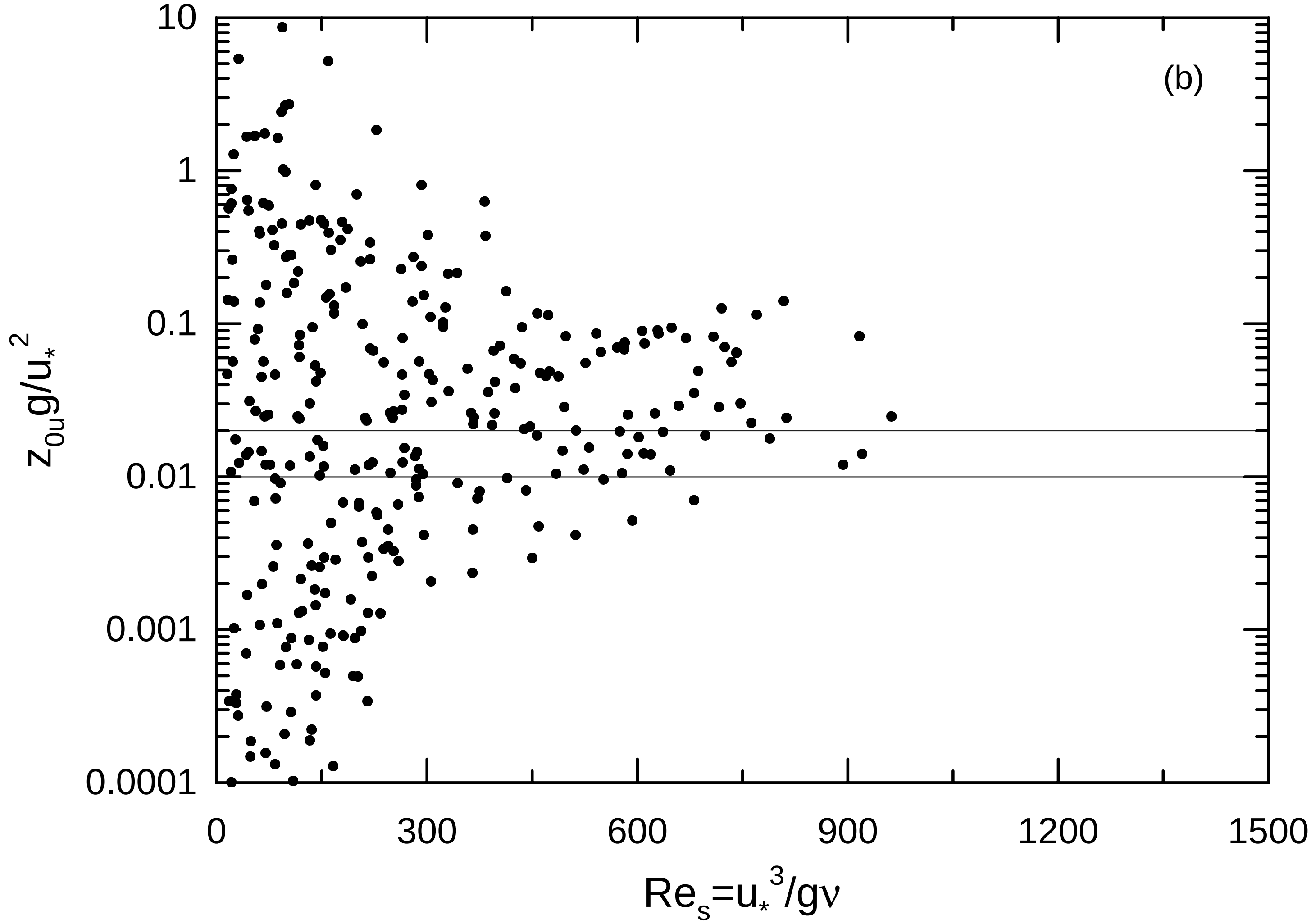


The roughness length with respect to the wind velocity made dimensionless with the Charnock length scale, $z_{0u}g/u_*^2$, versus the surface Reynolds number, $Re_s = u_*^3/g\nu$. Symbols show data from measurements taken over Kossenblatter See, Land Brandenburg, Germany, 6, 13 and 15 August and 12 October 1999. Horizontal line shows the Charnock formula with $C_{ch} = 0.1$.



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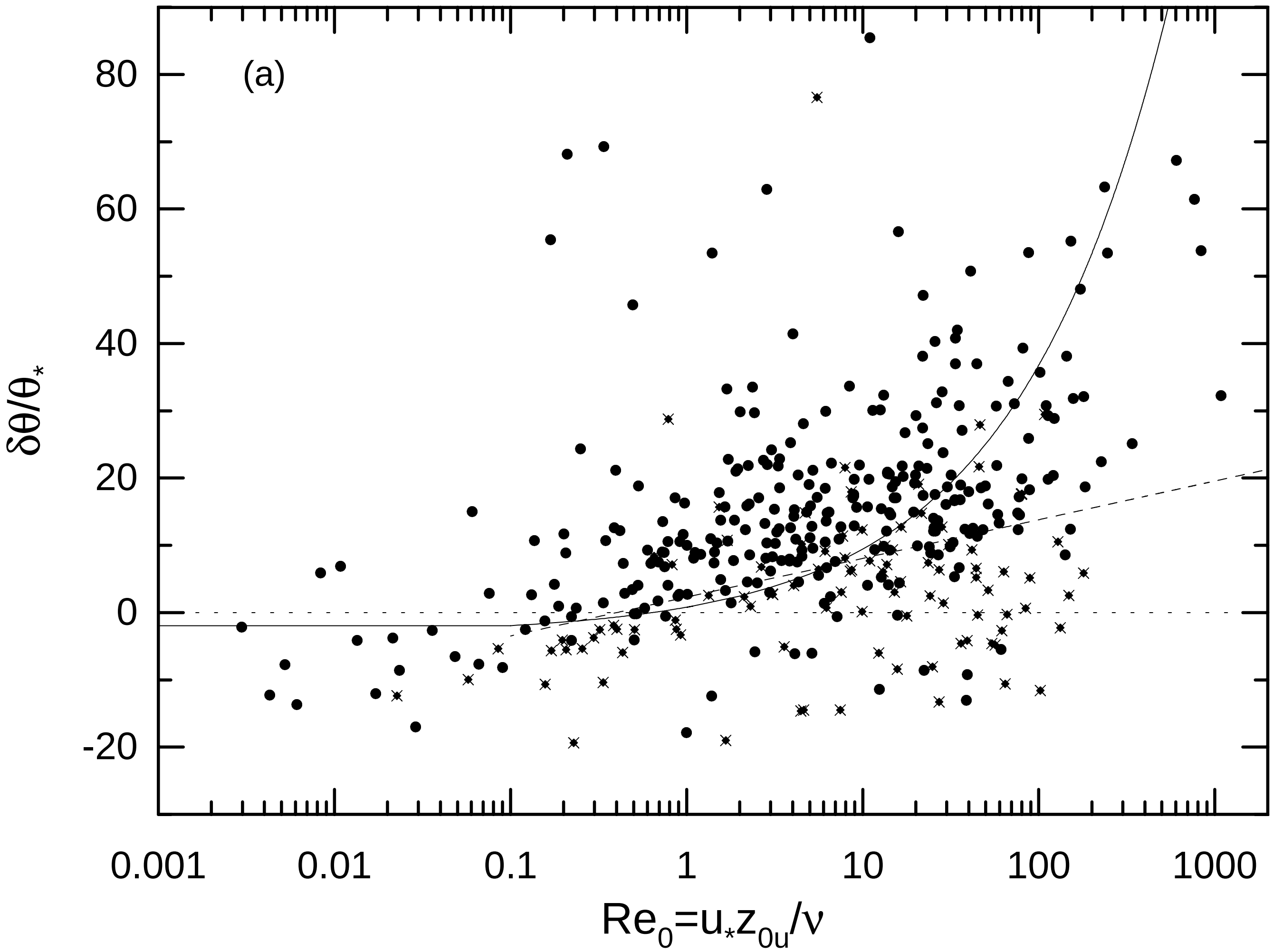
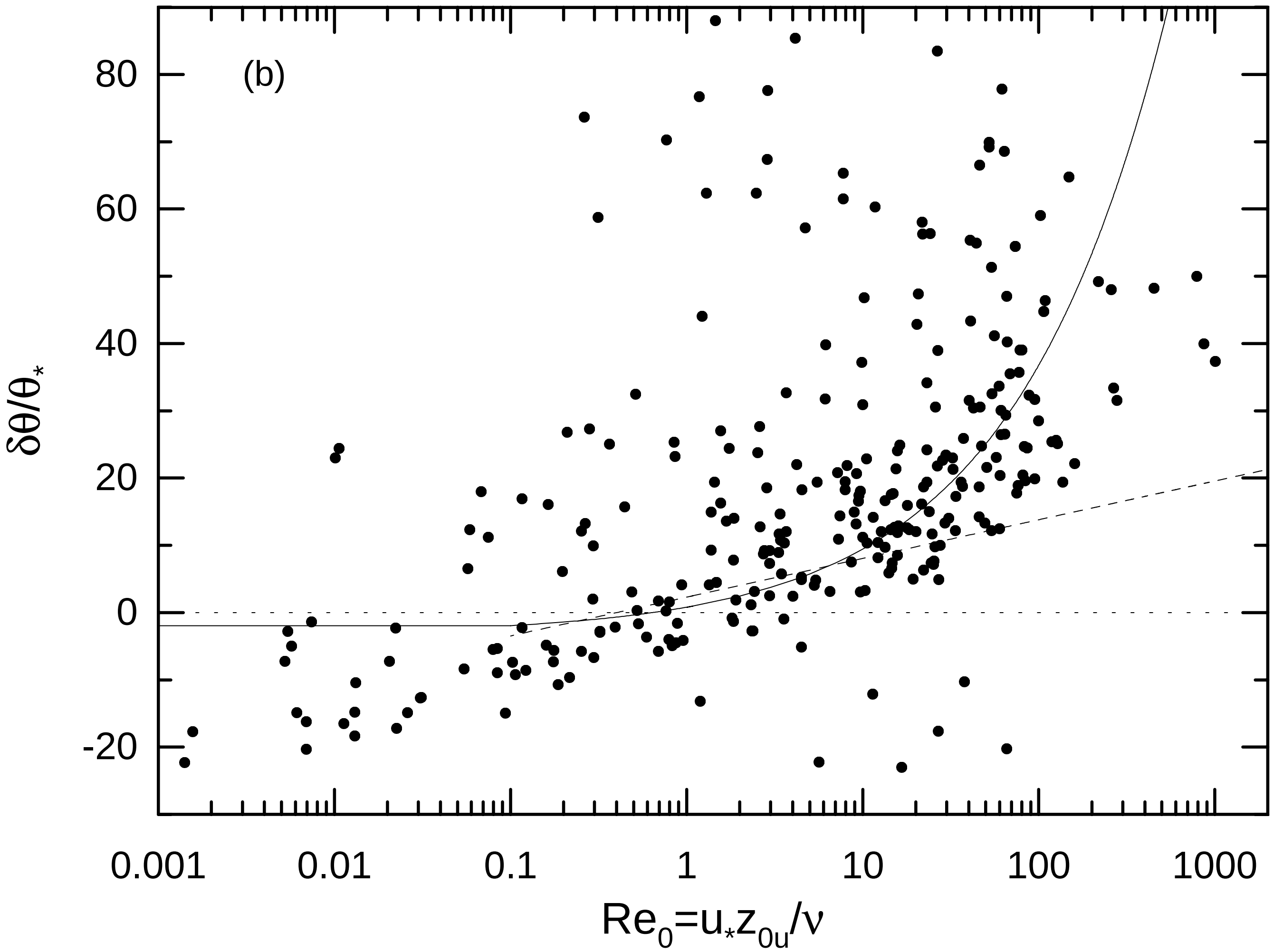


Dimensionless temperature increment across the roughness layer, $\delta\theta/\theta_* = (\mathrm{Pr}_n/\kappa)\ln(z_{0u}/z_{0T})$, versus the roughness Reynolds number, $Re_0 = z_{0u}u_*/\nu$. Solid curve shows the Zilitinkevich et al. (2001) formulation. Dashed curve shows the logarithmic law (Beljaars 1994), $\delta\theta/\theta_* = (Pr_n/\kappa)\,(0.92 + \ln Re_0)$. Symbols show data from measurements taken over Kossenblatter See, Land Brandenburg, Germany, 6, 13 and 15 August and 12 October 1999. Fancy dimonds depict values obtained with the vertual sensible heat flux, i.e. with the measured sensible heat flux that was not corrected for the effect of humidity.



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